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Congruences Between Modular Forms and Related Modules.

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Sunto. – *Fissiamo ℓ un primo e M un intero tale che $\ell \nmid M$; sia $f \in S_2(\Gamma_1(M\ell^2))$ una forma nuova supercuspidale di tipo fissato a ℓ e speciale in un insieme finito di primi. Per un'algebra di quaternioni indefinita su $\boldsymbol{Q}$, di discriminante che divide il livello di f, associamo a f un'algebra di Hecke locale quaternionica $\boldsymbol{T}$. L'algebra $\boldsymbol{T}$ agisce su un modulo M_f proveniente dalla coomologia di una curva di Shimura. Applicando il criterio di Taylor-Wiles e il teorema di Savitt, rivediamo $\boldsymbol{T}$ come l'anello di deformazione universale di un problema di deformazione globale di Galois associato a $\bar{\rho}_f$. In particolare M_f è libero di rango 2 su $\boldsymbol{T}$. Nel caso particolare in cui f sia di livello minimale, come conseguenza dei nostri risultati e grazie al lemma di Ihara classico, proviamo un teorema di alzamento di livello e un risultato sugli ideali di congruenza. L'estensione al caso non minimale è un problema aperto.*

Summary. – *We fix ℓ a prime and let M be an integer such that $\ell \nmid M$; let $f \in S_2(\Gamma_1(M\ell^2))$ be a newform supercuspidal of fixed type at ℓ and special at a finite set of primes. For an indefinite quaternion algebra over $\boldsymbol{Q}$, of discriminant dividing the level of f, there is a local quaternionic Hecke algebra $\boldsymbol{T}$ associated to f. The algebra $\boldsymbol{T}$ acts on a module M_f coming from the cohomology of a Shimura curve. Applying the Taylor-Wiles criterion and a recent Savitt's theorem, $\boldsymbol{T}$ is the universal deformation ring of a global Galois deformation problem associated to $\bar{\rho}_f$. Moreover M_f is free of rank 2 over $\boldsymbol{T}$. If f occurs at minimal level, as a consequence of our results and by the classical Ihara's lemma, we prove a theorem of raising the level and a result about congruence ideals. The extension of this results to the non minimal case is an open problem.*

Introduction.

The principal aim of this article is to study some congruence properties of modular forms studying the integer cohomology group coming from a Shimura curve. This work take place in a context of search which has its origin in the works of Wiles and Taylor-Wiles on the Shimura-Taniyama-Weil conjecture.
Conrad Diamond and Taylor [4] and Breuil Conrad Diamond and Taylor [1] allow one to hope that the congruence properties are predictable even in tha case of fixed type. This approach was follow by Terracini [20], in the case of trivial nebentypus. Our first result extends this work to a more general class of types and allows to work with modular forms having a non trivial nebentypus. Since we will

work with Galois representations which are not semistable at ℓ but only potentially semistable, we use a recent Savitt theorem [17], that prove a conjecture of Conrad, Diamond and Taylor ([4], conjecture 1.2.2 and conjecture 1.2.3), on the size of certain deformation rings parametrizing potentially Barsotti-Tate Galois representations, extending results of Breuil and Mézard (conjecture 2.3.1.1 of [2]) to the potentially crystalline case in Hodge-Tate weights $(0, 1)$.

1. – Deformation problem.

