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Uniform Kadec-Klee Property and Nearly Uniform Convexity in Köthe-Bochner Sequence Spaces.

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Sunto. – *Viene studiata la proprietà uniforme di Kadec-Klee in spazi sequenziali di Kothe-Bochner $E(X)$, dove E è uno spazio sequenziale di Kothe e X è un arbitrario spazio di Banach separabile. Precisamente, viene esaminato il problema se questa proprietà geometrica si può trasportare da X in $E(X)$. Ciò viene stabilito in contrasto con il caso in cui E è uno spazio di Kothe. Come corollario viene stabilito un criterio affichè $E(X)$ sia «nearly» uniformemente convesso.*

Summary. – *The uniformly Kadec-Klee property in Köthe-Bochner sequence spaces $E(X)$, where E is a Köthe sequence space and X is an arbitrary separable Banach space, is studied. Namely, the question of whether or not this geometric property lifts from X and E to $E(X)$ is examined. It is settled affirmatively in contrast to the case when E is a Köthe function space. As a corollary we get criteria for $E(X)$ to be nearly uniformly convex.*

1. – Introduction.

Köthe-Bochner spaces of vector valued functions $E(X)$ are generalizations of Lebesgue-Bochner and Orlicz-Bochner spaces. They have been investigated by many authors (see for example [3], [4], [5], [10], [13], [14] , [19], [20] and [21]). One of the fundamental problems in these spaces is the question of whether or not a geometric property lifts from X and E to $E(X)$. Although the answer to such a question is often expected, the proof of such a response is usually nontrivial. A survey of geometry in Köthe-Bochner spaces can be found in [16].

The property (**H**) is also known as the Radon-Riesz ([21]) or the Kadec-Klee property (**KK**) ([11]). Huff in [11] introduced two successively stronger notions and he called them nearly uniform convexity (**NUC**) and uniform Kadec-Klee property (**UKK**). He proved that a Banach space is nearly uniformly convex iff it has uniform Kadec-Klee property and it is reflexive. It is also known that nearly uniformly convex Banach space has the fixed point property of nonexpansive mappings.

The property (**H**) in Köthe-Bochner spaces was studied in [3], [5] and [14]. We consider uniform Kadec-Klee property in Köthe-Bochner sequence spaces. As far as we know, the earliest result concerning that subject is due to J. R. Partington [19]. He proved that Lebesgue-Bochner sequence space $l_p(X)$ for $1 \leqslant p < \infty$ is (**UKK**) if X is (**UKK**). Moreover Theorem 3.1 in [1] gives a result for general measure spaces that recovers Partington's theorem as a corollary.

First we prove a characterization of property (**UKK**) in an arbitrary Banach space. Basing on this we show that if X is a separable Banach space without the Schur property and E is a Köthe sequence space, then $E(X)$ is (**UKK**) iff X is (**UKK**) and E is uniformly monotone. Moreover, if X has the Schur property and E is uniformly monotone, then $E(X)$ is (**UKK**). Notice that our results are essentially stronger than that from Partington's paper [19] ($\rightarrow$if E is uniformly convex Köthe sequence spaces and X is (**UKK**), then $E(X)$ is (**UKK**)).

From our main result we also conclude that in Köthe sequence space the uniform monotonicity is stronger that property (**UKK**). Moreover we give an example of non symmetric Köthe sequence space which has uniform Kadec-Klee property and is not uniformly monotone. It corresponds to the result of Sukochev (Theorem 2 in [22]) from which among others implies that, in symmetric sequence spaces E with the shrinking basis, the uniform monotonicity and property (**UKK**) coincide.

As a corollary we also get that, if X is an infinite dimensional Banach space, then $E(X)$ is nearly uniformly convex iff both E and X possess the same property and E is uniformly monotone. Furthermore, if X is a finite dimensional Banach space, then $E(X)$ is nearly uniformly convex iff E is nearly uniformly convex. The same results have been obtained by D. Kutzarova and T. Landes in [15]. They consider (**NUC**) property in the substitution space $E(\chi)$ of family $\chi = (X_\omega)_{\omega \in \Omega}$ of Banach spaces, which in particular case, when all spaces X_ω are the same Banach space X, gets the Köthe-Bochner sequence space $E(X)$.

Denote by $\mathcal{N}$, $\mathcal{R}$ and $\mathcal{R}_+$ the sets of natural, real and non-negative real numbers, respectively. Let $(\mathcal{N}, 2^{\mathcal{N}}, m)$ be the counting measure space. By $l^0 = l^0(m)$ we denote the linear space of all real sequences.

Let $E = (E, \leqslant, \|\cdot\|_E)$ be a Banach sequence lattice over the measure space $(\mathcal{N}, 2^{\mathcal{N}}, m)$ (Köthe sequence space), where $\leqslant$ is semi-order relation in the space l^0 and $(E, \|\cdot\|_E)$ is a Banach sequence space, i.e. E is linear subspace of l^0, norm $\|\cdot\|_E$ is complete in E and the following two conditions are satisfied:

(*i*) if $x \in E$, $y \in l^0$, $|y| \leqslant |x|$, i.e. $|y(i)| \leqslant |x(i)|$ for every $i \in \mathcal{N}$, then $y \in E$ and $\|y\|_E \leqslant \|x\|_E$,

(*ii*) there exists a sequence x in E that is positive on whole $\mathcal{N}$ (see [12] and [18]).

Denote by E_+, l^0_+ the positive cone of E, l^0 respectively, i.e. $l^0_+ = \{x \in l^0 \colon x \geqslant 0\}$.

