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Equipartition of energy and heat production of earthquakes in the space of the phases

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Sismologia. — *Equipartition of energy and heat production of earthquakes in the space of the phases.* Nota di MICHELE CAPUTO (*), RODOLFO CONSOLE (**) e MARCO GASPERINI (**), presentata (***) dal Corrisp. M. CAPUTO.

RIASSUNTO. — Assumendo la validità del modello secondo il quale ogni terremoto può essere rappresentato su un piano delle fasi (l, p), si studiano le conseguenze dei risultati ottenuti in un precedente lavoro circa la distribuzione statistica dei terremoti sul piano stesso.

Si giunge alla conclusione che l'energia elastica liberata complessivamente dai terremoti è la stessa per ogni elemento $dl\, dp$ del piano (l, p), ed inoltre che il calore prodotto dai terremoti nell'intervallo $l, l + dl$ non dipende da l.

1. THE MODEL

As it is generally accepted we assume that earthquakes are due to elastic energy release in a fault with a linear dimension l and a stress drop p.

The formulae

$$W = \frac{k}{2\mu} l^3 p^2 \tag{1}$$

$$M_0 = \frac{l^3 p}{c} \tag{2}$$

where μ is the rigidity of the mean and k, c are geometrical factors, allow to establish that both the energy W and seismic moment M_0 assume a value univocally determined by the position of the event in the plane (l, p) of Caputo (1976, 1977).

The theory of Caputo (1976, 1977) assumes that the density distribution of the seismic events as function of l and p is:

$$D(l, p) = D l^{-\nu} p^{-1+\alpha} \tag{3}$$

where D, ν and α are parameters of the particular region.

The first interpretative works with this model were made by assuming $\alpha = 0$ because no information about the distribution of stress drop was available, and because by this assumption the validity of the values of ν obtained with this model was not disowned.

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According to the theory of Caputo (1976, 1977) the value of ν is related to the b value of the linear density distribution of earthquake of most seismic regions of the world; the relation is

$$b = -\frac{\nu - 1}{3} \gamma . \tag{4}$$

Since $\gamma \simeq 1.5$, from (4) it follows that in most seismic regions of the world $\nu \simeq 3$.

Caputo and Console (1977) in a recent work elaborated the data concerning the stress drop and seismic moment obtained by Thatcher and Hanks (1973) and by Tucker and Brune (1973) for the California, obtaining $\alpha \simeq 1$ and $\nu \simeq 3$. These values allow to draw important conclusions on the distribution of the elastic and thermal energies of the earthquakes in the space of the phases (l , p). They also allow to confirm, in first approximation, the mechanical model given by Caputo (1976).

In the following paragraph we shall show the implications resulting from $\alpha \simeq -1$.

2. Distribution of Energy.

Formula (1) gives us the value of the elastic energy released by an earthquake with linear dimension l and stress drop p. Therefore, the energy released by all earthquakes with dimensions in the range $l , l + dl$ and stress drop in the range $p , p + dp$ will be, according to (3):

$$dW_e = \frac{Dkl^3}{2 \mu} p^2 l^{-\nu} p^{-1+\alpha} \, dl \, dp . \tag{5}$$

Assuming, according to the results illustrated in the last paragraph, $\alpha \simeq -1$ and $\nu \simeq 3$, we have:

$$dW_e \simeq \frac{Dk}{2 \mu} \, dl \, dp = \text{const} \, dl \, dp \tag{6}$$

Formula (6) implies that in a given region, the earthquakes release the same amount of energy in all the elements of equal area $dl \, dp$ of the plane (l , p).

3. Mechanical Model.

The value of α determined by Caputo and Console (1977) allows to verify the mechanical model of earthquakes distribution based on the isotropic distribution of the directions of the fault planes in the seismic regions suggested by Caputo (1976); in that paper it is assumed that ν is the angle between the direction of the fault and the direction of the major tectonic force, the stress released p as the fault slips is then

$$p = \frac{f_a}{\dfrac{\sin 2\vartheta}{2} - f_c \cos^2 \vartheta} \tag{7}$$

where f_a represents the cohesive molecular force and f_c is the friction between the two sides of the fault. If we call ε the rate of stress accumulation, the number of stress drops p in the unit time and in the angle defined by ϑ, $\vartheta + \mathrm{d}\vartheta$, is

$$\frac{\varepsilon \, \mathrm{d}\vartheta}{p} = \frac{\varepsilon \, \mathrm{d}\vartheta}{f_a} \left(\frac{\sin 2\vartheta}{2} - f_c \cos^2 \vartheta \right) . \tag{8}$$

From (7) we obtain that the number of stress drops in the interval p, $p + \mathrm{d}p$ corresponding to ϑ, $\vartheta + \mathrm{d}\vartheta$ is

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}p} \frac{\varepsilon}{p} \mathrm{d}p . \tag{9}$$

A series expansion of $\dfrac{\varepsilon}{p} \dfrac{\mathrm{d}\vartheta}{\mathrm{d}p}$ to the first order in $\dfrac{f_a}{p}$ is obtained by differentiating (7). We obtain:

$$\frac{\varepsilon}{p} \frac{\mathrm{d}\vartheta}{\mathrm{d}p} \mathrm{d}p \simeq \varepsilon \frac{f_a}{p^3} \mathrm{d}p . \tag{10}$$

Formula (10) would therefore indicate that $\alpha = -2$, but we should take into account that the value of α has been determined using a set of values of p which is dominated by the smaller values of p; since the smaller values of p are of the order of f_a, formula (10) indicated that for the smaller values of p the distribution of stress drops is of the order of p^{-2}, which is agreement with the value $\alpha = -1$.

