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**Determination of creep, fatigue and activation
energy from constant strain-rate experiments**

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Meccanica dei solidi. — *Determination of creep, fatigue and activation energy from constant strain-rate experiments.* Nota (*) del Socio MICHELE CAPUTO.

RIASSUNTO. — Si interpretano i risultati degli esperimenti fatti su mezzi anelastici mediante applicazione di deformazioni crescenti linearmente nel tempo. Si trova che dai risultati di questi esperimenti, si può risalire alla funzione di *creep* che viene determinata con tre diversi modelli di approssimazione. Si trova inoltre che per decidere sul comportamento a lungo termine dei materiali sono necessari esperimenti a più lungo termine di quelli fatti.

Si dimostra poi che le relazioni classiche che rappresentano il comportamento dei materiali anelastici implicano un indurimento del mezzo e talora anche il fenomeno di fatica. Si suggerisce infine un metodo per determinare il numero di cicli che porta alla fatica. Come applicazione, usando dati di laboratorio a varie temperature e deformazioni linearmente crescenti nel tempo, si determinano la funzione di *creep* e l'energia di attivazione, entrambe dipendenti dalla temperatura, della Halite policristallina.

INTRODUCTION

Anelastic properties of materials are mathematically represented by the following relations (e.g. Caputo, Mainardi, 1971)

$$(1) \quad \begin{aligned} \varepsilon(t) &= \int_0^t h(t-\tau) \sigma(\tau) d\tau \\ \sigma(t) &= \int_0^t \bar{h}(t-\tau) \varepsilon(\tau) d\tau \end{aligned}$$

where $\varepsilon(t)$ is strain, $\sigma(t)$ is stress, h and \bar{h} are called the causal functions and represent the response of the system to a unit impulse $\delta(t)$ of stress and strain respectively. Instead of the impulse response in practice one observes the response of the stress ($m_1(t)$) to a step of strain or the response of the strain ($c_1(t)$) to a step of stress. We have in this case

$$(2) \quad \begin{aligned} \varepsilon(t) &= c_1(0) \sigma(t) + \int_0^t c_1'(t-\tau) \sigma(\tau) d\tau \\ \sigma(t) &= m_1(0) \varepsilon(t) + \int_0^t m_1'(t-\tau) \varepsilon(\tau) d\tau \end{aligned}$$

(*) Presentata nella seduta del 24 aprile 1982.

from which follows

$$(3) \quad \begin{aligned} h(t) &= c_1(0) \delta(t) + c_1'(t) \\ \bar{h}(t) &= m_1(0) \delta(t) + m_1'(t). \end{aligned}$$

The functions $c_1(t)$ and $m_1(t)$ are usually referred to as creep compliance and relaxation modulus. If we assume that the integrals appearing in (2) are higher order with respect to the other terms, then the stress-strain relations in which the integral term appears operating on linear combination of stress and strain (Caputo, Mainardi, 1971) are practically equivalent to (3).

Introducing the Laplace transforms in (1) and (2) we have with obvious notation

$$(4) \quad \begin{aligned} E(p) &= H(p) S(p) = p C_1(p) S(p) \\ S(p) &= \bar{H}(p) E(p) = p M_1(p) E(p) \end{aligned}$$

which imply the following compatibility condition.

$$(5) \quad H(p) \bar{H}(p) = p^2 C_1(p) M_1(p) = 1.$$

THE CONSTANT STRAIN RATE EXPERIMENT

Some recent laboratory experiment (Carter and Heard, 1970; Gangi *et al.*, 1981) have been made to observe the response of the system to a constant rate of strain instead of a step of stress.

In order to see the meaning of these experiments let us consider strains and stresses which are linear function of time

$$(6) \quad \varepsilon = t, \quad \sigma = t$$

and introduce the following relations in which appear the new functions $c_2(t)$ and $m_2(t)$

$$(7) \quad \begin{aligned} \sigma(t) &= m_2(0) \varepsilon'(t) + m_2'(0) \varepsilon(t) + \int_0^t m_2''(t-\tau) \varepsilon(\tau) d\tau \\ \varepsilon(t) &= c_2(0) \sigma'(t) + c_2'(0) \sigma(t) + \int_0^t c_2''(t-\tau) \sigma(\tau) d\tau. \end{aligned}$$

Introducing (6) in (7) we obtain

$$(8) \quad \begin{aligned} \sigma(t) &= m_2(t) \\ \varepsilon(t) &= c_2(t). \end{aligned}$$

Which imply that $m_2(t)$ and $c_2(t)$ represent the response of the system to a constant rate of strain and stress respectively.