We fix a prime $\ell > 2$. Let $\boldsymbol{Z}_{\ell^2}$ denote the integer ring of $\boldsymbol{Q}_{\ell^2}$, the unramified quadratic extension of $\boldsymbol{Q}_\ell$. Let $M \neq 1$ be a square-free integer not divisible by ℓ. We fix f an eigenform in $S_2(\Gamma_1(M\ell^2))$, then $f \in S_2(\Gamma_0(M\ell^2), \psi)$ for some Dirichlet character $\psi : (\boldsymbol{Z}/M\ell^2\boldsymbol{Z})^\times \to \overline{\boldsymbol{Q}}^\times$ of order prime to ℓ. For abuse of notation, let ψ be the adelisation of the Dirichlet character ψ and we denote by ψ_p the composition of ψ with the inclusion $\boldsymbol{Q}_p^\times \to \boldsymbol{A}^\times$.
We fix a regular character $\chi : \boldsymbol{Z}_{\ell^2}^\times \to \overline{\boldsymbol{Q}}^\times$ of conductor ℓ such that $\chi|_{\boldsymbol{Z}_\ell^\times} = \psi_\ell|_{\boldsymbol{Z}_\ell^\times}$ and we extend χ to $\boldsymbol{Q}_{\ell^2}^\times$ by putting $\chi(\ell) = -\psi_\ell(\ell)$. We observe that χ is not uniquely determined by ψ and, if we fix an embedding of $\overline{\boldsymbol{Q}}$ in $\overline{\boldsymbol{Q}}_\ell$, we can ragard the values of χ in this field. We consider the type $\tau = \chi \oplus \chi^\sigma$, where σ denotes the complex conjugation.
We fix a decomposition $M = N\Delta'$ where Δ' is a product of an odd number of primes and we shoose $f \in S_2(\Gamma_1(M\ell^2))$ such that the automorphic representation associated to f is supercuspidal of type $\tau = \chi \oplus \chi^\sigma$ at ℓ and special at primes $p|\Delta'$.
Let $\rho_f : G_{\boldsymbol{Q}} \to GL_2(\overline{\boldsymbol{Q}}_\ell)$ be the Galois representation associated to f and $\overline{\rho}$ be its reduction modulo ℓ. We impose the following conditions on $\overline{\rho}$:

$$\overline{\rho} \text{ is absolutely irreducible;} \tag{1}$$

$$\text{if } p|N \text{ then } \overline{\rho}(I_p) \neq 1; \tag{2}$$

$$\text{if } p|\Delta' \text{ and } p^2 \equiv 1 \bmod \ell \text{ then } \overline{\rho}(I_p) \neq 1; \tag{3}$$

$$\mathrm{End}_{\overline{\boldsymbol{F}}_\ell[G_\ell]}(\overline{\rho}_\ell) = \overline{\boldsymbol{F}}_\ell. \tag{4}$$

$$\text{if } \ell = 3,\ \overline{\rho} \text{ is not induced from a character of } \boldsymbol{Q}(\sqrt{-3}). \tag{5}$$

Let K be a finite extension of $\boldsymbol{Q}_\ell$ containing $\boldsymbol{Q}_{\ell^2}$, $\mathrm{Im}(\psi)$ and the eigenvalues for f of all Hecke operators. Let $\mathcal{O}$ be the ring of integers of K, λ be a uniformizer of $\mathcal{O}$, $k = \mathcal{O}/(\lambda)$ be the residue field.

Let $\mathcal{B}$ denote the set of normalized newforms h in $S_2(\Gamma_0(M\ell^2), \psi)$ which are supercuspidal of type χ at ℓ and whose associated representation ρ_h is a deformation of $\overline{\rho}$. For $h \in \mathcal{B}$, let $h = \sum_{n=1}^{\infty} a_n(h)q^n$ be the q-expansion of h and let $\mathcal{O}_h$ be the $\mathcal{O}$-algebra generated in $\boldsymbol{Q}_\ell$ by the Fourier coefficients of h. Let $\boldsymbol{T}$ denote the sub-$\mathcal{O}$-algebra of $\prod_{h \in \mathcal{B}} \mathcal{O}_h$ generated by the elements $\widetilde{T}_p = (a_p(h))_{h \in \mathcal{B}}$ for $p \not| M\ell$.

1.1 – *The global deformation condition of type* (sp, τ, ψ).