E is said to be strictly monotone ($E \in \mathbf{SM}$) if for every $0 \leqslant y \leqslant x$ with $y \neq x$ we have $\|y\|_E < \|x\|_E$. We say that a Banach lattice E is *uniformly monotone* ($E \in \mathbf{UM}$) if for every $q \in (0, 1)$ there exists $p \in (0, 1)$ such that for all $0 \leqslant y \leqslant x$ satisfying $\|x\|_E = 1$ and $\|y\|_E \geqslant q$ we have $\|x - y\|_E \leqslant 1 - p$ (see [9]). Then the modulus $p(\cdot)$ of the uniform monotonicity of E is defined as follows

$$p(q) = \inf\{1 - \|x - y\|_E \colon \|x\|_E = 1,\ \|y\|_E \geqslant q,\ 0 \leqslant y \leqslant x\}.$$

A Banach lattice E is called *order continuous* ($E \in \mathbf{OC}$) if for every $x \in E$ and every sequence $(x_m) \in E$ such that $0 \leftarrow x_m \leqslant |x|$ we have $\|x_m\|_E \to 0$ (see [12] and [18]).

Recall that E satisfies the *Fatou property* ($E \in \mathbf{FP}$) if $x \in l^0$ and $(x_m) \in E$ are such that $0 \leqslant x_m \nearrow x$ and $\sup_m \|x_m\|_E < \infty$, then $x \in E$ and $\|x\|_E = \lim_{m \to \infty} \|x_m\|_E$ (see [2], [12] and [18]).

Let $(X, \|\cdot\|_X)$ be a real Banach space, $B(X)$ and $S(X)$ be the closed unit ball, unit sphere of X, respectively. For any subset A of X, we denote by $\operatorname{conv}(A)$ the convex hull of A. The symbol $x_n \xrightarrow{w} x$ denotes that x_n converges to x weakly in X.

We say that a sequence $\{x_n\} \subset X$ is an *ε-separated* for some $\varepsilon > 0$ if

$$\operatorname{sep}\{x_n\}_X = \inf\{\|x_n - x_m\|_X \colon n \neq m\} > \varepsilon .$$

We say that X has the *Kadec-Klee property* provided on the unit sphere sequences converge in norm whenever they converge weakly. Huff in [11] presented an equivalent formulation: X has the *Kadec-Klee property* if

$$(\mathbf{KK})\colon \quad \begin{matrix} (x_n) \subset B(X) \\ x_n \xrightarrow{w} x \\ \operatorname{sep}\{x_n\}_X > 0 \end{matrix} \quad \Rightarrow \ \|x\|_X < 1 .$$

A Banach space X is called to have *uniform Kadec-Klee property* ($X \in$ (**UKK**) for short) if for every $\varepsilon > 0$ there exists $\delta \in (0, 1)$ such that

$$(\mathbf{UKK})\colon \quad \begin{matrix} (x_n) \subset B(X) \\ x_n \xrightarrow{w} x \\ \operatorname{sep}\{x_n\}_X \geqslant \varepsilon \end{matrix} \quad \Rightarrow \ \|x\|_X < 1 - \delta . \tag{1}$$

Recall that a Banach space X has the Schur property (write $X \in (\mathbf{SP})$ for short) if every weakly null sequence is norm null. Every Schur space is $(\mathbf{UKK})$ and the converse is not true ([11]).

A Banach space is said to be *nearly uniformly convex* (write $X \in (\mathbf{NUC})$) if for every $\varepsilon > 0$ there exists $\delta \in (0, 1)$ such that for every sequence $\{x_n\} \subseteq B(X)$ with $\operatorname{sep}\{x_n\}_X \geqslant \varepsilon$ we have

$$\operatorname{conv}(\{x_n\}) \cap (1-\delta) B(X) \neq \phi .$$

For any Banach space we have $(\mathbf{NUC}) \Rightarrow (\mathbf{UKK}) \Rightarrow (\mathbf{KK})$. Moreover $X \in (\mathbf{NUC})$ iff $X \in (\mathbf{UKK})$ and X is reflexive ([11]).

Now, let us define the type of spaces to be considered in this paper. For a real Banach space $\langle X, \|\cdot\|_X \rangle$, denote by $\mathcal{M}(\mathcal{N}, X)$, or just by $\mathcal{M}(X)$, the space of sequences $x = (x(i))_{i=1}^{\infty}$ such that $x(i) \in X$ for all $i \in \mathcal{N}$. Define

$$E(X) = \{x \in \mathcal{M}(X) \colon \|x(\cdot)\|_X \in E\} .$$

Then $E(X)$ becomes to be a Banach space with the norm

$$\|x\| = \| \|x(\cdot)\|_X \|_E$$

and it is called a *Köthe-Bochner sequence space.*

2. – Auxiliary lemmas.

Lemma 1. – *If $x, y \in X \setminus \{0\}$, then*

$$\|x+y\|_X \leqslant \frac{1}{2}\|\widehat{x}+\widehat{y}\|_X(\|x\|_X + \|y\|_X) + \left(1 - \frac{1}{2}\|\widehat{x}+\widehat{y}\|_X\right) | \|x\|_X - \|y\|_X | ,$$

where $\widehat{x} = x/\|x\|_X$ (Lemma 1.1 in [10]).

Lemma 2. – *Let X be a separable Banach space and E be an order continuous sequence Köthe space. If $f_n, f \in E(X)$ and $f_n \xrightarrow{w} f$ in $E(X)$, then $f_n(i) \xrightarrow{w} f(i)$ in X for every $i \in \mathcal{N}$ (Lemma 1 in [14]).*

Lemma 3. – *Let E be any Banach sequence lattice. Then $E \in (\mathbf{UM})$ iff for every $\alpha \in (0, 1)$ there is $\eta(\alpha) \in (0, 1)$ such that for any $x \in E_+$ with $\|x\|_E = 1$ and for any $A \in 2^{\mathcal{N}}$ such that $\|x\chi_A\|_E \geqslant \alpha$ there holds $\|x\chi_{\mathcal{N}\setminus A}\|_E \leqslant 1 - \eta$ (Theorem 7 in [9], see also Lemma 6 in [15]).*

3. – Results.