If we substitute

$$(9) \quad \begin{aligned} \varepsilon &= 1(t) \\ \sigma &= 1(t) \end{aligned}$$

in (7) we obtain the relaxation modulus and the compliance respectively,

$$(10) \quad \begin{aligned} m_1(t) &= m_2(0) \delta(t) + m_2'(t) \\ c_1(t) &= c_2(0) \delta(t) + c_2'(t) \end{aligned}$$

which imply that the laboratory experiments made with constant rate of strain or stress in practice give the same information, as the experiment with a step of strain or stress. The compatibility conditions for $m_2(t)$ and $c_2(t)$ are obtained by taking the Laplace transforms of (7)

$$(11) \quad \begin{aligned} S &= [p^2 M_2(p) - \varepsilon(0) m_2(0)] E \\ E &= [p^2 C_2(p) - \sigma(0) c_2(0)] S \\ C_2(p) &= \frac{c_2(0) \sigma(0)}{p^2} + \frac{1}{p^2} \frac{1}{p^2 M_2(p) - m_2(0) \varepsilon(0)}. \end{aligned}$$

If $\sigma(0) = \varepsilon(0) = 0$, we obtain using (10 and 11)

$$(12) \quad \begin{aligned} c_2(t) &= \frac{(t - \tau)^3}{3!} * \text{LT}^{-1} \{ [M_2(p)]^{-1} \} \\ c_1(t) &= \frac{(t - \tau)^2}{2} * \text{LT}^{-1} \{ [M_2(p)]^{-1} \}. \end{aligned}$$

Therefore, the laboratory experiments giving the response to a constant strain rate supply also the response to a constant stress rate and to a step of stress.

Laboratory experiments (Gangi 1981) show that $m_2(0) = 0$ and that $m_2(t)$ with good approximation is

$$(13) \quad m_2(t) = a_0 e^{-\alpha t} + b_0 e^{-\beta t} + c_0, \quad a_0 + b_0 + c_0 = 0$$

by substitution in (12) we find

$$(14) \quad \begin{aligned} c_1(t) &= -(a_0 \alpha + b_0 \beta)^{-1} - (a_0 + b_0)^{-1} t + \\ &+ \frac{a_0 b_0 (\alpha - \beta)^2}{(a_0 + b_0)^2 \alpha \beta (\alpha a_0 + \beta b_0)} \left(1 - e^{-\frac{\alpha \beta (\alpha_0 + b_0) t}{\alpha a_0 + \beta b_0}} \right). \end{aligned}$$

Many authors tried to fit the experimental data, obtained applying constant strain rate, by assuming that $b_0 = 0$. In this case one obtains

$$(15) \quad c_1(t) = \frac{-1}{\alpha a_0} (1 + \alpha t) \quad , \quad c_1'(t) = -\frac{1}{a_0} .$$

It is thus clear that the second exponential in (13) causes an increase of the initial rate of creep, as it is seen from

$$(16) \quad c_1'(t) = -\frac{1}{a_0 + b_0} \left\{ 1 - \frac{a_0 b_0 (\alpha - \beta)^2}{(\alpha a_0 + \beta b_0)^2} e^{-\frac{\alpha\beta(a_0 + b_0)}{\alpha a_0 + \beta b_0} t} \right\} .$$

Since $a_0 < 0$, $b_0 < 0$ and $c_1''(t) > 0$, the zero of (16) gives a minimum for (14). For large values of t (14) is linear as (15). From $m_2(t)$ one may obtain the complex index of refraction n (e.g. Caputo Mainardi: 1971) by setting $p = i\omega$ ($\omega = \text{frequency}$)

$$n(\omega) = \left\{ \frac{i\omega (C_1(i\omega))}{c_1(0)} \right\}^{+1/2} .$$

In turn n gives the Q^{-1} and the dispersion of the phase velocity. The results of this section, namely formulae (10) through (16), can also be obtained by direct use of (1) or (2).

