Barbara Fagiolini

Holistic ICT environments for effective mathematics teaching and learning

Dottorato in Computer Science and Computational Mathematics, Varese (2020).
<http://www.bdim.eu/item?id=tesi_2020_FagioliniBarbara_1>

UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA

Dipartimento di Scienza e Alta Tecnologia

Anno Accademico 2016-2017

Dottorato in Informatica e Matematica del Calcolo - XXXII Ciclo

HOLISTIC ICT ENVIRONMENTS FOR EFFECTIVE MATHEMATICS TEACHING AND LEARNING

Author: Barbara Fagiolini
ID Number: 731854

Advisor:	Prof.ssa Daniela Lucangeli *Università degli Studi di Padova*
Advisor:	Prof.ssa Silvia Elena Benavides Varela *Università degli Studi di Padova*
Advisor:	Prof. Stefano Serra Capizzano *Università degli Studi dell'Insubria*
PhD Coordinator:	Prof. Marco Donatelli *Università degli Studi dell'Insubria*

Università degli Studi dell'Insubria
Dipartimento di Scienza e Alta Tecnologia
Sede di Como

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PhD Thesis in Computer Science and Computational Mathematics
AUTHOR: Dott.ssa Barbara Fagiolini

ADVISOR: Prof.ssa Daniela Lucangeli
ADVISOR: Prof.ssa Silvia Benavides Varela
ADVISOR: Prof. Stefano Serra Capizzano
PHD COORDINATOR: Prof. Marco Donatelli

Como, September 23rd 2019

We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time
...
Thomas Stearns Eliot, Four Quartets

To Lavinia Elena

Contents

Abstract **1**

Introduction **3**

1 The Mathematical Profiles **15**
1.1 Introduction . . . 15
1.1.1 Identify children's mathematics profiles as early as possible . 15
1.1.2 Mathematical Learning Difficulties (MLD) . . . 17
1.1.3 Purpose of this study . . . 21
1.2 The Mathematical Profile & Dyscalculia Test (MathPro) . . . 22
1.2.1 The four-pronged model . . . 22
1.2.2 The MathPro Test . . . 23
1.3 The Italian standardization of the MathPro Test . . . 26
1.3.1 Participants . . . 26
1.3.2 The MathPro Battery . . . 27
1.3.2.1 Reaction time Test on the ability to use the instrument 27
1.3.2.2 Single-digit numbers comparison and Multi-digit numbers comparison (Number magnitude Comparison) . 28
1.3.2.3 Dots Comparison (Dots Magnitude Comparison) . . 28
1.3.2.4 Subitizing (Flashed dots) . . . 30
1.3.2.5 Enumeration (Fast counting) . . . 32
1.3.2.6 Numbers dictation (Number-word to Arabic digits transcoding) . . . 32
1.3.2.7 Next number (Subsequent consecutive number) . . 33
1.3.2.8 Previous number (Preceding consecutive number) . 33
1.3.2.9 Addition facts retrieval (Simple additions) . . . 34
1.3.2.10 Multiplication facts retrieval (Simple multiplications) 35
1.3.2.11 Mental calculations (Additions, Subtractions, Multiplications, Divisions) . . . 35
1.3.2.12 Word problems . . . 35
1.3.2.13 Calculation principles (Solve the second based on the first) . . . 37
1.3.2.14 Numerical patterns (Patterns' recognition - Find out the missing number) . . . 37
1.3.2.15 Number lines 0-100 . . . 38
1.3.2.16 Number lines 0-1000 . . . 38
1.3.2.17 Squares (2D Shapes) . . . 39

1.3.2.18 Building blocks (3D Shapes) 40
1.3.3 Statistical analyses 40
1.3.4 The Results of the Italian experimentation and standardization 41
1.3.5 Some emerging mathematical profiles 44
1.3.6 Qualitative analysis 48
1.3.6.1 Individual Report 48
1.3.6.2 'Bravo in Maths': Student Profile 49
1.3.7 Comparison of Year 1student profiles belonging to the same class 54
1.4 Teacher's attitude 55
1.4.1 Traditional school and technological school 55
1.4.2 Teachers' opinion and qualitative evaluation of the battery . . 62
1.4.3 Survey 64
1.4.4 Discussion 68
1.5 Conclusion 69