THEOREM 1. – *Let X be a Banach space. Then $X \in$ (**UKK**) iff for every $\eta > 0$ there exists $\sigma \in (0, 1)$ such that*

$$\left.\begin{array}{l}(x_n) \subset X\backslash\{0\} \\ x_n \xrightarrow{w} x \in X\backslash\{0\} \\ \operatorname{sep}\{x_n/\|x_n\|_X\} \geqslant \eta\end{array}\right\} \rightarrow \|x\|_X < (1-\sigma) \liminf_{n \rightarrow \infty} \|x_n\|_X. \tag{2}$$

PROOF OF NECESSITY. – Take $\eta > 0$. Let the sequence (x_n) in $X\backslash\{0\}$ be such that $x_n \xrightarrow{w} x \in X\backslash\{0\}$ and $\operatorname{sep}\{x_n/\|x_n\|_X\} \geqslant \eta$. By the weak convergence of (x_n) to x we get $a = \liminf_{n\rightarrow\infty}\|x_n\|_X < \infty$. Hence, passing to a subsequence, if necessary, we may assume that $\|x_n\|_X \rightarrow a$. Define $y = x/a$. and $y_n = x_n/\|x_n\|_X$. Then $y_n \in B(X)$ and $\operatorname{sep}\{y_n\} \geqslant \eta$. We claim that $y_n \xrightarrow{w} y$. Indeed, by the lower semicontinuity of the norm with respect to the weak topology, we conclude that $a \geqslant \|x\|_X > 0$. Consequently, for every $x^* \in X^*$, we get

$$|x^*(y_n - y)| =$$

$$\left| x^*\left(\frac{x_n}{\|x_n\|_X} - \frac{x}{a}\right)\right| \leqslant \left| x^*\left(\frac{x_n}{\|x_n\|_X} - \frac{x}{\|x_n\|_X}\right)\right| + \left| x^*\left(\frac{x}{\|x_n\|_X} - \frac{x}{a}\right)\right| =$$

$$\frac{1}{\|x_n\|_X}|x^*(x_n - x)| + \left|\frac{1}{\|x_n\|_X} - \frac{1}{a}\right| |x^*(x)| \rightarrow 0\,.$$

Take the number $\sigma = \delta(\eta)$ from (1). Then $\|x/a\| = \|y\| < 1 - \sigma$.

PROOF OF SUFFICIENCY. – Let $\varepsilon > 0$. Take a sequence (x_n) in $B(X)$ with $\operatorname{sep}\{x_n\} \geqslant \varepsilon$. Assume that $x_n \xrightarrow{w} x$. Passing to subsequence, if necessary, we may assume that $\|x_n\|_X \rightarrow b$, $b \in [\varepsilon/2, 1]$ and $\|x_n\|_X \geqslant \varepsilon/4$ for every $n \in \mathcal{N}$. Then, applying Lemma 1, we conclude that there exist a number $\eta = \eta(\varepsilon) > 0$ and a subsequence $(x_{n_j})_{j=1}^{\infty} \subset (x_n)_{n=1}^{\infty}$ such that $\operatorname{sep}\{x_{n_j}/\|x_{n_j}\|_X\} \geqslant \eta$. Taking $\delta = \sigma(\eta)$ from (2), we can finish the proof.

THEOREM 2. – *Let E be a Köthe sequence space and X be a separable Banach space.*

(*i*) *If $X \notin$ (**SP**), then $E(X) \in$ (**UKK**) iff $X \in$ (**UKK**) and $E \in$ (**UM**).*

(*ii*) *If $X \in$ (**SP**), $E \in$ (**UM**), then $E(X) \in$ (**UKK**).*

(*i*). PROOF OF NECESSITY. – Since E and X are embedded isometrically into $E(X)$ and the property (**UKK**) is inherited by subspaces, E and X have the (**UKK**) property. Since $E \in$ (**UKK**), so $E \in$ (**OC**) ([6]). Moreover, if Banach space X does not have the Schur property, then there exists a sequence $(x_n)_{n=1}^{\infty} \subset S(X)$ such that $x_n \to 0$ weakly in X. Then, applying Hahn-Banach theorem, it is easy to prove that for every $a \in (0, 1)$ there exists a subsequence $(y_n)_{n=1}^{\infty} \subset (x_n)_{n=1}^{\infty}$ such that $\operatorname{sep}\{y_n\}_X \geq a$. Using this observation it is easy to show that $E \in$ (**UM**) the same way as in the proof of Theorem 1 (the implication (1) $\Rightarrow$ (2)) in [15].

PROOF OF SUFFICIENCY. – Assume that $X \in$ (**UKK**) and $E \in$ (**UM**). Let $\varepsilon > 0$. In view of Theorem 1, in order to prove that $E(X) \in$ (**UKK**), it is enough to consider the elements f_n from the unit sphere of $E(X)$. Suppose that the sequence $(f_n) \subset S(E(X))$ is such that $\operatorname{sep}\{f_n\}_{E(X)} \geq \varepsilon$ and $f_n \xrightarrow{w} f$ in $E(X)$. Since (**UM**) $\Rightarrow$ (**OC**) in any Banach function lattice, in view of Lemma 2, we conclude that $f_n(i) \xrightarrow{w} f(i)$ in X for every $i \in \mathcal{N}$. Consequently for every $i \in \mathcal{N}$ the sequence $(\|f_n(i)\|_X)_{n=1}^{\infty}$ is bounded, so it contains a convergence subsequence. Using well known diagonal method, we can find a subsequence $(f_{n_k})_{k=1}^{\infty} \subset (f_n)_{n=1}^{\infty}$ and a sequence $g \in l_+^0$ such that

$$\|f_{n_k}(i)\|_X \xrightarrow{k \to \infty} g(i) \text{ for every } i \in \mathcal{N} . \tag{3}$$

Denote still this subsequence by $(f_n)_{n=1}^{\infty}$. Notice that uniform monotonicity of E implies that E has the Fatou property (see [2]). Hence $g \in E$ and $\|g\|_E \leq 1$. Take $\eta = \eta(\varepsilon/8)$ from Lemma 3. Let the number $\gamma \in \mathcal{R}$ satisfy

$$0 < \gamma < \eta/3 . \tag{4}$$

We consider two cases:

I. Suppose that $\|g\|_E \leq 1 + 3\gamma - \eta$. By the lower semicontinuity of the norm with respect to the weak topology, we conclude that $\|f(i)\|_X \leq \liminf \|f_n(i)\|_X$ for every $i \in \mathcal{N}$. Hence $\|f(i)\|_X \leq g(i)$ for every $i \in \mathcal{N}$. Then $\|f\| \leq 1 - p_1$, where $p_1 = \eta - 3\gamma$.