THE GENERALIZED EXPERIMENTS

The previous reasoning can be extended to the response of the system to strain and stress of the following type:

$$(17) \quad \varepsilon = \varepsilon_n t^{n-1} \quad , \quad \sigma = \sigma_n t^{n-1} \quad , \quad n > 0, \quad n \text{ integer}$$

introducing the following relations

$$(18) \quad \sigma = \sum_{i=1}^n i \left[\frac{\partial^{(i-1)} \varepsilon}{\partial t^{i-1}} \cdot m_n^{(n-i)}(0) \right] + \int_0^t m_n^{(n)}(t - \tau) \varepsilon(\tau) d\tau$$

$$\varepsilon = \sum_{i=1}^n i \left[\frac{\partial^{(i-1)} \sigma}{\partial t^{i-1}} \cdot c_n^{(n-i)}(0) \right] + \int_0^t c_n^{(n)}(t - \tau) \sigma(\tau) d\tau$$

where $m_n(t)$, $c_n(t)$ are functions to be determined. Substituting (17) in (18) we obtain

$$(19) \quad \sigma = (n-1)! m_n(t) \quad , \quad \varepsilon = (n-1)! c_n(t) .$$

Therefore $m_n(t)$ and $c_n(t)$ represent the response of the system to strain and stresses given by (17).

The compatibility condition, if $\varepsilon^{(i)}(0) = \sigma^{(i)}(0) = 0$ (for $i = 0, 1, \dots, n-2$), is

$$(20) \quad p^{2n} M_n(p) C_n(p) = 1.$$

If we assume in (18) $\varepsilon(t)$ and $\sigma(t)$ as given in (9) we obtain the relaxation modulus and the compliance respectively

$$(21) \quad \begin{aligned} \sigma &= m_n^{(n-1)}(t) \\ \varepsilon &= c_n^{(n-1)}(t). \end{aligned}$$

Therefore,

$$(22) \quad \begin{aligned} m_1(t) &= m_n^{(n-1)}(t) \\ c_1(t) &= c_n^{(n-1)}(t) \end{aligned}$$

which imply that the response of the system to a stress, or strain represented in the time domain by a $(n-1)$ power of the time is given by the $(n-1)$ -integral of the material functions $c_1(t)$ and $m_1(t)$.

ANOTHER STRESS STRAIN RELATION

A satisfactory fit of the data obtained with constant rate of strain is obtained using the stress strain relation introduced by Caputo (1982).

$$(23) \quad \left(a + b \frac{\partial^{z_1}}{\partial t^{z_1}} \right) \sigma = \left(c + d \frac{\partial^{z_2}}{\partial t^{z_2}} \right) \varepsilon$$

where z_i , ($i = 1, 2$), $0 < z_i < 1$, are real. Caputo (1982) showed that relation (23) implies dissipation with (considering that $m \gg b$, $d \gg a$)

$$(24) \quad Q^{-1} = \frac{b}{m} \omega^{v_2} \sin \frac{\pi v_2}{2} - \frac{a}{d} \sin \frac{\pi v_1}{2}$$

dispersion of the phase velocity v of waves

$$(25) \quad v^{-1} = \sqrt{\frac{\rho d}{m}} \left(1 - \frac{\omega^{v_2}}{2} \frac{b}{m} \cos \frac{\pi v_2}{2} + \frac{\omega^{v_1}}{2} \frac{a}{d} \cos \frac{\pi v_1}{2} \right)$$

and also fatigue (Caputo 1979) giving the number of cycles which may bring the medium to fatigue. The experimental data obtained with constant strain rate can be advantageously represented using (23). In fact, assuming $b = c = 0$, $a = 1$, $\varepsilon = \varepsilon_0 t$, (23) gives

$$(26) \quad \sigma = \varepsilon_0 t^{1-z_2} / (1 - z_2) !$$

which in turn implies that the creep $c_1(t)$ is

$$(27) \quad c_1(t) = \varepsilon_0 t^{2z_2} / (z_2) !$$

It is to be noted that the basic difference between (15), (16) and (27) is that in (15) and (16), after a sufficiently long time, the creep rate is constant, while (27) represents a mechanism with a creep rate decreasing with time. Only experimental data covering a sufficiently long time interval can decide which creep law considered is more realistic.