2 Technology and emotions 73
2.1 Introduction 73
2.1.1 Psychophysiological assessment 75
2.1.2 Purpose of this study 76
2.2 Skin Sensor 77
2.3 Session: June 13, 2019 4.30 pm 77
2.3.1 First session results 79
2.3.2 Second session results 83
2.4 Conclusion and future directions 85

3 Effectiveness of digital tools 87
3.1 Introduction 87
3.1.1 Review of the previous summative studies into teaching math via digital tools 90
3.1.2 Purpose of this study 92
3.2 Methods 93
3.2.1 Literature search and inclusion criteria 93
3.2.2 Coding procedure 94
3.2.3 Statistical Analysis 95
3.2.3.1 Sensitivity analysis and evaluation of publication bias 98
3.2.3.2 Effects of possible moderators 98
3.3 Results and Discussion 98
3.3.1 Search results 98
3.3.2 Included studies, participant details and descriptive statistics . 99
3.3.3 Random-effects model Meta-analysis 102
3.3.3.1 Overall effects 102
3.3.4 Evaluating sensitivity of the analysis 104
3.3.4.1 Evaluating publication bias 104
3.3.4.2 Evaluating moderators 106
3.4 Discussion 107
3.5 Conclusion 111
3.5.1 Limitations 112

4 Educational videogames at home **113**
4.1 Introduction 114
4.1.1 Purpose of this study 114
4.2 Methods 115
4.2.1 Literature search and inclusion criteria 115
4.3 Results 116
4.3.1 Search results 116
4.4 Discussion 119
4.5 Conclusion 124

Conclusions and future directions **127**

Appendix **133**

List of Figures **155**

List of Tables **159**

References **161**

Ringraziamenti **189**

Abstract

To understand the numbers world, each child must face a path that includes propaedeutic phases and only overcoming these phases will allow the child to consolidate processes before tackling new ones. If this does not happen, the child becomes self-convinced of ”not being able to understand math” and becomes increasingly aware of it throughout primary school. The transition to middle school may have a significant impact on a child, with specific difficulties or disabilities in learning mathematics, especially if his/her difficulties have not yet been identified. The risk for him/her is to accumulate bad grades in mathematics and, above all, psychological distress due to the many failures. Some people find that they are dyscalculic in high school, or adulthood: by then they may have severely undermined self-esteem and faced significant emotional and/or psychopathological disorders. It is therefore evident how the early identification of difficulties and learning disabilities has a fundamental role to avoid this lack of recognition becoming a cause of the child's discomfort. Moreover a disorder such as dyscalculia is a neurobiological feature so it does not change during the lifetime.

Once having identified pupils who present difficulties, it is important to activate courses aimed at recovering learning difficulties. Families should always be involved in the paths taken by their children and intervention plans should adopt a shared line of action. Building an educational alliance is an advantage for the activities that the teacher must undertake and fundamental for the child, to affirm his/her safety by going through an even more stimulating path. Besides, the family, supported by learning experts, can avoid anxious attitudes with respect to their child's difficulties or choose not adequate solutions.

The present thesis studies the contributions of Information and Communication Technology (ICT) in supporting various aspects of mathematical teaching and learning. A multidimensional approach was used. In the first part, empirical studies assessed the effectiveness of digital tools to identify individual differences based on cognitive profiles (Chapter 1) and emotional responses (Chapter 2) associated with math performance in children from Northern Italy. In the second part a meta-analysis and systematic review analysis were carried out to evaluate the effectiveness of interventions supporting math learning by means of ICT in the school (Chapter 3) and home environments (Chapter 4) respectively. The results revealed that the teachers' evaluation may be completed with the support of a profiling software (Chapter 1). Chapter 2 proposes to recognize the children's emotions using the physiological signal generated by the skin conductance and the results can give clues about the "child/teacher" and "child/parents" relationship that can be used in pedagogical strategies. The results of the meta-analyses of 15 studies about the effectiveness of digital-based interventions (Chapter 3) indicate that digital tools positively impact the mathematics achievement of students with or at risk for dyscalculia. Finally, the literature review that analyzes the effects of educational math games at home and the parents' contribution to Educational Technology (Chapter 4) suggests that the interactions among home, classroom, and children, influence children and allow them to learn basic skills, including math skills. In this chapter the literature review provides a summative overview of the current evidence to date missing in the literature.