II. Let

$$\|g\|_E > 1 + 3\gamma - \eta . \tag{5}$$

Since $\|f(\cdot)\|_X \leq g(\cdot)$, so $\operatorname{supp}\|f(\cdot)\|_X \subset \operatorname{supp} g(\cdot)$. Define the sets

$$A = \left\{ i \in \operatorname{supp} g : \frac{\|f(i)\|_X}{g(i)} < \frac{1}{2} \right\} \text{ and } B = \left\{ i \in \operatorname{supp} g : \frac{\|f(i)\|_X}{g(i)} \geq \frac{1}{2} \right\}.$$

Obviously $A \cup B = \operatorname{supp} g$ and $A \cap B = \emptyset$. We divide the proof into two parts.

1. Assume that $\|g\chi_A\|_E \geq \gamma$. Notice that $\|f(\cdot)\|_X \leq g(\cdot) - \frac{1}{2} g(\cdot)\chi_A$. It is easy to see that if $E \in$ (**UM**), then for each $0 < q < b$ and $x, y \in E_+$ such that $y \leq x$, $\|x\|_E \leq b$ and $\|y\|_E \geq q$ there holds $\|x - y\|_E \leq \|x\|_E(1 - p(q/b))$, where $p(\cdot)$ is the modulus of uniform monotonicity of E given in the definition (see the proof of Theorem 7 in [9]). Hence we get $\|f\| \leqslant 1 - p_2$, where $p_2 = p(\gamma/2)$.

2. Suppose that $\|g\chi_A\|_E < \gamma$. Then $\|g\chi_B\|_E \geqslant \|g\|_E - \gamma$. Denote $B = \{i_1, i_2, \dots\}$ and $B_k = \{i_1, i_2, \dots, i_k\}$. Let $g_k = g\chi_{B \cap B_k}$. Then $0 \leqslant g_k \nearrow g\chi_B$ and $\sup_k \|g_k\|_E < \infty$. By the Fatou property of E we get $\|g\chi_B\|_E = \lim_{k\to\infty} \|g_k\|_E$. Consequently we may assume that $\operatorname{card} B < \infty$ and

$$(6) \qquad \|g\chi_B\|_E \geqslant \|g\|_E - 2\gamma\,.$$

By (3) we get $\|f_n(\cdot)\|_X \chi_B \xrightarrow{n\to\infty} g(\cdot)\,\chi_B$. Applying again the Fatou property we obtain $\|g\chi_B\|_E \leqslant \liminf_{n\to\infty} \| \, \|f_n(\cdot)\|_X \chi_B\|_E$. Hence, passing to a subsequence, if necessary, we may assume that

$$(7) \qquad \|\,\|f_n(\cdot)\|_X \chi_B\|_E \geqslant \|g\|_E - 3\gamma$$

for every $n \in \mathcal{N}$. We claim that

$$(8) \qquad \|\,\|f_n(\cdot)\|_X \chi_{\mathcal{N}\setminus B}\|_E < \varepsilon/8$$

for every $n \in \mathcal{N}$. If not, then $\|\,\|f_n(\cdot)\|_X \chi_{\mathcal{N}\setminus B}\|_E \geqslant \varepsilon/8$ for some $n \in \mathcal{N}$. By Lemma 3 we conclude $\|\,\|f_n(\cdot)\|_X \chi_B\|_E \leqslant 1 - \eta(\varepsilon/8)$. Consequently, in view of (5) and (7), we get a contradiction, so (8) is true. We will show that

$$(9) \qquad \operatorname{sep}\{f_n \chi_B\}_{E(X)} \geqslant \varepsilon/8\,.$$

Otherwise, in view of (8) and the triangle inequality, for some $n \neq m$, we would get

$$\varepsilon \leqslant \|f_n - f_m\| \leqslant \|(f_n - f_m)\,\chi_B\| + \|(f_n - f_m)\,\chi_{\mathcal{N}\setminus B}\| \leqslant \frac{3\varepsilon}{8}\,.$$

This contradiction proves that (9) is true. Take $\lambda \in \mathcal{R}$ such that

$$(10) \qquad 0 < \lambda < \varepsilon/16\,.$$

Then, in view of (9) and (10), it is easy to see that for every $n \neq m$ there exists $i_0 \in B$ satisfying $\|f_n(i_0) - f_m(i_0)\|_X \geqslant \lambda\|f(i_0)\|_X$. Moreover we will prove that the following condition holds:

(+) there exist a subset $B_0 \subset B$ and a subsequence $(z_n) \subset (f_n)$ such that

$$\|z_n(i) - z_m(i)\|_X \geqslant \lambda\|f(i)\|_X$$

for all $n \neq m$, $i \in B_0$ and

$$\|z_n(i) - z_m(i)\|_X < \lambda\|f(i)\|_X$$

for every $n \neq m$ and $i \in B \setminus B_0$.

Denote by F_B the family of all nonempty subsets of the set B. We have $\operatorname{card} B < \infty$, so $\operatorname{card} F_B < \infty$.