THE FATIGUE

In a recent paper (Hsi-Ping Liu and Louis Peselnik 1979) demonstrate that mechanical hysteresis loop shapes for a linear anelastic solid satisfying constant over a wide frequency range are sensitive to cycling stress waveform. Cusped or asymmetrical hysteresis loops are compatible with linear anelastic behavior for nonsinusoidal loading and do not necessarily imply a nonlinear attenuation mechanism.

The same was proved in a previous paper (Caputo 1976) using the same saw tooth strain model of Hsi-Ping Liu and Louis Peselnik (1979) and the almost frequency independent model of Caputo (1967). In another paper Caputo (1979) also proved that the same almost frequency independent model (Caputo 1967) using the Boltzmann after effect equation, explains the phenomenon of fatigue of the elastic materials with almost frequency independent implying that also the phenomenon of fatigue does not necessarily imply a non linear mechanism and is strictly related to that of attenuation. We shall see here that fatigue may occur in the linear field not only in the media which behave according to the model of Caputo (1967) but could occur also in the media behaving according to the more general model described by (2).

We shall show that the stress strain relations imply a time variation of the ratio between the applied strain and the resulting stress (or of the ratio between the applied stress and the resulting strain, which in turn imply fatigue). To see this let us apply to (2) a cyclic input and substitute for $c_1(t)$ and $m_1(t)$ the creep and relaxation functions $\psi(t)$ and $\bar{\psi}(t)$

$$(28) \quad c_1(t) = c_0 + \psi(t) \quad , \quad m_1(t) = m_\infty + \bar{\psi}(t)$$

with

$$\begin{aligned} \psi'(t) > 0 \quad , \quad \psi''(t) \leq 0 \quad , \quad \psi(0) = 0 \\ \bar{\psi}'(t) < 0 \quad , \quad \bar{\psi}''(t) \geq 0 \quad , \quad \bar{\psi}(\infty) = \text{finite} . \end{aligned}$$

If the cyclic inputs are

$$(29) \quad \varepsilon = \varepsilon_0 \sin \omega_0 t \quad , \quad \sigma = \sigma_0 \sin \omega_0 t \quad , \quad \omega_0 = 2\pi/T_0 .$$

We may compute the output at the time $A = (n + 1/4) T_0$, $n \geq 0$

$$\begin{aligned}
 \sigma(n) &= \varepsilon_0 m_1(0) \sin \frac{2\pi A}{T_0} + \varepsilon_0 \int_0^A m_1'(A - \tau) \sin \omega_0 \tau \, d\tau = \\
 &= \varepsilon_0 m_\infty + \varepsilon_0 \omega_0 \int_0^A \sin \omega_0 z \bar{\psi}(z) \, dz \\
 \varepsilon(n) &= \sigma_0 c_1(0) \sin \omega_0 A + \sigma_0 \int_0^A c_1'(A - \tau) \sin \omega_0 \tau \, dz = \\
 &= \sigma_0 c_2 + \sigma_0 \omega_0 \int_0^A \sin \omega_0 z \psi(z) \, dz .
 \end{aligned}
 \tag{30}$$

Let us consider $\sigma(n)$ first, $\bar{\psi}(t)$ is an analitic positive and decreasing function with $\bar{\psi}''(t) = 0$ only on a finite set and with $\lim_{t \rightarrow \infty} \bar{\psi}(t) = 0$ then it can be shown that $\sigma(n)$ is an increasing function of n . This can be proved as follows. Let us consider

$$\begin{aligned}
 U(n) &= \varepsilon_0 \int_{(n-3/4)T_0}^{(n-1/4)T_0} \sin \omega_0 z \bar{\psi}(z) \, dz \\
 V(n) &= \varepsilon_0 \int_{(n-1/4)T_0}^{(n+1/4)T_0} \sin \omega_0 z \bar{\psi}(z) \, dz
 \end{aligned}
 \tag{31}$$