We let Δ_1 be the product of primes $p|\Delta'$ such that $\overline{\rho}(I_p) \neq 1$, and Δ_2 be the product of primes $p|\Delta'$ such that $\overline{\rho}(I_p) = 1$. We define the global deformation condition of type (sp, τ, ψ):

DEFINITION 1.1. – *We consider the functor $\mathcal{F}$ which associate to a local complete noetherian $\mathcal{O}$-algebra A with residue field k, the set of strict equivalence classes of continuous homomorphisms $\rho : G_{\boldsymbol{Q}} \to GL_2(A)$ lifting $\overline{\rho}$ and satisfying the following conditions:*

a) *ρ is unramified outside $M\ell$;*
b) *if $p|\Delta_1 N$ then $\rho(I_p) \simeq \overline{\rho}(I_p)$;*
c) *if $p|\Delta_2$ then ρ_p satisfies the sp-condition, that is for a lift F of* Frob_p *in G_p*

$$\mathrm{tr}\,(\rho_p(F))^2 = (p\mu(p) + \mu(p))^2 = \psi_p(p)(p+1)^2;$$

d) *ρ_ℓ is weakly of type τ;*
e) $\det(\rho) = \varepsilon\psi$, *where $\varepsilon : G_{\boldsymbol{Q}} \to \boldsymbol{Z}_\ell^\times$ is the cyclotomic character.*

We observe that our local Galois representation $\rho_{f,\ell} = \rho_\ell$ is of type τ ([4]); let $\boldsymbol{R}^D_{\mathcal{O},\ell}$ be the local universal deformation ring associated to a local deformation problem of being weakly of type τ. Since, in dimension 2, potentially Barsotti-Tate is equivalent to potentially crystalline (ence potentially semi stable) of Hodge-Tate weight $(0,1)$ ([9], theorem C2), this allow us to apply Savitt's result ([17], theorem 6.22) thus $\mathcal{O}[[X]] \simeq \boldsymbol{R}^D_{\mathcal{O},\ell}$. Moreover, the space of deformations of $\overline{\rho}_p$ satisfying the sp-condition includes the restrictions to G_p of representations coming from forms in $S_2(\Gamma_0(N\Delta'\ell^2), \psi)$ which are special at p, but it does not contain those coming from principal forms in $S_2(\Gamma_0(N\Delta'\ell^2), \psi)$. The corresponding versal ring is $\mathcal{O}[[X,Y]]/(X,XY) = \mathcal{O}[[Y]]$. The functor $\mathcal{F}$ is representable; let $\mathcal{R}$ be the universal ring associated to $\mathcal{F}$.

2. – Shimura curves and cohomology.

We put $\Delta = \ell\Delta'$. Let B be the indefinite quaternion algebra over $\boldsymbol{Q}$ of discriminant Δ. Let R be a maximal order in B. If p is a finite place we put $B_p = B \otimes_{\boldsymbol{Q}} \boldsymbol{Q}_p$ and $R_p = R \otimes_{\boldsymbol{Z}} \boldsymbol{Z}_p$. If $p \not| \Delta$, we fix an isomorphism $i_p : B_p \to$

$M_2(\boldsymbol{Q}_p)$ such that $i_p(R_p) = M_p(\boldsymbol{Z}_p)$ and we define

$$V_0(N) = \prod_{p \nmid N} R_p^\times \times \prod_{p|N} K_p^0(N), \quad V_1(N) = \prod_{p \nmid N\ell} R_p^\times \times \prod_{p|N} K_p^1(N) \times (1 + u_\ell R_\ell)$$

where

$$K_p^0(N) = i_p^{-1}\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(\boldsymbol{Z}_p) \mid c \equiv 0 \bmod N \right\}$$

$$K_p^1(N) = i_p^{-1}\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(\boldsymbol{Z}_p) \mid c \equiv 0 \bmod N,\ a \equiv 1 \bmod N \right\}.$$

Let $\widehat{\psi} := \prod_{p|N} \psi_p \times \chi$ be a character of $V_0(N)$ with kernel $V_1(N)$. For $i = 0, 1$ we shall consider the Shimura curves $\boldsymbol{X}_i(N) = B_{\boldsymbol{Q}}^\times \setminus B_{\boldsymbol{A}}^\times / K_\infty^+ \times V_i(N)$, where $K_\infty^+ = \boldsymbol{R}^\times SO_2(\boldsymbol{R})$ and let $H^1(\boldsymbol{X}_1(N), \mathcal{O})^{\widehat{\psi}}$ be the sub-Hecke-module of $H^1(\boldsymbol{X}_1(N), \mathcal{O})$ on which the subgroup of $V_0(N)/V_1(N)$ with order prime to ℓ acts as $\widehat{\psi}$.