1. Consider the element f_1 and the sequence $(f_n)_{n=2}^{\infty}$. Then there exist a subsequence $(f_n^{(1)})_{n=1}^{\infty} \subset (f_n)_{n=2}^{\infty}$ and a subset $B_1 \in F_B$, such that

$$\|f_1(i) - f_n^{(1)}(i)\|_X \geqslant \lambda \|f(i)\|_X$$

for every $n \in \mathcal{N}$, $i \in B_1$ and

$$\|f_1(i) - f_n^{(1)}(i)\|_X < \lambda \|f(i)\|_X$$

for every $i \in B \backslash B_1$ and $n \in \mathcal{N}$. Denote $h_1^{(1)} = f_1$ and $h_{n+1}^{(1)} = f_n^{(1)}$ for every $n \in \mathcal{N}$.

2. Consider the element $f_1^{(1)}$ and the sequence $(f_n^{(1)})_{n=2}^{\infty}$. Then there exist a subsequence $(f_n^{(2)})_{n=1}^{\infty} \subset (f_n^{(1)})_{n=2}^{\infty}$ and a subset $B_2 \in F_B$ such that

$$\|f_1^{(1)}(i) - f_n^{(2)}(i)\|_X \geqslant \lambda \|f(i)\|_X$$

for every $n \in \mathcal{N}$, $i \in B_2$ and

$$\|f_1^{(1)}(i) - f_n^{(2)}(i)\|_X < \lambda \|f(i)\|_X$$

for every $i \in B \backslash B_2$ and $n \in \mathcal{N}$. Denote $h_1^{(2)} = f_1^{(1)}$ and $h_{n+1}^{(2)} = f_n^{(2)}$ for every $n \in \mathcal{N}$. Since $\operatorname{card} F_B < \infty$, so taking the next steps analogously analogously we conclude that there exist a set $B_0 \in F_B$, the sequence $(j_k)_{k=1}^{\infty}$ of natural numbers and the sequence of subsequences $(h_n^{(j_k)})_{n=1}^{\infty}$, $k = 1, 2, \ldots$ such that

$$(h_n^{(j_1)})_{n=1}^{\infty} \supset (h_n^{(j_2)})_{n=1}^{\infty} \supset \ldots$$

and for every $k \in \mathcal{N}$ we get

$$\|h_1^{(j_k)}(i) - h_n^{(j_k)}(i)\|_X \geqslant \lambda \|f(i)\|_X$$

for every $n \in \mathcal{N}$, $n \geqslant 2$, $i \in B_0$ and

$$\|h_1^{(j_k)}(i) - h_n^{(j_k)}(i)\|_X < \lambda \|f(i)\|_X$$

for every $n \in \mathcal{N}$, $n \geqslant 2$, $i \in B \backslash B_0$. Define $z_n = h_1^{(j_n)}$ for every $n \in \mathcal{N}$. In such a way we have constructed the sequence $(z_n)_{n=1}^{\infty}$ satisfying the condition $(+)$. Denote still this subsequence by $(f_n)_{n=1}^{\infty}$. Furthermore we will prove that

$$\|f_n \chi_{B_0}\| \geqslant \varepsilon/32 \tag{11}$$

for every $n \in \mathcal{N}$ except at most two elements. Suppose conversely that $\|(f_n)\chi_{B_0}\| < \varepsilon/32$ for $n \in \{n_1, n_2\}$. By condition $(+)$ we obtain $\|f_{n_1}(i) - f_{n_2}(i)\|_X <$

$\lambda\|f(i)\|_X$ for every $i \in B\backslash B_0$. Hence, by (9) and (10), we get

$$\frac{\varepsilon}{8} \leqslant \|(f_{n_1}-f_{n_2})\chi_B\| \leqslant \|(f_{n_1}-f_{n_2})\chi_{B_0}\| + \|(f_{n_1}-f_{n_2})\chi_{B\backslash B_0}\| <$$

$$\|f_{n_1}\chi_{B_0}\| + \|f_{n_2}\chi_{B_0}\| + \lambda < \frac{\varepsilon}{8},$$

which is a contradiction. Moreover, we will show that

$$\|g\chi_{B_0}\| \geqslant \varepsilon/64\,. \tag{12}$$

Take $a \in E$ such that $a(i) > 0$ for every $i \in \mathcal{N}$ and $\|a\|_E < \varepsilon/64$. Let $l = \operatorname{card} B_0$. Denote $B_0 = \{i_1, i_2, \ldots, i_l\}$. In view of (3), for every $j = 1, 2, \ldots, l$ there exists number $N_j \in \mathcal{N}$ such that $|\|f_n(i_j)\|_X - g(i_j)| < a(i_j)$ for every $n \geqslant N_j$. Denote $N_0 = \max\limits_{1 \leqslant i \leqslant l}\{N_i\}$. Then

$$\|\|f_n(\cdot)\|_X\chi_{B_0} - g(\cdot)\chi_{B_0}\|_E < \varepsilon/64$$

for every $n \geqslant N_0$. Consequently, by (11), we conclude that (12) is true.