and that $\sin \omega_0 z$ is antisymmetric with respect to $z = (n - \frac{1}{2}) T_0$ (and $z = nT_0$ respectively) one may see that, since $\bar{\psi}''(z) \leq 0$ then $U(n) > 0$, ($V(n) < 0$ respectively). Also considering that $\sin \omega_0 z$ is symmetric with respect to the point $z = (n - \frac{1}{2}) T_0$ and $\bar{\psi}''(z) \geq 0$ we may see that

$$|V(n)| < U(n) \quad , \quad U(n) + V(n) > 0 .
 \tag{32}$$

Consider now

$$\sigma(m) = \varepsilon_0 \int_0^{T/4} \sin \omega_0 z \bar{\psi}(z) \, dz + m_\infty + \sum_1^\infty m (U(m) + V(m))
 \tag{33}$$

for $n \rightarrow \infty$ we obtain a series of positive and negative terms decreasing in absolute values which converges, then $\lim_{n \rightarrow \infty} \sigma(n) = \bar{\sigma}$

This implies that the ratio between $\sigma(n)$ and $\varepsilon(n)$ is increasing or there is hardening of the material. This hardening can be defined as follows.

Let $\bar{\Delta}(n)$ be

$$\bar{\Delta}(n) = \frac{2\pi\varepsilon_0}{T_0} \int_{(n-3/4)T_0}^{(n+1/4)T_0} \sin \omega_0 z \bar{\psi}(z) \, dz
 \tag{34}$$

$\bar{\Delta}(n)$ is positive; the ratio

$$(35) \quad \frac{\bar{\Delta}(n)}{\sigma(n)}$$

can be defined the stress hardening of the material. Since $\bar{\Delta}(n)$ is decreasing function of n with $\lim_{n \rightarrow \infty} \bar{\Delta}(n) = 0$ and $\sigma(n)$ is an increasing function of n then the hardening is a decreasing function of n .

If $\bar{\sigma}$ is larger than the stress σ^* which gives failure then fatigue will occur.

Consider now $\varepsilon(n)$. Since $\psi(t)$ is a positive monotonic increasing function of t , $\varepsilon(n)$ is decreasing. The ratio of $\sigma(n)$ to $\varepsilon(n)$ is increasing and the material becomes harder. As for $\sigma(n)$ we may see that $\lim_{n \rightarrow \infty} \varepsilon(n) = \bar{\varepsilon}$. If $\bar{\varepsilon}$ is larger than the strain ε^* which give failure with $\sigma = \bar{\sigma}$ then fatigue would occur.

We may define strain hardening of the ratio

$$(36) \quad \frac{\Delta(n)}{\varepsilon(n)}, \quad \Delta(n) = \frac{2\pi\sigma_0}{T_0} \int_{(n-3/4)T_0}^{(n-1/4)T_0} \sin \omega_0 z \psi(z) dz.$$

If $\bar{\sigma} > \sigma^*$ (or $\bar{\varepsilon} > \varepsilon^*$) the value of n which gives $\bar{\sigma} = \sigma^*$ ($\bar{\varepsilon} = \varepsilon^*$) is of great practical interest because it indicates the number of cycles which would bring material to fatigue. An estimate of it can be obtained from the knowledge of the functions $c_1(t)$ or $\psi(t)$ which in turn can also be estimated as follows introducing the index of refraction $n = n_r - in_i$

$$(37) \quad \begin{aligned} [p C_1(p)]^{1/2} &= n_r - in_i = n_r \left(1 - i \frac{Q^{-1}}{2}\right) \\ v^{-1} &= n_r (\rho c_0)^{1/2} \\ p C(p) &= \frac{1}{\rho v^2} \left(1 - \frac{Q^{-1}}{2}\right)^2 \end{aligned}$$

where v is the phase velocity.

The inversion of (37) gives

$$(38) \quad c_1(t) = \int_{Br} [\rho p v(p)]^{-1} (1 - Q^{-1}(p)) e^{pt} dt$$

where Br indicates a Bromwich path.

