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ATTI ACCADEMIA NAZIONALE DEI LINCEI  
CLASSE SCIENZE FISICHE MATEMATICHE NATURALI  
**RENDICONTI**

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PAOLO BALDI, MARCO UNGUENDOLI, PAOLO GASPERINI

**Planning and optimization of geodetic networks for  
determining fault movements**

*Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche,  
Matematiche e Naturali. Rendiconti, Serie 8, Vol. 75 (1983), n.3-4, p.  
127-143.*

Accademia Nazionale dei Lincei

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**Geodesia.** — *Planning and optimization of geodetic networks for determining fault movements.* Nota (\*) di PAOLO BALDI (\*\*), MARCO UNGUENDOLI (\*\*\*) e PAOLO GASPERINI (\*\*\*\*), presentata dal Corrisp. E. BOSCHI.

RIASSUNTO. — Le possibilità di applicazione delle moderne tecniche di ottimizzazione delle reti geodetiche, istituite per lo studio delle deformazioni del suolo in aree sismiche, vengono analizzate prendendo in considerazione vari parametri, quali la precisione, l'affidabilità, i costi e le indicazioni disponibili a priori sulle caratteristiche del fenomeno geofisico che si intende studiare.

#### INTRODUCTION

One of the aims that geophysics sets itself is to study recent crustal deformation activity at tectonic plate boundaries or in seismic areas, accumulating quantitative data relating to as many of the parameters that characterize the spatial, temporal and energy spectra of these movements, and to deduce from this information the nature of deep crustal processes. Study of movements in seismically dangerous regions or near active faults and volcanoes will make it possible to relate the surface effects to the stress pattern inside the crust.

This research may be carried out using geodetic methods, i.e. a set of techniques designed to measure lengths, directions, heights, gravity acceleration intensity and direction, as well as the variation of these quantities over time.

The degree of accuracy that may reasonably be attributed to geodetically measured crustal deformations depends on the amount of time intervening between measurements, on the rate of motion and on the precision of the measuring technique used.

There are a lot of problems that may arise during geodetic data analysis; in seismic areas, for example, it is necessary to distinguish between deformations that are directly and temporally related to the seismic event and, on the other hand, slow displacements which can be recorded over a lengthy period both before and after the earthquake (Rikitake, 1976; Meissner, 1978). Whereas

(\*) Pervenuta all'Accademia il 29 ottobre 1983.

(\*\*) Dipartimento di Fisica, Settore Geofisica. Università di Bologna.

(\*\*\*) Istituto di Topografia, Geodesia e Geofisica Mineraria. Università di Bologna.

(\*\*\*\*) Istituto Nazionale di Geofisica. Roma.

pre-seismic and co-seismic strain accumulation are generally related to accumulating stress field and sudden faulting, the time-dependent deformation which follows the sudden slip on large earthquake faults may be related to the viscous relaxation of the asthenosphere. Furthermore, it is necessary to take into account all the processes which distort the strains and tilts produced by stresses in the earth, such as inhomogeneities in elastic constants, cavities and topography, which give rise to deviations from the values theoretically obtainable on the basis of simple earth models.

#### CO-SEISMIC DEFORMATIONS

Important features of the earthquake mechanism include the size and shape of the rupture surface, its orientation, the faulting motion on this surface, and

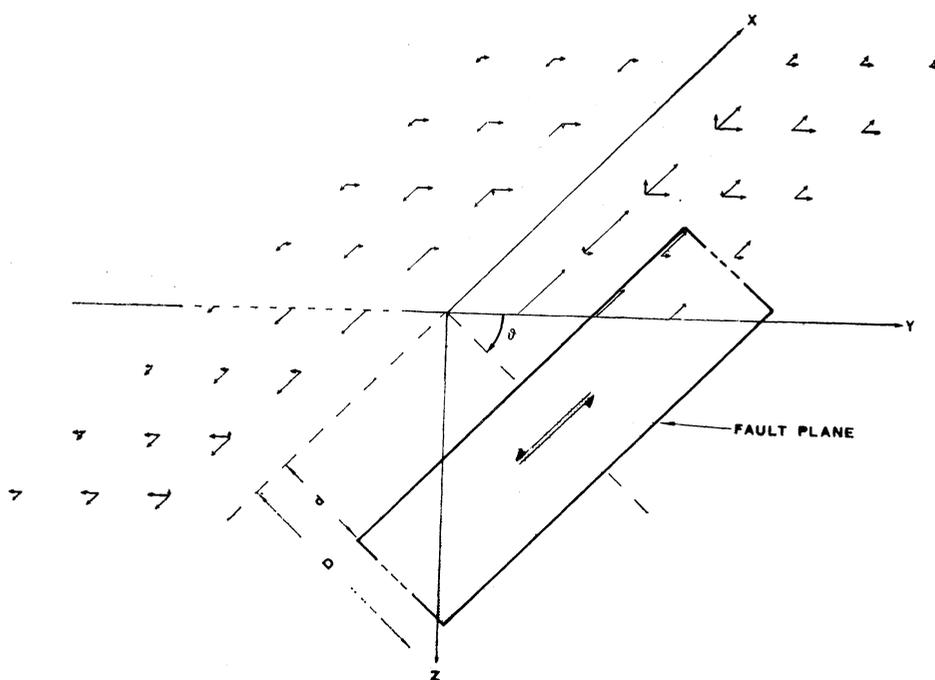


Fig. 1. - Vertical and horizontal components of displacement, for a finite rectangular strike-slip fault.

the time history of the process. Seismic data, in general, provide information about seismic source mechanisms; geodetic surveys could provide important advances in our knowledge of earthquake source models.