4. Distribution of Heat Production

The phenomenon of heat production on the surface defining the fault is complex. It depends mostly on the friction between the two opposite surfaces of the fault but also on other parameters. Here we limit the estimate to a simple model where the heat production does not produce a significant melting which in turn would lubricate the fault. The heat produced in this model is proportional to the surface of the fault, proportional to the dislocation dD and proportional to the force locking the fault. If ϑ is the angle between the detection of the fault and the major tectonic force $\bar{p}$ (Caputo, 1976) the force locking the fault is $\bar{p} \cos \eta$ and the heat produced by each earthquakes in the cell $\mathrm{d}l$, $\mathrm{d}p$ is proportional to

$$l^2 \,\mathrm{dD}\bar{p}\,\mathrm{tg}\,\vartheta = l^2\,\mathrm{dD}\bar{p}\cos\vartheta \tag{11}$$

since

$$\mathrm{dD} = \frac{\bar{p}l}{\mu}\sin\vartheta \tag{12}$$

with ε the deformation prior to the earthquake, the total heat produced by the faults of linear dimension l and direction ϑ is

$$\mathrm{N}l^3\,\bar{p}^2\sin\vartheta\cos\vartheta \tag{13}$$

where N is the number of faults in the cell $\mathrm{d}l$, $\mathrm{d}p$.

The corresponding number of faults in the cell $\mathrm{d}l$, $\mathrm{d}p$ is

$$\begin{aligned}\mathrm{N} &= l^{-\nu}\,p^{-1+\alpha}\,\mathrm{d}l\,\mathrm{d}p = l^{-\nu}\,\bar{p}^{-1+\alpha}(\cos\vartheta)^{-1+\alpha}\,\mathrm{d}l\,\frac{\mathrm{d}p}{\mathrm{d}\vartheta}\,\mathrm{d}\vartheta = \\ &= l^{-\nu}\,\bar{p}^{-1+\alpha}\left\{(\cos\vartheta)^{-1+\alpha}\frac{\mathrm{d}p}{\mathrm{d}\vartheta}\,\mathrm{d}\vartheta\right\}\mathrm{d}l\,.\end{aligned} \tag{14}$$

The heat produced by the faults in the range $l, l+\mathrm{d}l$ is then proportional to

$$l^{-\nu}\,\bar{p}^{-1+\alpha}(\cos\vartheta)^{-1+\alpha}\,\mathrm{d}l\,\frac{\mathrm{d}p}{\mathrm{d}\vartheta}\,\mathrm{d}\vartheta\,l^3\,\bar{p}^2\sin\vartheta\cos\vartheta\,. \tag{15}$$

Integrating over all the directions of the active faults ϑ_1, ϑ_2 (Caputo, 1976) we obtain for the range $l, l+\mathrm{d}l$

$$l^{-\nu}\,\bar{p}^{-1+\alpha}\,\mathrm{d}l\,l^3\,\bar{p}^2\int_{\vartheta_1}^{\vartheta_2}(\cos\vartheta)^{-1+\alpha}\sin\vartheta\cos\vartheta\,\frac{\mathrm{d}p}{\mathrm{d}\vartheta}\,\mathrm{d}\vartheta \tag{16}$$

or

$$\mathrm{W}_n = \mathrm{const}\,l^{-\nu}\,\bar{p}^{-1+\alpha}\,\bar{p}^2\,l^3\,\mathrm{d}l\,. \tag{17}$$

Since $\nu \simeq 3$, $\alpha \simeq -1$ we finally obtain

$$\mathrm{W}_n = \mathrm{const}\,\mathrm{d}l \tag{18}$$

or that the heat produced by the faults in the range $\mathrm{d}l$ does not depend on l.

5. Earth Rotation.

To see the possible implications of the determination of $\alpha = -1$ on the effect of earthquakes on the irregularities of the Earth rotation we may reasonably assume that the distribution of the directions of the displacements of the sides of the faults does not depend on the dimension of the faults nor on the stress drop. Then the variation $\Delta\mathrm{I}$ of the moment of inertia caused by the earthquakes in the time interval T in the ranges $l, l+\mathrm{d}l$ and $p, p+\mathrm{d}p$

is proportional to

$$\eta_2 \, \mathrm{T} \, \frac{l^3 \, p}{\mu} \, \mathrm{N} \, \mathrm{d}l \, \mathrm{d}p \tag{19}$$

where η_2 is a coefficient which depends on the latitude: substituting for N the expression (14) and assuming again $\alpha = -1$, $\nu = 3$ we find

$$\Delta \mathrm{I} \simeq \eta_2 \, \frac{\mathrm{T}}{p\mu} \, \mathrm{d}l \, \mathrm{d}p \tag{20}$$

which suggests that each cell $\mathrm{d}l$, $\mathrm{d}p$ of the space of the phases gives a contribution to the perturbation of the rotation of the Earth which is independent on the size of the faults of the cell and is inversely proportional to the stress drop of the cell. This in turn implies that a large contribution of earthquakes to the irregularities of the rotation of the Earth does not come necessarily, as generally believed, from the earthquakes with large magnitude but that it could come also from the region of the space of the phases where the stress drop is small.

For instance O'Connel and Dziewonski (1975) considered the earthquakes with magnitude larger than a given threshold assuming a one to one relation between M and M_0

$$\log \mathrm{M}_0 = 8.8 + 2.5 \, \mathrm{M} \tag{21}$$

But one may verify from formula (30) of Caputo (1977) that the value of p corresponding to a given $\overline{\mathrm{M}}$ could be larger than that corresponding to $\overline{\mathrm{M}} + 1$; therefore the shift of the pole caused by the former could be larger than that caused by the latter, independently from the other pertinent parameters. In conclusion the problem of the Chandler wobble should be revisited.

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