Note that $\|f(i)\|_X > 0$ for every $i \in B$. For every $i \in B_0$ define the sequence

$$(h_n(i))_{n=1}^{\infty} = (f_n(i)/\|f(i)\|_X)_{n=1}^{\infty} \subset X\,.$$

By condition $(+)$ we conclude that for every $i \in B_0$ we have $\operatorname{sep}\{h_n(i)_X\} \geqslant \lambda$. Let $i_1 \in B_0$. Then, in view of the definition of the set B, we get that

$$\lim_{n\to\infty}\|h_n(i_1)\|_X = \frac{g(i_1)}{\|f(i_1)\|_X} = h_1 \in [1, 2].$$

Furthermore, applying Lemma 1, we conclude that there exist a number $\lambda_1 = \lambda_1(\lambda, h_1)$ and a subsequence $(h_{n_k})_{k=1}^{\infty} \subset (h_n)_{n=1}^{\infty}$ such that

$$\operatorname{sep}\{h_{n_k}(i_1)/\|h_{n_k}(i_1)\|_X\}_X \geqslant \lambda_1.$$

Moreover the function $\lambda_1(\lambda, \cdot)$ is nonincreasing and $\lambda_1(u, v) > 0$ for every $u, v > 0$. Let $\lambda_0 = \lambda_1(\lambda, 2)$. Then

$$\operatorname{sep}\{h_{n_k}(i_1)\|h_{n_k}(i_1)\|_X\}_X \geqslant \lambda_0.$$

Take $i_2 \in B_0$ and consider the sequence $(h_{n_k}(i_2))_{k=1}^{\infty}$. Similarly we deduce that there exists a subsequence $(h_{n_{k_j}})_{j=1}^{\infty} \subset (h_{n_k})_{k=1}^{\infty}$ such that

$$\operatorname{sep}\{h_{n_{k_j}}(i_2)/\|h_{n_{k_j}}(i_2)\|_X\}_X \geqslant \lambda_0.$$

In such a way we can find a sequence $(v_n)_{n=1}^{\infty} \subset (h_n)_{n=1}^{\infty}$ satisfying

$$\operatorname{sep}\{v_n(i)/\|v_n(i)\|_X\}_X \geqslant \lambda_0$$

for every $i \in B_0$. Denote still this subsequence by $(h_n)_{n=1}^{\infty}$. But

$$\operatorname{sep}\{h_n(i)/\|h_n(i)\|_X\}_X = \operatorname{sep}\{f_n(i)/\|f_n(i)\|_X\}_X.$$

Basing on Theorem 1 take a number $\sigma = \sigma(\lambda_0)$. Then

$$\|f(i)\|_X < g(i)(1-\sigma)$$

for every $i \in B_0$. Then $\|f(\cdot)\|_X \leq g(\cdot) - \sigma g(\cdot)\chi_{B_0}$. By (12) we have $\|\sigma g(\cdot)\chi_{B_0}\|_E \geq \varepsilon\sigma/64$. Finally $\|f\| \leq 1 - p_3$, where $p_3 = p(\varepsilon\sigma/64)$ and $p(\cdot)$ is the modulus of uniform monotonicity of E.

(ii). Suppose that $X \in$ (**SP**), $E \in$ (**UM**). Let $\varepsilon > 0$. We will show that $E(X) \in$ (**UKK**). Take a sequence $(f_n) \subset S(E(X))$ such that $\operatorname{sep}\{f_n\}_{E(X)} \geq \varepsilon$ and $f_n \xrightarrow{w} f$ in $E(X)$. In view of Lemma 2, we conclude that $f_n(i) \xrightarrow{w} f(i)$ in X for every $i \in \mathcal{N}$. By the Schur property of X we get $f_n(i) \to f(i)$ in X for every $i \in \mathcal{N}$. We will use the same notation and the same steps as in the proof of (i). Then $\|f(i)\|_X = g(i)$ for every $i \in \mathcal{N}$. Moreover $A = \emptyset$ and $B = \operatorname{supp} g$. Consequently $\operatorname{sep}\{f_n\chi_B\}_{E(X)} \geq \varepsilon/8$. On the other hand $\operatorname{card} B < \infty$ and $\|f_n(\cdot) - f(\cdot)\|_X \to 0$ in E. Hence $\|\|f_n(\cdot) - f(\cdot)\|_X \chi_B\|_E \to 0$. This contradiction shows that case I is the only one to consider and finishes the prove. ■

REMARK 1. – *The implication $E(X) \in$ (**UKK**) $\Rightarrow E \in$ (**UM**) is not true if X has the Schur property. It is enough to take $X = \mathcal{R}$ and E from Example* 1. *We do not know whether that implication holds if X is an infinite dimensional Banach space with the Schur property, i.e. not reflexive. However, in that case, we present precise criteria for a sequence Köthe space E with the shrinking basis (see Theorem* 3 *below).*

REMARK 2. – *It is worth to mention that property* (**UKK**) *does not lift from X into $E(X)$ in the case when E is a Köthe function space. It is enough to consider the Lebesgue-Bochner space $L_p(\mu, X)$ when $1 < p < \infty$ and μ is the Lebesgue measure on* $[0, 1]$. *Then if X is not uniformly convex then $L_p(\mu, X)$ has not uniform Kadec Klee property* (*Theorem* 3.4.9 *in* [16]). *This fact also follows from the proof of Theorem* 2 *in* [19].

Denote by $\{e_n\}_{n=1}^{\infty}$ the basis of E, by $\{e_n^*\}_{n=1}^{\infty}$ the sequence of biorthogonal functionals to the $\{e_n\}_{n=1}^{\infty}$. Let Γ be the weak topology and $\Gamma_0 = \sigma(E, [\{e_n^*\}])$ be the topology generated by the closed linear span of $\{e_n^*\}$. Obviously Γ_0 is weaker than Γ. Denote by E^* the Banach dual of E. The basis $\{e_n\}_{n=1}^{\infty}$ is called shrinking when $\{e_n^*\}_{n=1}^{\infty}$ form a basis of E^* (in particular this is the case when E^* is separable or E is reflexive, see also Proposition 1.b.1 in [17]). If the basis $\{e_n\}_{n=1}^{\infty}$ is shrinking, then the topology Γ_0 coincide with Γ.

It follows from Proposition 8 in [8] that if X is a reflexive Köthe function space over the complete, σ-finite measure space (Ω, Σ, μ) and $x_n \to x$ μ-a.e., then $x_n \to x$ weakly in X. Notice also that if X has a Schauder basis, then X is reflexive iff the basis is both shrinking and boundedly complete (Theorem 1.b.5 in [17]).