Theoretical expressions for surface and subsurface deformations accompanying faulting, obtained on the basis of the elastic theory of dislocation (Chinnery, 1961; Maruyama, 1964), have been given by Press (1965), Mansinha and

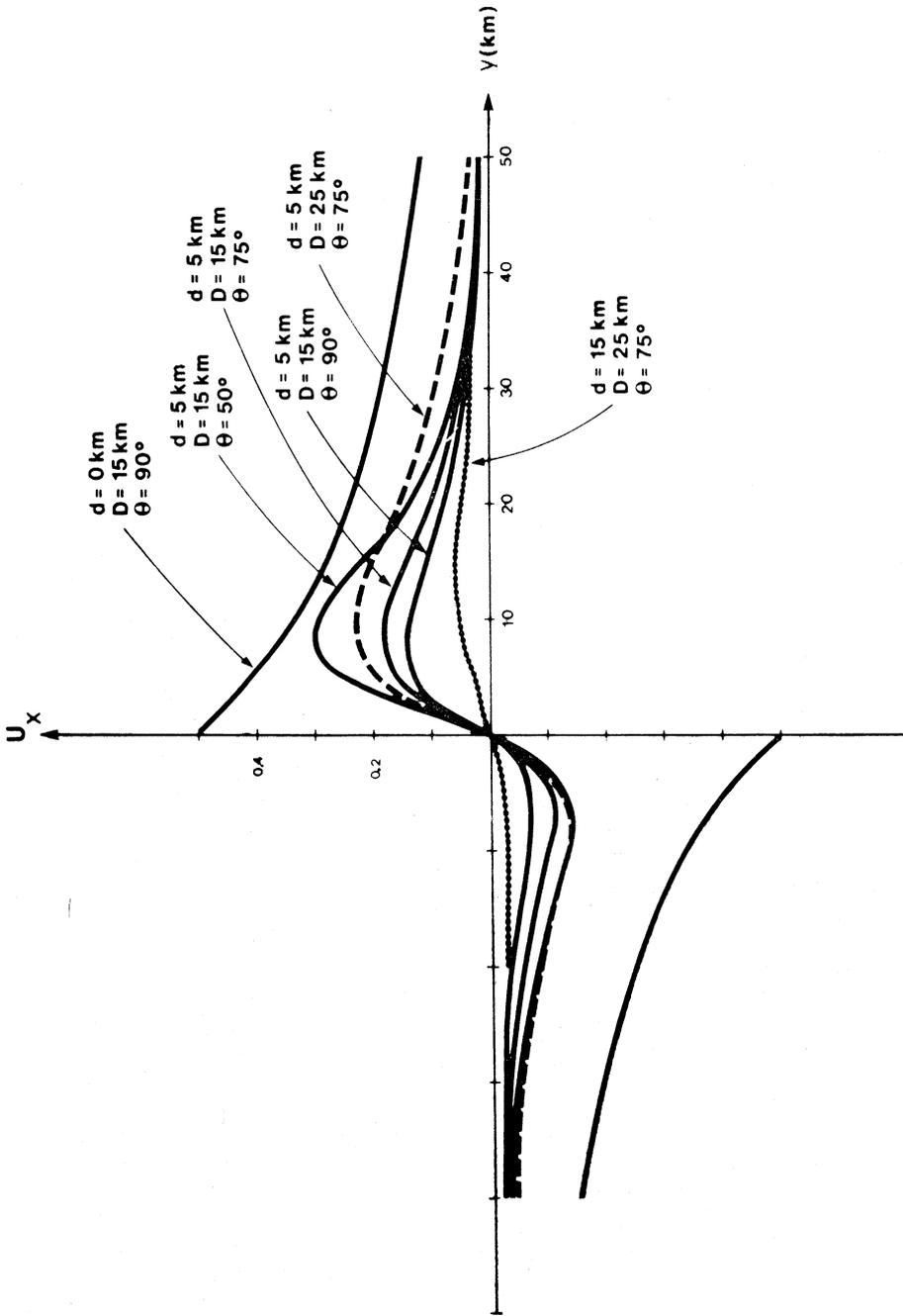


Fig. 2. -  $U_x$  component of displacement for different parameters of a strike-slip fault. The displacement is given as a fraction of the slip of the fault.