Altogether the thesis indicates that designing holistic ICT environments proves successful for effective mathematics teaching and learning not only for typically developing children but also for students in disadvantaged situations, including those suffering from dyscalculia.

Introduction

Mathematics in historical perspective

Different cultures have different traditions and histories. Mathematics has seen the contribution of different cultures that have developed along different paths.

Marcia Ascher, 'Mathematics elsewhere. An Exploration of ideas across cultures' (Ascher, 2002), offers a comprehensive view of mathematics that shows how, even in cultures that do not distinguish mathematics as a separate category, the mathematical concepts exist. The mathematical ideas, for example, are the basis of the concept of time of the different cultures and the different calendars express their conceptualization of the world view. Not surprisingly, even the forms of divination, in almost all cultures, start with discrete random processes clearly formulated. One example is the astragalomancy, practiced by the ancient Greeks, which can be considered a forerunner of craps (David, 1998). Even the study, and understanding, of the world of physics would be inconceivable without its mathematical modeling (Kline, 2012).

Abstraction and generalization come into play when we have to read any type of maps: meteorological, geological, automotive, hiking, navigation, etc.. To read a map one needs to know the use of a scale and the meaning of the contour lines and to interpret a reference grid. The display and graphical representation are very important also in the teaching of mathematics.

The diagrams have a fundamental role, especially with the advent of computer technology, but they were present already in the Greek mathematics (Netz, 2003). Even relationships are fundamental in mathematical systems ('father of...', 'taller than...', ‘less than…’, ‘equal’, ‘not equal’, etc.) as they are in social organizations.

Mathematics in everyday life

The etymology of the word ’mathematics’ derives from the Greek (‘mathema’ translatable as “science”, “knowledge” or “learning”): the word ‘mathematikós’ can be interpreted as ’inclined to learn’ and therefore seems to promote a positive attitude towards learning.

Mathematics is useful in everyday life, it helps to understand the world and it is a discipline at the service of all the other disciplines (it is usually said that math is the language of all the sciences). Although mathematicians are experts who apply their skills to solve problems in astrophysics, biology, medicine, etc., at school, considering the way maths topics are faced, often mathematics seems considered far from reality and useless: you study maths just to get good marks while at school what is hoped is that the student will achieve not only a ’competence in mathematics’, centered on the mathematical discipline, but also a real ’mathematical competence’ (Arzarello & Robutti, 2002), (D’Amore, 2003) intended as the ability to understand the role that mathematics plays in the real world (*PISA Assessing Scientific, Reading and Mathematical Literacy A Framework for PISA 2006: A Framework for PISA 2006*, 2006). This attitude makes teaching mathematics really difficult and, in turn, for many students the learning process becomes really hard. Learning in Italian schools, for example, takes place in a very formal way, using little the so-called “reality tasks”. Yet, according to some studies performed on informal learning, children seem able to spontaneously develop strategies to move in the world of quantities. ‘Street Mathematics and School Mathematics’ (Nunes, Schliemann, & Carraher, 1993) highlights how this discipline seems to have two parallel lives, the one in the school environment and the one that is the ’street mathematics’, the one we use daily. In the study of Nūnes and colleagues, the mathematical skills that Brazilian street children had developed to sur-

vive were analyzed. They were able to do very complex mental calculations, so outside the school context, they had developed and used effective mathematical skills that became concrete, real, useful and, therefore, simple.
There is a lot of attention on this discipline even concerning emotional features. “Understanding Emotions in Mathematical Thinking and Learning” (Xolocotzin Eligio, 2017) is one of the very authoritative publications on mathematics and emotions.