3. – Construction of a Taylor-Wiles system.

Let $\boldsymbol{T}_0^{\widehat{\psi}}(N)$ be the $\mathcal{O}$-algebra generated by the Hecke operators T_p, $p \neq \ell$ and the diamond operators, acting on $H^1(\boldsymbol{X}_1(N), \mathcal{O})^{\widehat{\psi}}$. By the Jacquet-Langlands correspondence, the form f determines a character $\boldsymbol{T}_0^{\widehat{\psi}}(N) \to k$. The kernel of this character is a maximal ideal $\mathfrak{m}$ in $\boldsymbol{T}_0^{\widehat{\psi}}(N)$; we define $M = H^1(\boldsymbol{X}_1(N), \mathcal{O})_{\mathfrak{m}}^{\widehat{\psi}}$. By combining proposition 4.7 of [5] with the Jacquet-Langlands correspondence, we see that there is a natural isomorphism $\boldsymbol{T} \simeq \boldsymbol{T}_0^{\widehat{\psi}}(N)_{\mathfrak{m}}$. Therefore $M \otimes_{\mathcal{O}} K$ is free of rank 2 over $\boldsymbol{T} \otimes_{\mathcal{O}} K$. Since $\mathcal{R}$ is topologically generated by the traces of $\rho^{\text{univ}}(\text{Frob}_p)$ for $p \neq \ell$, [14] §1.8, there is a surjective homomorphism of $\mathcal{O}$-algebras $\Phi : \mathcal{R} \to \boldsymbol{T}$. Applying the Taylor-Wiles criterion in the version of Diamond and Fujiwara, we prove:

THEOREM 3.1. – a) *$\mathcal{R}$ is complete intersection of dimension* 1;
b) *$\Phi : \mathcal{R} \to \boldsymbol{T}$ is an isomorphism;*
c) *M is a free $\boldsymbol{T}$-module of rank* 2.

4. – A generalization of the Conrad, Diamond and Taylor's result using Savitt's theorem.

Savitt's theorem allows to suppress the assumption of acceptability in the definition of strong acceptability. In particular, if S is a set of rational prime not dividing $N\Delta'\ell$, we consider a newform $f \in S_2(\Gamma_0(SM\ell^2), \psi)$ with nebentypus ψ, supercuspidal of type $\tau = \chi \oplus \chi^\sigma$ at ℓ and such that $\overline{\rho}_f$ satisfies the conditions (1), (2), (4) and (5) of section 1. We assume that f occurs with type τ and minimal level. We consider deformations of type (S, τ) of $\overline{\rho}$, [4], such that $\det(\rho) = \varepsilon\psi$ and we will call this deformation problem of type (S, τ, ψ). Let $\mathcal{R}_S^{\mathrm{mod},\psi}$ be classical type (S, τ, ψ) universal deformation ring and let $\boldsymbol{T}_S^{\mathrm{mod},\psi}$ be the classical Hecke algebra acting on the space of the modular forms of type (S, τ, ψ). Let M_S^{mod} be the cohomological module defined in §5.3 of [4], (the "τ-part" of the first cohomoly group of a modular curve of level depending on S) and let $M_S^{\mathrm{mod},\psi}$ be the ψ-part of M_S^{mod}. Then by Savitt's theorem and by the Ihara's lemma, $\Phi_S^{\mathrm{mod},\psi}: \mathcal{R}_S^{\mathrm{mod},\psi} \to \boldsymbol{T}_S^{\mathrm{mod},\psi}$ is a complete intersection isomorphism and $M_S^{\mathrm{mod},\psi}$ is a free $\boldsymbol{T}_S^{\mathrm{mod},\psi}$-module of rank 2. We observe that $\mathcal{R} \simeq \mathcal{R}_\emptyset^{\psi,\mathrm{mod}}$, $\boldsymbol{T} \simeq \boldsymbol{T}_\emptyset^{\psi,\mathrm{mod}}$, $M \simeq M_\emptyset^{\psi,\mathrm{mod}}$ so we find teorem 3.1. As a consequences we find the following results.