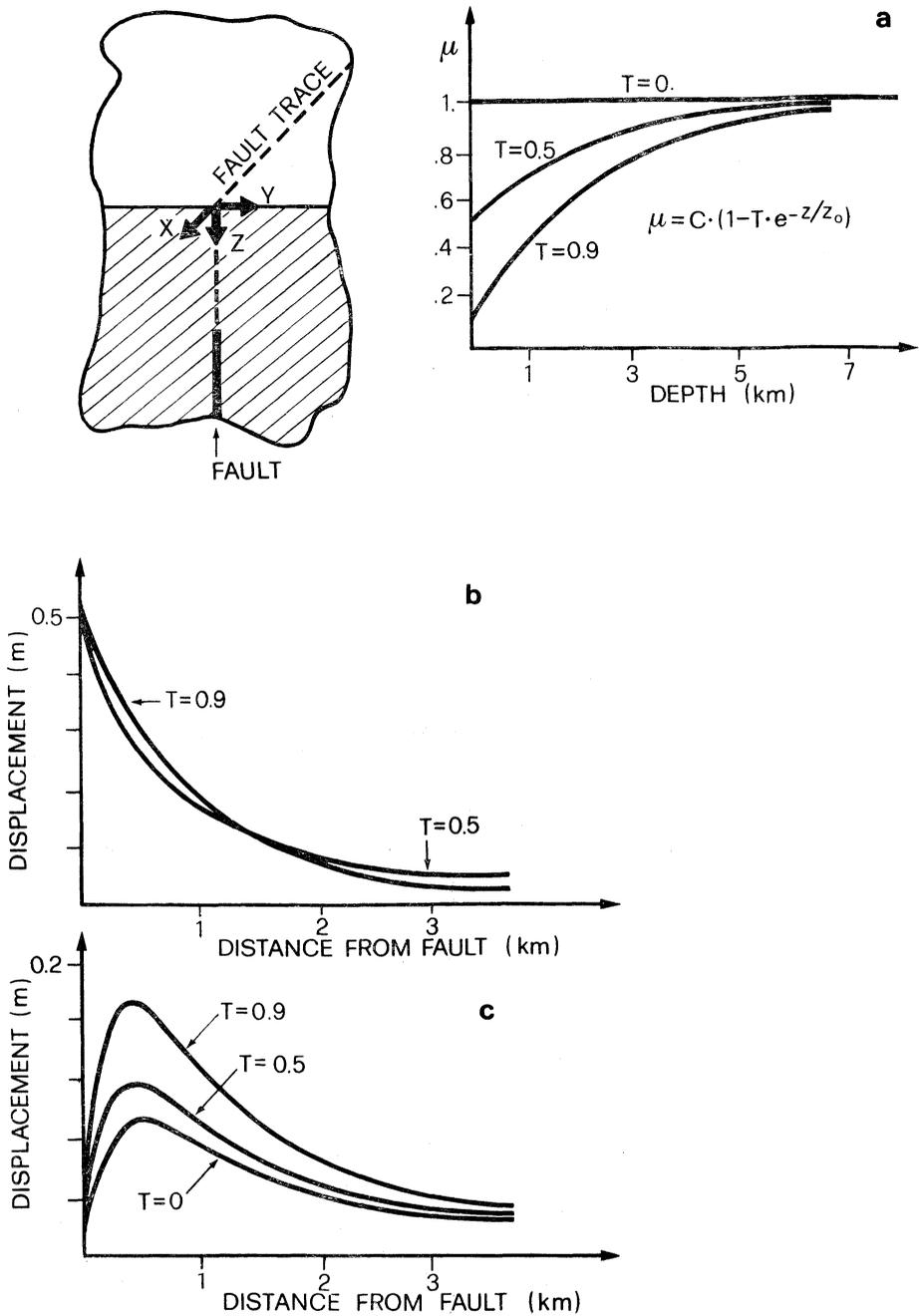


Fig. 3. - Normalized shear modulus as a function of depth (a); surface displacement as a function of distance from the fault: (b) surface fault case; (c) buried case. (Mahrer and Nur, 1979).

Smyle (1971) and others. For example, the vertical and horizontal displacement components are derived for a finite rectangular strike-slip fault in a uniform elastic half-space with the geometry shown in fig. 1. In particular the

$U_x$  displacement component along a median line perpendicular to the fault strike is shown in fig. 2.

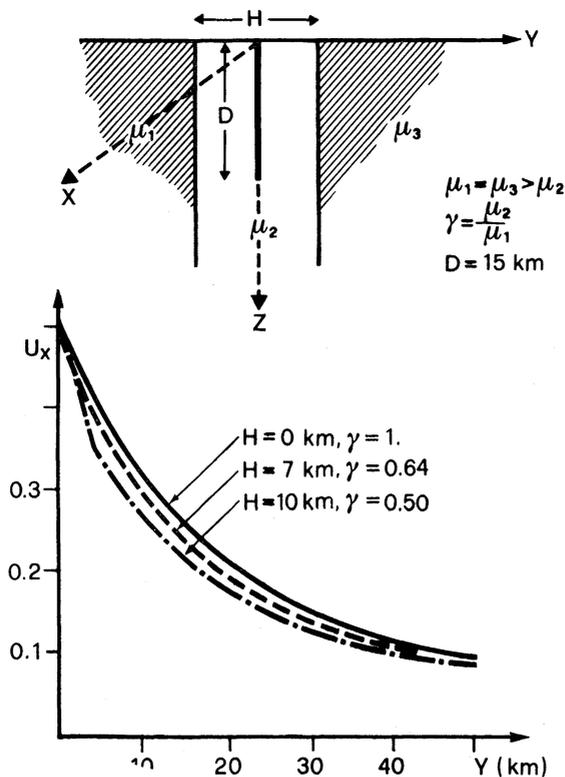


Fig. 4. - Displacement  $U$  in units of slip as a function of distance from a strike-slip fault, for different models of laterally inhomogeneous medium. (Rybicki and Kasahara, 1977).