APPLICATION TO EXPERIMENTAL RESULTS

Laboratory experiments (Gangi 1981, Heard 1972) give the values of a_0 , b_0 , c_0 , α , β for transient and steady state deformation of polycrystalline Halite at pressures from 20 MPa to 200 MPa, strain rates very near $1.1 \cdot 10^{-4} \text{ sec}^{-1}$

and the temperatures indicated in Fig. 1. Formula (16) allows to estimate the steady state creep $-(a_0 + b_0)^{-1}$ which is shown in Fig. 1 and indicates that, in the temperature ranges considered, the creep is a linear function of the temperature.

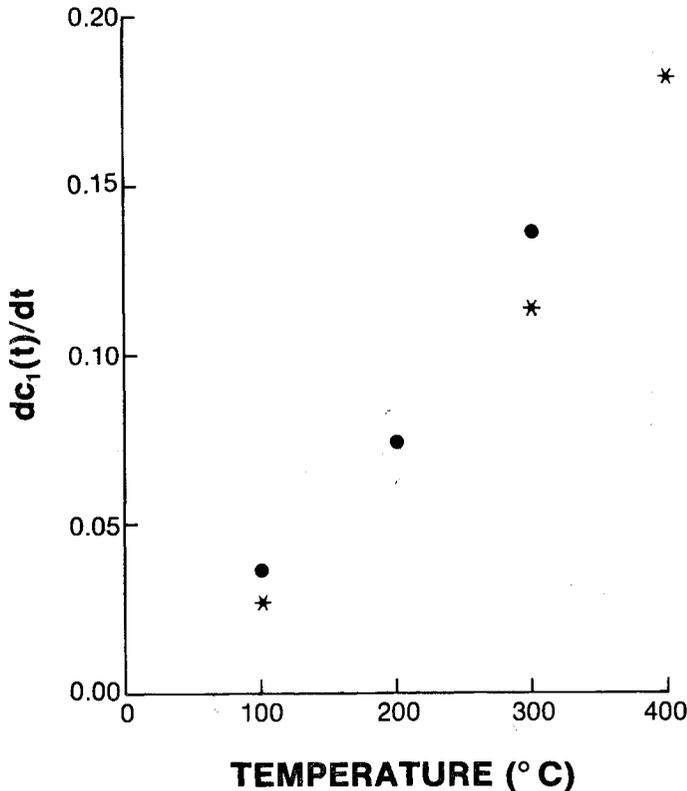


Fig. 1. - Steady state creep of polycrystalline Halite as function of the temperature at 20 MPa (dots) and 200 MPa (asterisk). The ordinate is $(\text{MPa sec})^{-1}$.

It is generally accepted that the steady state rate of creep is related to the temperature as follows

$$(39) \quad \frac{dc_1(t)}{dt} = K e^{-\frac{\bar{Q}}{R(T+273)}}$$

where \bar{Q} is the activation energy, k is a factor with dimension $(\text{MPa sec})^{-1}$ which depends on the material and the stress, and R is the gas constant. If $dc_1(t)/dt$ is a function of the temperature as indicated in Fig. 1 then

$$\frac{dc_1(t)}{dt} = f(T)$$

and from (39) follows that

$$\bar{Q} = -R(273 + T) \ln [f(T)/T].$$

For polycrystalline Halite and $100\text{ }^{\circ}\text{C} \lesssim T \lesssim 300\text{ }^{\circ}\text{C}$, assuming $K = 10^{9.40}$ (MPa sec) $^{-1}$ from Heard's (1972) data, (associated to $\bar{Q}/R = 11.5$ (K Cal mole $^{-1}$) at 173 MPa and $T = 200\text{ }^{\circ}\text{C}$) we obtain tentatively

$$\frac{\bar{Q}}{R} = (273 + T) [25.33 - \ln(0.49 + 0.39 \cdot 10^{-2} T + 0.31 \cdot 10^{-4} T^2)].$$

Analysis of the data shown in figures 1, 2, 3, 4 and 5 of Heard's (1972) paper on the laboratory tests on polycrystalline Halite show that for this material the relations (7) are valid only in limited ranges of strain rate as for instance it is the case in geologic phenomena. The results of this section are valid for strain rates around $1.1 \cdot 10^{-4}$ sec $^{-1}$.

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