4.1 – *Raising the level.*

If f occurs with type τ and minimal level, the following result hold:

PROPOSITION 4.1. – *Let $f = \sum a_n q^n$ be a normalized newform in $S_2(\Gamma_0(M\ell^2), \psi)$ supercuspidal of type $\tau = \chi \oplus \chi^\sigma$ at ℓ, special at primes in a finite set Δ', there exist $g \in S_2(\Gamma_0(qM\ell^2), \psi)$ supercuspidal of type τ at ℓ, special at every prime $p|\Delta'$ such that $f \equiv g \bmod \lambda$ if and only if $a_q^2 \equiv \psi(q)(1+q)^2 \bmod \lambda$ where q is a prime such that $(q, M\ell^2) = 1$, $q \not\equiv -1 \bmod \ell$.*

4.2 – *Congruence ideals.*

Let g be a newform in $S_2(\Gamma_0(N\Delta_1\ell^2), \psi))$, supercuspidal of type τ at ℓ. We suppose that $\overline{\rho}$ is ramified at every prime in Δ_1.

Let Δ_2 be a finite set of primes p, not dividing $\Delta_1\ell$ such that $p^2 \not\equiv 1 \bmod \ell$ and $\mathrm{tr}\,(\overline{\rho}(\mathrm{Frob}_p))^2 \equiv \psi(p)(p+1)^2 \bmod \ell$. We let $\mathcal{B}_{\Delta_2}$ denote the set of newforms h of weight 2, character ψ and level dividing $N\Delta_1\Delta_2\ell$ which are special at Δ_1, supercuspidal of type χ at ℓ and such that $\overline{\rho}_h = \overline{\rho}$. We choose an ℓ-adic ring $\mathcal{O}$ with residue field k, sufficiently large, so that every representation ρ_h for $h \in \mathcal{B}_{\Delta_2}$ is defined over $\mathcal{O}$ and $Im(\psi) \subseteq \mathcal{O}$. For every pair of disjoint subset S_1, S_2 of Δ_2 we denote by $\mathcal{R}_{S_1,S_2}$ the universal solution over $\mathcal{O}$ for the deformation problem of $\overline{\rho}$ consisting of the deformations ρ satisfying conditions $b), d), e)$ of definition 1.1 and

a) ρ is unramified outside $N\Delta_1 S_1 S_2 \ell$;

c) if $p|S_2$ then ρ_p satisfies the sp-condition.

Let $\mathcal{B}_{S_1,S_2}$ be the set of newforms in $\mathcal{B}_{\Delta_2}$ of level dividing $N\Delta_1 S_1 S_2 \ell$ which are special at S_2 and let $\boldsymbol{T}_{S_1,S_2}$ be the sub-$\mathcal{O}$-algebra of $\prod_{h \in \mathcal{B}_{S_1,S_2}} \mathcal{O}$ generated by the elements $\widetilde{T}_p = (a(h))_{h \in \mathcal{B}_{S_1,S_2}}$ for p not in $\Delta_1 \cup S_1 \cup S_2 \cup \{\ell\}$. As a consequence of the generalization of Conrad, Diamond and Taylor's result, we have that $\mathcal{R}_{S_1,\emptyset} \to \boldsymbol{T}_{S_1,\emptyset}$ is an isomorphism of complete intersections, for any subset S_1 of Δ_2.

If $\Delta_1 \neq 1$ then each $\boldsymbol{T}_{\emptyset,S_2}$ acts on a local component of the cohomology of a suitable Shimura curve, obtained by taking an indefinite quaternion algebra of discriminant $S_2\ell$ or $S_2\ell p$ for a prime p in Δ_1. Let η_{h,S_1,S_2} be the congruence ideal of h relatively to $\mathcal{B}_{S_1,S_2}$; we know that η_{h,S_1,S_2} controls congruences between h and linear combinations of forms different from h in $\mathcal{B}_{S_1,S_2}$. Theorem 3.1 gives the following:

THEOREM 4.2. – *Suppose $\Delta_1 \neq 1$ and Δ_2 as above. Then*

a) $\mathcal{B}_{\emptyset,\Delta_2} \neq 0$;

b) *for every subset $S \subseteq \Delta_2$, the map $\mathcal{R}_{S,\Delta_2/S} \to \boldsymbol{T}_{S,\Delta_2/S}$ is an isomorphism of complete intersection;*

c) *for every $h \in \mathcal{B}_{\emptyset,\Delta_2}$, $\eta_{h,S,\Delta_2/S} = (y_S(h))\eta_{h,\emptyset,\Delta_2}$ where $y_S(h)$ is a well defined element of $\mathcal{O}$ coming from the deformation problem.*

5. – Problem: extension of results to the non minimal case.

Let S be a finite set of primes not dividing $M\ell$; we fix $f \in S_2(\Gamma_0(N\Delta'\ell^2 S), \psi)$ supercuspidal of type τ at ℓ, special at primes $p|\Delta'$. If we modify our Galois deformation problem allowing ramification at primes in S, we obtain a new universal deformation ring $\mathcal{R}_S$ and a new Hecke algebra acting on the newforms giving rise to such representation. We make the following conjecture:

CONJECTURE 5.1. – • *$\mathcal{R}_S \to \boldsymbol{T}_S$ is an isomorphism of complete intersection;*
• *let M_S be the module $H^1(\boldsymbol{X}_1(NS), \mathcal{O})^{\widehat{\psi}}_{\mathfrak{m}_S}$ coming from the cohomology of the Shimura curve $\boldsymbol{X}_1(NS)$ associated to the open compact subgroup of $B^{\times,\infty}_{\boldsymbol{A}}$, $V_1(NS) = \prod_{p \nmid NS\ell} R_p^\times \prod_{p|NS} K_p^1(N) \times (1 + u_\ell R_\ell)$ where $K_p^1(N)$ is defined in section 2, and u_ℓ is a uniformizer of $B_\ell^\times$. M_S is a free $\boldsymbol{T}_S$-module of rank 2.*

Conjecture 5.1 easily follows from the following conjecture:

CONJECTURE 5.2. – *Let q be a prime number such that $q \nmid N\Delta'\ell^2$. We fix a maximal non Eisenstein ideal of the Hecke algebra $\widehat{\boldsymbol{T}^{\psi}_0}(N)$ acting on the group $H^1(\boldsymbol{X}_1(N), \mathcal{O})^{\widehat{\psi}}$. Let $\boldsymbol{X}_1(N)$ be the Shimura curve $\boldsymbol{X}_1(N) = B^\times \setminus B^\times_{\boldsymbol{A}}/K^+_\infty V_1(N)$ where $V_1(N) = \prod_{p \nmid N\ell} R_p^\times \prod_{p|N} K_p^1(N) \times (1 + u_\ell R_\ell)$ where $K_p^1(N)$ is defined in section 2, and u_ℓ is a uniformizer of $B_\ell^\times$. The map*

$$a_{\mathfrak{m}} : H^1(\boldsymbol{X}_1(N), \mathcal{O})^{\widehat{\psi}}_{\mathfrak{m}} \times H^1(\boldsymbol{X}_1(N), \mathcal{O})^{\widehat{\psi}}_{\mathfrak{m}} \to H^1(\boldsymbol{X}_1(N_q), \mathcal{O})^{\widehat{\psi}}_{\mathfrak{m}^q}$$

is such that $a \otimes_{\mathcal{O}} k$ is injective, where $\mathfrak{m}^q$ is the inverse image of the ideal $\mathfrak{m}$ under the natural map $\widehat{\boldsymbol{T}_0^{\psi}}(Nq) \to \widehat{\boldsymbol{T}_0^{\psi}}(N)$ and $k = \mathcal{O}/\lambda$.

This conjecture would provide an analogue for the Shimura curves of the Ihara's lemma in case $\ell | \Delta$. In [6] and in [7], Diamond and Taylor show that if ℓ not divides the discriminant of the indefinite quaternion algebra, then the analogue of conjecture 5.2 holds.

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