In general the asymmetry of deformations at the two sides of the fault is strictly correlated to the slip and inclination of the fault plane, whereas the displacement field near to and far from the fault are indicative respectively of the upper and lower limits of the fault.

The results obtained by substituting the uniform elastic half space with a variable shear modulus are quite significant. Assuming that rigidity  $\mu(z)$  is a continuous and smooth function of depth:

$$(1) \quad \mu(z) = c(1 - Te^{z/z_0})$$

where the value of  $T$  is derived from seismic data ( $T=0.9$ ) as well as from laboratory measurements ( $T=0.5$ ) (Steward and O'Neil, 1972; Brace, 1965), Mahrer and Nur (1979) obtain theoretical displacements almost identical for different values of  $T$  in the case of surface faults; in contrast, in the case of a buried fault, a noticeable dependence is present (fig. 3).

For a model of a fault in a laterally inhomogeneous medium and in particular when a low rigidity zone is present around the fault, a strong distortion of the strain field is introduced (fig. 4).

#### GEODETIC MEASUREMENTS

A series of high-precision levelling surveys is the most fruitful method for investigating vertical land movements; using this technique, it is possible to examine in detail and with a high degree of accuracy (s.e. of height difference 1. (mm)  $\sqrt{D}$  (Km)) the displacements, even over sizeable regions.

Vertical control networks should be planned bearing in mind a considerable number of geophysical parameters, such as the area affected by the deformation, the characteristics of the displacement and its supposed magnitude, the density of the bench-mark, also taking into account the high cost of measurements, the various possible level routes and the degree of accuracy expected.

As far as the accuracy of the measurements is concerned, there are in theory no problems, given that high precision levelling only gives rise to errors that are proportionately negligible both in terms of the size of the co-seismic deformations, as well as in terms of background noise. The measurement of slow deformations, such as pre-seismic and post-seismic ones, is a conceptually different problem, given that the accumulation of stress can cause surface deformations varying in intensity, distribution and speed.

It is therefore necessary, when planning the net, to bear in mind, not only the importance of a homogeneous distribution of datum points throughout the zone in question, and of the optimal configuration of the net itself, but also the costs and execution time involved which may make it impossible to repeat the survey frequently. Problems which may arise in network adjustment, caused by vertical crustal movements active during the period in which the survey is carried out, are overcome by combining this technique with continuous tilt components recorded by tiltmeters (Savage *et al.*, 1979), and tidegauges.

Gravity remeasurements may to some extent substitute a levelling survey: the accuracy of the results thus obtained is less satisfactory but costs are lower and less time is involved. Simultaneous monitoring of elevation and gravitational change can contribute further elements to the interpretation of the phenomena in question (Baldi and Postpischl, 1981). Generally speaking, the gravity variation corresponding to a vertical displacement caused by an accumulating stress field in a seismic area, could be expressed as the sum of the free air and the Bouguer effect plus the gravity contribution made by the subsurface density change related to the volumetric strain (Walsh and Rice, 1979).

The use of modern microgravimeters (e.g. LaCoste Romberg MOD. D) makes it possible to achieve extremely accurate results; standard errors smaller than  $5.10^{-8}$  m/sec<sup>2</sup> may be obtained with very well constrained gravity nets (Marson and Morelli, 1979; Baldi and Marson, 1981). Improvement in the accuracy of this technique depends to a very large extent on the elimination

of effects caused by the earth tide, tidal loading, local water table level and atmospheric mass movements.

Horizontal crustal deformations may be measured by means of survey in which angles and distances are involved; electromagnetic distance-measuring instruments can determine distances to an accuracy of 1 ppm or less, according to the ability to estimate the average propagation speed of light in the atmosphere, using meteorological measurements (Owens, 1967; Baldi and Unguendoli, 1982) or by means of the simultaneous measurement of optical path length at two or three different wavelengths (Slater and Hugget, 1976).

With a view to developing adequate geodetic networks, some criteria, such as accuracy, reliability and expense need to be considered as regards the optimization of the net. Such optimization is usually classified in four different orders, starting from a parametric adjustment based on the observation equation:

$$(2) \quad Ax - d = v$$

which, by means of a least square solution and following the free adjustment scheme, gives:

$$(3) \quad x = (A^T P A)^+ A^T P d$$

with the co-variance matrix:

$$(4) \quad K_{xx} = \sigma_0^2 Q_{xx} = \sigma_0^2 (A^T P A)^+$$

where A is the configuration matrix, P the weight matrix,  $\sigma_0$  is the variance of the unit weight and ( )<sup>+</sup> denotes the Moore-Penrose inverse.

In the case of planimetric networks, for example, disregarding the "Zero order design" relating to the optimal reference system, the optimization process is carried out by means of "First, Second and Third order design", which relate respectively to the configuration of the net and the observational plan, to the search for an optimal distribution of observation weights, and to the improvement of the net by the inclusion of additional points and/or observations (Schmitt, 1982).

As regard the "First order design", the theoretical approach is severely constrained by topography and by information relating to the geophysical phenomenon in question. On the basis of point distribution, measurements should be planned while seeking the greatest reliability and precision possible, without forgetting the question of cost. The most practical approach is the interactive simulation.