LEMMA 4. – *Let E be a sequence Köthe space with the shrinking basis $\{e_n\}_{n=1}^{\infty}$. If $x_n \to 0$ pointwisely in E and $(\|x_n\|_E)_{n=1}^{\infty}$ is bounded, then $x_n \to 0$ weakly in E.*

PROOF. – Suppose that $x_n \to 0$ pointwisely in E and $(\|x_n\|_E)$ is bounded. Under our assumption it is enough to show that $x_n \to 0$ in topology Γ_0 in E. Let $x^* \in \Gamma_0$. If $x^* = \sum_{i \in I} \alpha_i e_i^*$ for $(\alpha_i)_{i \in I} \subset \mathcal{R}$ and $I \subset \mathcal{N}$ with card $I < \infty$, then obviously $x^* x_n \to 0$ as $n \to \infty$. Suppose that x^* is such that $x_k^* \to x^*$ in E^* and for every $k \in \mathcal{N}$ we have $x_k^* = \sum_{i \in I_k} \alpha_i^{(k)} e_i^*$ for $(\alpha_i^{(k)})_{i \in I_k} \subset \mathcal{R}$ and $\operatorname{card} I_k < \infty$. Then

$$|x^* x_n| = |(x^* - x_k^* + x_k^*)\, x_n| \leqslant$$

$$|(x^* - x_k^*)\, x_n| + |x_k^* x_n| \leqslant \|x^* - x_k^*\|_{E^*} \|x_n\|_E + |x_k^* x_n| .$$

Hence $x^* x_n \to 0$ as $n \to \infty$.

The next result corresponds to Theorem 2(ii) and define precisely in particular case the criteria for $E(X)$ to be (**UKK**).

THEOREM 3. – *Let E be a sequence Köthe space with the shrinking basis $\{e_n\}_{n=1}^{\infty}$ and X be a separable Banach space with the Schur property. Then $E(X) \in$ (**UKK**) iff $E \in$ (**UKK**).*

PROOF OF NECESSITY. – It is clear, since E is embedded isometrically into $E(X)$ and the property (**UKK**) is inherited by subspaces.

PROOF OF SUFFICIENCY. – Suppose that $E \in$ (**UKK**). Then $E \in$ (**OC**). Let $\varepsilon > 0$. Take a sequence $(f_n) \subset B(E(X))$ such that $\operatorname{sep}\{f_n\}_{E(X)} \geqslant \varepsilon$ and $f_n \xrightarrow{w} f$ in $E(X)$. In view of Lemma 2, we conclude that $f_n(i) \xrightarrow{w} f(i)$ in X for every $i \in \mathcal{N}$. By the Schur property of X we get $f_n(i) \to f(i)$ strongly in X for every $i \in \mathcal{N}$. Consequently for every $I \subset \mathcal{N}$ with $\operatorname{card} I < \infty$ we have $\|\|f_n(\cdot) - f(\cdot)\|_X \chi_I\|_E \to 0$. Since $E \in$ (**OC**), so there exists $A \subset \mathcal{N}$ with

$$\operatorname{card} A < \infty \quad \text{and} \quad \|\|f(\cdot)\|_X \chi_{\mathcal{N} \setminus A}\|_E < \varepsilon/16 . \tag{13}$$

Moreover there exists a number $N_1 \in \mathcal{N}$ such that

$$\|(\|f_n(\cdot)\|_X - \|f_m(\cdot)\|_X)\, \chi_A\|_E < \varepsilon/2$$

for every $n, m \geqslant N_1$. Then $\operatorname{sep}\{(f_n \chi_{\mathcal{N}\setminus A})_{n=N_1}^{\infty}\} \geqslant \varepsilon/2$. Consequently

(14) $\quad \|\|f_n(\cdot)\|_X \chi_{\mathcal{N}\setminus A}\|_E \geqslant \varepsilon/4$ for every $n \geqslant N_1$ excluding at most two elements.

Denote $g_n(\cdot) = (\|f_{n+N_1}(\cdot)\|_X - \|f(\cdot)\|_X)\, \chi_{\mathcal{N}\setminus A}$. Then $g_n \to 0$ pointwisely in E and

$(\|g_n\|_E)_{n=1}^{\infty}$ is bounded. By Lemma 4 we get $g_n \xrightarrow{w} 0$ in E. Furthermore, by (13) and (14), we get $\|g_n\|_E \geq \varepsilon/8$ for every $n \in \mathcal{N}$. Consequently, by Hahn-Banach theorem, passing to a subsequence if necessary, we may assume that $\operatorname{sep}\{g_n\}_E \geq \varepsilon/16$. But

$$\operatorname{sep}\{g_n\}_E = \operatorname{sep}\{\|f_{n+N_1}(\cdot)\|_X \chi_{\mathcal{N}\setminus A}\}_E \leq \operatorname{sep}\{\|f_{n+N_1}(\cdot)\|_X\}_E .$$

Moreover $\|f_{n+N_1}(\cdot)\|_X \xrightarrow{w} \|f(\cdot)\|_X$ in E. Applying uniform Kadec Klee property of E we get

$$\|f\| = \|\|f(\cdot)\|_X\|_E \leq 1 - \delta ,$$

where $\delta = \delta(\varepsilon/16)$ is from (1).

Since (**KK**) $\Rightarrow$ (**OC**) ([6]) and every order continuous Köthe sequence space has a natural basis $\{e_n\}_{n=1}^{\infty}$ it is natural to consider the uniform Kadec-Klee property $(\mathbf{UKK})_\tau$ with respect to convergence in the topology τ, where $\tau = \Gamma$ or $\tau = \Gamma_0$. Note that Theorem 1 remains true if we replace the property $(\mathbf{UKK})_\tau$ for $\tau = \Gamma$ by the property $(\mathbf{UKK})_\tau$ for $\tau = \Gamma_0$. Obviously $(\mathbf{UKK})_{\Gamma_0} \Rightarrow (\mathbf{UKK})_{\Gamma}$. Moreover, since for reflexive spaces $(\mathbf{UKK})_{\Gamma_0}$ and $(\mathbf{UKK})_{\Gamma}$ are equivalent, we conclude that $X \in$ (**NUC**) iff $X \in (\mathbf{UKK})_{\Gamma_0}$ and X is reflexive.