One of the criteria used to evaluate the reliability of a net, that is to say its resistance to gross observation errors, is based on the analysis of the diagonal elements  $r$  of the matrix R, where:

$$(5) \quad R = A Q_{xx} A^T P - I.$$

These elements may vary from 0 to 1; good reliability can be obtained for high values of  $r_i$ , as well as with a homogeneous distribution of such values throughout the entire net (Winner, 1981).

Once the network has been drawn up and the theoretical aspect of the measurements established, it is possible to take steps to improve the accuracy of the single measurements, with a view to obtaining a better overall network design. This may be done by seeking a method of weighting the measurements themselves which responds more satisfactorily to the needs for which the measurements are taken (Second order design).

In effect, the analytic approach is founded on the solution of the following equation, as it relates to P:

$$(6) \quad (A^T PA)^+ = Q_{xx}$$

in which the criterion matrix Q must be defined in advance.

In general, a geodetic net ought to tend towards homogeneity and isotropy. In this case, the Q is normally constructed using the Tailor-Karman structure solution (Grafarend, 1972, Wimmer, 1981) the simplest case of which may be reduced to the equation:

$$(7) \quad Q = I$$

where I is the unit matrix, involving the drawback that the co-variance between the coordinates is assumed to be zero.

A different approach, which appear to yield results more in line with real needs, consists in the use of the singular value decomposition of the matrix Q of the net:

$$(8) \quad Q_{xx} = V\lambda V^T$$

where the diagonal matrix  $\lambda$  contains the eigenvalues of  $Q_{xx}$ , and V is the matrix of the corresponding orthonormalized eigenvectors. A possible derivation of the criterion matrix might consist in contracting the eigenvalue spectrum, without changing the eigenvectors, with a consequent diminution in the semiaxes of the error ellipses. A different approach, of particular interest in the study of deformations, might be that of obtaining a variation in the orientations of the semiaxes of the error ellipses, by rotating the eigenvectors matrix V (Crosilla, 1982). In any case this method is severely limited by the shape of the net, and therefore the results obtainable are generally negligible.

#### EXAMPLES

In the case of a fault, even supposing that the terrain is such as to permit one to choose the distribution of points in total freedom, there nevertheless

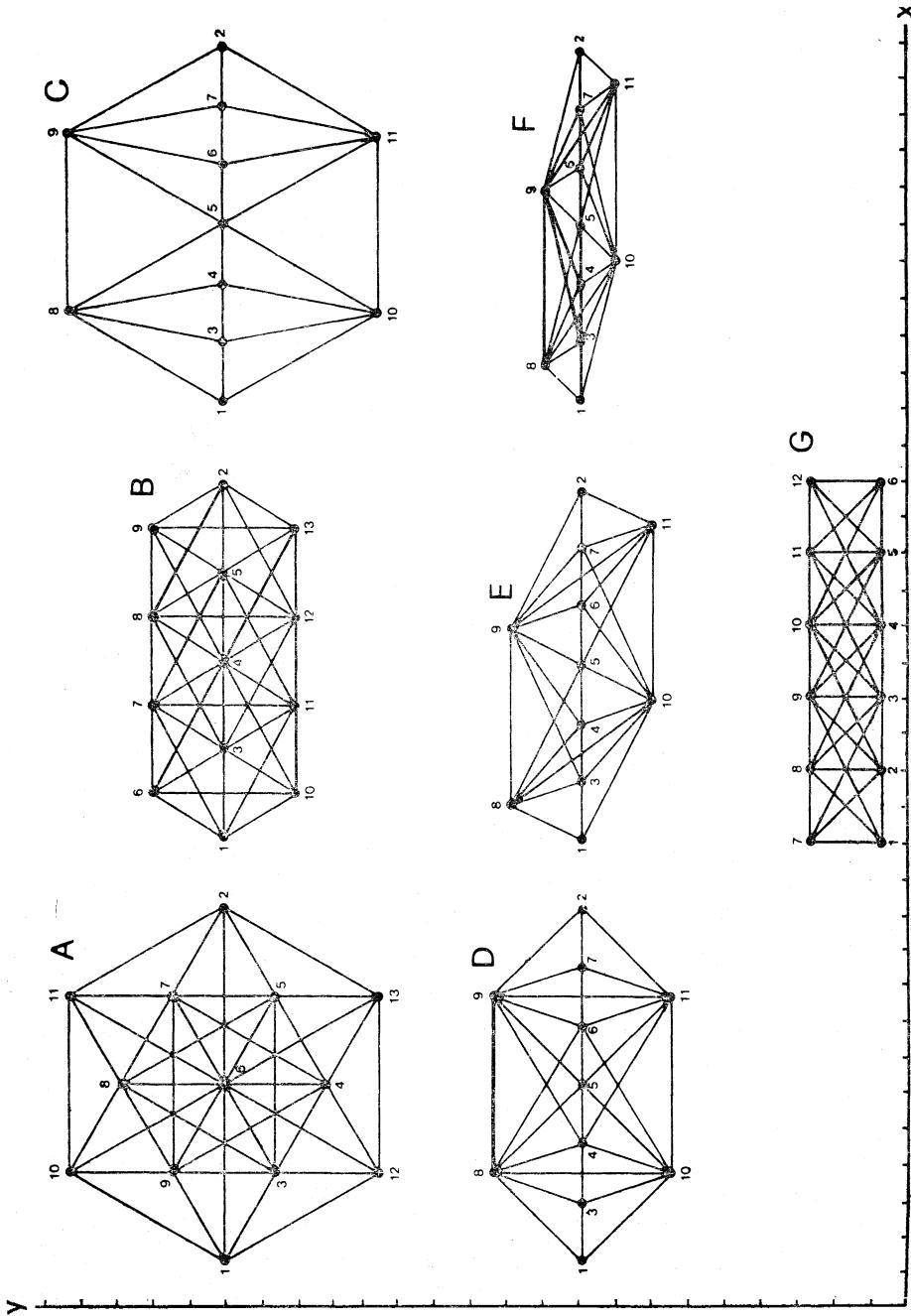


Fig. 5. - Different scheme of planimetric network.

remain many other factors that have to be borne in mind. These include: the size of the area to be surveyed and, on the other hand, the time available for taking the measurements and considerations of costs, whether the direction of movements is known, etc. Fig. 5 shows various different schemes of nets designed for the study of co-seismic movements in a region crossed by a fault. These nets were elaborated on the assumption that only measurements of distance were to be made and that it would be possible to measure all those distances not in excess of a pre-established length  $d$ , related to the type of instrument to be used. Some possible solutions are given in Table I.

TABLE I  
*Selection of electro-optical distance measuring devices.*

D (Km)	Instrument	Standard deviation (mm)
2	Mekometer (Xenon-flash-lamp)	+/- (0.2 + 1.10 <sup>-6</sup> D)
15	Terrameter (Two wave lengths)	+/- (1.10 <sup>-7</sup> D)
50	AGA Mod. 8 (He-Ne Laser)	+/- (5. + 1.10 <sup>-6</sup> D)
150	Sial MD 60 (Micro-wave)	+/- (10. + 3.10 <sup>-6</sup> D)

Net A can provide a description of the deformation trend over a huge area and may therefore be used, for example, when it is impossible to localize the fault in advance; the other kinds of nets described in fig. 5 may be used to concentrate research on the deformations along a traverse of the strike fault. Supposing that all the sides are measured following the same procedure and that errors are therefore approximately proportional to distances, the various nets yield different results. This is made apparent by the dimensions and orientation of the error ellipses obtainable when using an AGA mod. 8 geodimeter to take the measurements (Table II). As can be seen from the Table, the initial condition of near-homogeneity and isotropy (equal and circular error ellipses) gives way to more flattened-out ellipses, the semi-major axes of which are prevalently oriented perpendicularly to the net's longitudinal axis.

Assuming that the accuracy of the measurements, and the weights, may be varied at will, the application of the second order design would lead to an improvement in the shape of the ellipses. The values of the ellipse semi-axis ratios of the two nets with the greatest and the least isotropy are given in Table III. The method followed involves the contraction of the eigenvalues according to the formula:

$$(9) \quad \lambda_i = \lambda_i - t (\lambda_i - \lambda_{\min}), \quad 0 < t < 1$$