Sukochev ([22]) proved that for Banach lattice whose order is induced by the unconditional basis $\{e_n\}_{n=1}^{\infty}$ the uniform monotonicity implies the property $(\mathbf{UKK})_{\Gamma_0}$. Taking $X = \mathcal{R}$ in Theorem 2(*ii*) we get easily the following

COROLLARY 1. – *Let E be a sequence Köthe space. If $E \in$ (**UM**), then $E \in (\mathbf{UKK})_{\Gamma}$.*

Recall that E is symmetric sequence space if for every $x \in E$ there holds $\|x\|_E = \|x^*\|_E$, where x^* is a nonincreasing rearrangement of x, i.e. $x^* = (x(n_1), x(n_2), \ldots, x(n_k), \ldots)$ and the permutation n_k of $\mathcal{N}$ is such that $|x^*| = (|x(n_1)|, |x(n_2)|, \ldots, |x(n_k)|, \ldots)$ is nonincreasing.

Sukochev (Theorem 2 in [22]) proved that for Banach lattice whose order is induced by the symmetric basis $\{e_n\}_{n=1}^{\infty}$ the uniform monotonicity and the property $(\mathbf{UKK})_{\Gamma_0}$ coincide. We give an example of non symmetric Köthe sequence space which has the $(\mathbf{UKK})_{\Gamma_0}$ property and is not uniformly monotone.

EXAMPLE 1. – *For $i = 1, 2, \ldots,$ let X_i denote R_i with the norm $\|(x_1, x_2, \ldots, x_i)\|_i = \sup_{1 \leq j \leq i} |x_j|$. Then define*

$$Y = \left\{ y = (y_i) : y_i \in X_i \textit{ for every } i = 1, 2, \ldots \textit{ and } \sum_{i=1}^{\infty} \|y_i\|_i^2 < \infty \right\}$$

equipped with the norm $\|y\| = \left(\sum_{i=1}^{\infty} \|y_i\|_i^2 \right)^{1/2}$. *By Theorem* 2 *in* [11], $Y \in$ (**NUC**), *so* $Y \in$ (**UKK**). *In view of reflexivity of* Y *we conclude that* $Y \in (\mathbf{UKK})_{\Gamma_0}$. *But it is easy to observe that* Y *is not even strictly monotone.*

COROLLARY 2. – *Let* E *be a symmetric sequence Köthe space with the shrinking basis* $\{e_n\}_{n=1}^{\infty}$ *and* X *be a separable Banach space. Then* $E(X) \in$ (**UKK**) *iff* $X \in$ (**UKK**) *and* $E \in$ (**UKK**) *iff* $X \in$ (**UKK**) *and* $E \in$ (**UM**).

PROOF. – From the remarks given above we conclude that under our assumptions we have that $E \in (\mathbf{UKK})_{\Gamma_0}$ iff $E \in (\mathbf{UKK})_{\Gamma}$. Moreover Theorem 2 in [22] states that $E \in (\mathbf{UKK})_{\Gamma_0}$ iff $E \in$ (**UM**). Thus thesis is an immediate consequence of our Theorem 2.

The following Corollary was also obtained in [15]

COROLLARY 3. – (i) *Let* E *be a sequence Köthe space and* X *be an infinite dimensional Banach space. Then* $E(X) \in$ (**NUC**) *iff* $X \in$ (**NUC**), $E \in$ (**NUC**) *and* $E \in$ (**UM**).

(ii) *Let* E *be a sequence Köthe space and* X *be a finite dimensional Banach space. Then* $E(X) \in$ (**NUC**) *iff* $E \in$ (**NUC**).

(i) Proof of necessity. – Since spaces X and E are embedded isometrically into $E(X)$ and the property (**NUC**) is inherited by subspaces, so $X \in$ (**NUC**) and $E \in$ (**NUC**). Then X is reflexive. But X is also infinite dimensional, so X fails to have the Schur property. By Theorem 2(i) we conclude that $E \in$ (**UM**).

PROOF OF SUFFICIENCY. – If $X \in$ (**NUC**), then $X \in$ (**UKK**) and X is reflexive (Theorem 1 in [11]). Since $E \in$ (**UM**), then by Theorem 2(i) we get $E(X) \in$ (**UKK**). Moreover E and X are reflexive. From Theorem 5.3 in [7] it follows that $(E(X))^* = E'(X^*)$, where X^* is a Banach dual of X and E' is a Köthe dual of E, i.e.

$$E' = \left\{ y \in l^0 \colon \|y\|_{E'} = \sup_{\|x\|_E \leq 1} \sum_{i=1}^{\infty} x(i) y(i) < \infty \right\}.$$

Furthermore $E \in$ (**OC**) iff $E' = E^*$ (see [18]). Consequently $E(X)$ is reflexive. Thus $E(X) \in$ (**NUC**).

(ii) We prove only the sufficiency. If $E \in$ (**NUC**), then $E \in$ (**UKK**) and E is reflexive. So it has the shrinking basis. Moreover, every finite dimensional space is reflexive and has the Schur property. Then $E(X)$ is reflexive and thesis is a consequence of Theorem 3 and Theorem 1 in [11].

Taking $E = l^p$ in Theorem 2 and Corollary 3 we get a result of Partington [19]

COROLLARY 4. – (*i*) *Let* $1 \leqslant p < \infty$. *The space* $l^p(X) \in$ (**UKK**) *iff* $X \in$ (**UKK**).

(*ii*) *Let* $1 < p < \infty$. *The space* $l^p(X) \in$ (**NUC**) *iff* $X \in$ (**NUC**).

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