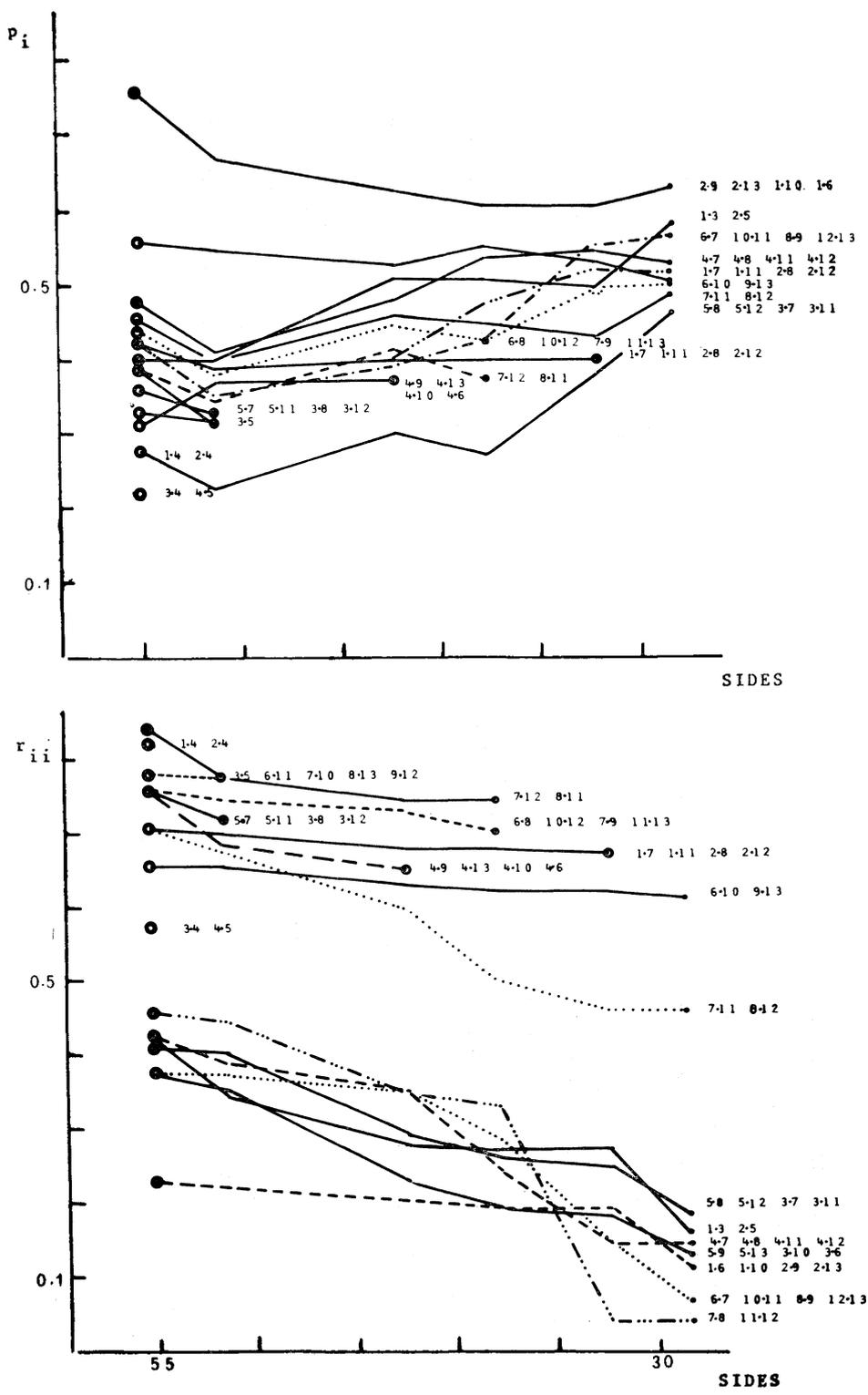


Fig. 6. - Schema of reduction of sides of net B. The weight distribution is derived by contracting the spectrum of eigenvalues with  $t = 1$ .

TABLE IV  
*Semiaxes and orientation of error ellipses of points (net B) for the different configurations shown in fig. 6*

N	1		2		3		4		5		6							
	a	b	a	b	a	b	a	b	a	b	a	b						
		$\varphi$																
1,2	1.9	1.7	90°	1.9	1.9	90°	2.0	2.0	0°	2.1	2.0	0°	2.2	2.2	179°	2.7	2.4	179°
3,5	1.4	1.3	90	1.6	1.5	160	2.1	1.5	180	2.2	1.6	180	2.3	1.6	180	2.4	1.7	180
4	1.7	1.1	90	1.8	1.7	0	1.8	1.8	179	3.0	1.9	0	3.0	2.1	0	3.0	2.2	0
6,13	1.9	1.5	17	1.9	1.6	16	2.0	1.7	10	2.1	1.8	170	2.6	1.8	0	2.6	1.9	3
7,12	1.7	1.3	103	1.8	ö.3	102	2.0	1.5	103	2.1	1.5	103	2.3	1.6	110	2.4	1.8	108
8,11	1.7	1.3	77	1.8	1.3	77	2.0	1.5	76	2.1	1.5	76	2.3	1.6	69	2.4	1.8	71
9,10	1.9	1.5	163	1.9	ö.6	163	2.0	1.7	169	2.1	1.8	9	2.6	1.8	179	2.6	1.9	176

in which  $\lambda_{\min}$  stands for the minimum value of the  $\lambda_i$ . As may be seen, in the case of net F, no degree of homogeneity and isotropy is reached, and this is obviously due to the shape of the net.

The second order design and reliability criteria may also be used to carry out a rational measurement reduction operation involving only the slightest losses in accuracy in relation to the reduction in costs and to the speed of the measurements themselves, and without modifying the weight matrix P. As an example of such an application, we took into consideration net (B) which has extremely high redundancy, and we eliminated, on the basis of the application of the second order design, the least weighted sides one after another, even if they were characterized by high reliability. This criterion, which may be easily programmed for automatic interactive processing, has yielded good results, as may be observed from the data reported in fig. 6 and Table IV, in which it may be seen that the reduction of the number of sides from 55 to 32 leads

TABLE V

*Net F: ellipses for single trilateration, and those obtained by adding the set of angles (16) in a round from points 4,5 and 6.*

<i>n</i>	1			2		
	<i>a</i>	<i>b</i>	$\varphi$	<i>a</i>	<i>b</i>	$\varphi$
1	4.3	1.4	96°	2.7	1.3	102°
2	4.3	1.4	96	2.7	1.3	102
3	2.4	1.1	77	2.2	1.0	75
4	3.3	0.9	83	1.8	0.9	85
5	3.6	0.9	91	2.0	0.8	90
6	3.3	1.0	83	1.8	0.9	85
7	2.4	1.1	77	2.2	1.0	75
8	2.8	1.6	42	1.8	1.4	21
9	2.8	1.4	80	1.9	1.0	79
10	2.8	1.4	80	1.9	1.0	79
11	2.8	1.6	42	1.8	1.4	21

to only negligible changes in the size of the error ellipses and that even the minimum reliability values maintain perfectly acceptable values. Any further diminution, however, in the number of sides leads to serious losses in reliability.

One can avail oneself of the so-called third order design, by adding measurements of points, to improve a pre-existing net of a particular type. For

example, type F of fig. 5, which is obviously unsuitable for accurate detection of the normal movement component in the direction of its main trend, may be improved by adding the set of angles in a round from points, 4, 5 and 6. The results of this computation are shown in Table V. This procedure is obviously of the empirical kind and can be pursued in the form of a series of attempts, dictated by experience and by trial and error.

#### CONCLUSION

All the currently known methods of designing and optimizing geodetic networks have drawbacks which make them hard to apply to concrete problems; indeed, the choice of the configuration is greatly restricted by background factors such as topography, geology, costs, etc. The availability of *a priori* knowledge concerning possible movements and their direction may help to identify the most practical network possible from the geophysical and geodetic point of view.

If we consider the surface displacements accompanying faulting, the field measurements throughout the area in which one might expect elastic strain to build up may be too slow and expensive to perform; on the other hand, more frequent measurements, more closely spaced sites distributed over a narrow area perpendicular to the fault strike, the extension of the network to a considerable distance from the fracture zone, may make it possible to obtain a correct interpretation of co-seismic deformation. As regards the measurements of slow deformations, pre-seismic and post-seismic, varying in intensity, distribution and speed, the combination of the classical geodetic techniques with continuous tilt and strain recorded by tiltmeters and strain-meters may partially overcome the problems connected with frequent repetition of field measurements.

In any case a realistic approach to the planning of a net is to apply the interactive method and trial and error procedure, taking into account, at the same time, all environmental conditions as well as theoretical optimization criteria.

#### REFERENCES

- BALDI P. and MARSON I. (1981) - *Gravity and geodetic networks for the study of crustal deformations in seismic area.* « Boll. Geod. Sc. Aff. », 3.  
 BALDI P. and POSTPISCHL D. (1981) - *Gravity Variations during pre-seismic crustal deformations.* « Il Nuovo Cimento », 10.  
 BALDI P. and UNGUENDOLI M. (1982) - *Some problems in monitoring crustal deformation by geodetic methods.* « Quaterniones Geodesiae », 3.  
 BRACE W.F. (1965) - *Some new measurements of linear compressibility of rocks.* « J. Geophys. Res. », 70.  
 CHIMNERY M.A. (1961) - *The deformation of the ground around surface faults.* « Bull. Seism. Soc. Am. », 51.

- CROSILLA F. (1982) – *A criterion matrix for the second order design of control networks*. Proceedings of the Meeting on Survey Control Networks, Aalborg, ASPW, München.
- GRAFAREND E. (1972) – *Genavigkeitsmase geodatischer netze*. Deutsche Geodatische Kommission, Reihe A, Heft 73, München.
- HOLDAHL S.R. (1980) – *Crustal movements and redefinition of heights*. Proceedings NAD Symposium, Ottawa.
- MAHRER K.D. and NUR A. (1979) – *Strike slip faulting in a downward varying crust*. « J. Geophys. Res. », 84.
- MANSINHA L. and SMYLYE D.E. (1971) – *The displacement fields of inclined faults*. « Bull. Seism. Soc. Am. », 61.
- MARUYAMA T. (1964) – *Statistical elastic dislocation in an infinite and semi-infinite medium*. « Bull. Earthquake Res. Inst. », Tokyo Univ., 42.
- MARSON I. and MORELLI C. (1979) – *A microgravimetric network for studying local gravity variations in North-eastern Italy (Friuli) connected with earthquake activity: institution of the net*. « Boll. Geod. Sc. Aff. », 2.
- McHUGH S. and JOHNSTON M.J.S. (1979) – *A review of Observations and dislocation modeling of some creep-related tilt perturbations from Central California*. Proc. of the International Workshop on Monitoring Crustal Dynamics in Earthquake Zones, Strasbourg.
- MEISSNER R. (1978) – *What kind and what magnitude of creep movements take place at the earth's surface?* Proc. of the European Workshop on Space Oceanography, Navigation and Geodynamics, Germany, ESASP-137.
- OWENS J.C. (1967) – *Optical refractive index of air: dependence on pressure, temperature and composition*. « Appl. Opt. », 6.
- PRESS F. (1965) – *Displacements, strains and tilts at teleseismic distances*. « J. Geophys. Res. », 70.
- RIKITAKE I. (1976) – *Earthquake Prediction*. Elsevier, Amsterdam, Niederlande.
- RYBIKI K. and KASAHARA K. (1977) – *A Strike-slip fault in a laterally inhomogeneous medium*, « Tectonophysics », 42.
- SCHMITT G. (1982) – *Optimisation of geodetic networks*. « Review of Geophysics and Space Physics », 4.
- SLATER L.E. and HUGGETT G.R. (1976) – *A multiwavelength distance-measuring instrument for geophysical experiments*. « J. Geophys. Res. », 35.
- STEWART S.W. and O'NEIL M.E. (1972) – *Seismic travel times and near surface crustal velocity structure bounding the San Andreas Fault zone near Parkfield*, « California, U.S. Geol. Surv. Prof. Pap. », 800-C, C117-C118.
- WALSH J.B. and RICE J.R. (1979) – *Local change in gravity resulting from deformation*. « J. Geophys. Res. », 84.
- WIMMER H. (1981) – *Second order design of geodetic networks by an iterative approximation of a given criterion matrix*, Proceedings of the IAG Symposium, München.