Mathematical Cognition

Mathematics is a discipline in which several generations of students seem to have highlighted learning difficulties Researchers believe that understanding underlying mathematical processes can help us to identify why so many individuals struggle with mathematics. Mathematical cognition is an interdisciplinary field of research that seeks to understand the processes by which individuals come to understand mathematical ideas. Mathematical cognition researchers are anthropologists, mathematics education researchers, neuroscientists, philosophers and developmental or cognitive psychologists (Bruce et al., 2016) that are interested in a variety of complex mental activities such as the identification of relevant quantities, encoding or transcribing those quantities into an internal representation, mental comparisons, calculations and ”those mechanisms or processes that are involved in the control, regulation, and active maintenance of task-relevant information in the service of complex cognition” (Miyake & Shah, 1999). Research in mathematical cognition (Ashcraft & Christy, 1995), (anne LeFevre, Shanahan, & DeStefano, 2004), (DeStefano & LeFevre, 2004) is interested in ‘how mathematical understanding and performance develops from infancy to adulthood, the factors that explain individual differences in mathematical performance, why some individuals find mathematics so difficult’ (Gilmore, Göbel, & Inglis, 2018), and consequently how to improve mathematical performance. Learning is a process of encoding, strengthening, and proceduralizing knowledge and this process happens gradually (J. R. Anderson, Lebiere, Lovett, & Reder, 1998). Experimental cognitive, neuroimaging and intervention are the methodologies employed by researchers to investigate cognitive systems and their development and to individuate the formal and informal activities that individuals engage in when learning mathematics (Gilmore et al., 2018). Anderson et al. (R. Anderson, 2007) believe that the combination of a dense data stream of student behavior and a large sample of students will expand our knowledge of students' mathematical cognition and will advance our ability to help students learn mathematics. Studying mathematical cognition may have a positive impact on mathematics education and performance.

Individual Differences in Mathematics

The mathematical ideas are spread in the every-day life and therefore the understanding of math concepts becomes important not only to 'do well in school' but also to solve a range of problems in everyday situations. Unfortunately, there are many children with mathematical difficulties and it is necessary to dwell on what you mean by problems. The student may have sensory deficits, psychological or socio-cultural origins, may simply have difficulty with the characteristics of mathematics as a discipline and, at times, the difficulties can also arise from the relationship with the math teacher or the used materials (Zan, 2006).
There is still an open debate about developmental trajectories and reasons for a failure in learning mathematics. However, there seems to be an agreement on the fact that environmental, cultural, or economic disadvantaged situations increase the likelihood of experiencing poor mathematical outcomes throughout the educational path, particularly when a child has neurobiological dispositions to number deficits. When such contextual factors are verified, children are considered at risk for learning difficulties in mathematics. Low mathematics attainment can be often accompanied by psychological and behavioral problems, which might in turn intensify the negative

academic outcomes. There is also a specific neuro-developmental disorder of learning mathematics, developmental dyscalculia, which is of neuro-biological origin. In fact, many individuals with this developmental disturb present differences in brain structure, function or connectivity, particularly in the parietal lobes (Kucian & von Aster, 2014), (Kaufmann, Kucian, & von Aster, 2014), (Kucian, Kaufmann, & von Aster, 2014), (Ansari, 2016). Furthermore, several studies have reported that many children with mathematical difficulties have problems not only with domain-specific functions but also with domain-general ones and genetic research suggests that mathematical ability is heritable (Kovas, Harlaar, Petrill, & Plomin, 2005). There are numerous problems that children with developmental dyscalculia can experience and the debate continues to enrich the studies in this field of research.

To explore this area, clinicians use standardized tests, measuring the reaction times and the accuracy of the performances relative to the numerical intelligence and the executive and calculation procedures. The analysis of any delays, or difficulties in the acquisition of skills in numerical intelligence, makes it possible to identify the students at early risk, even in the pre-school period. However, the minimum age to perform a diagnosis of developmental dyscalculia, in Italy, is from the end of the third year of primary school. The tests investigate:

- quantification mechanisms, comparison, serialization;
- the strategies of mental calculation;
- the functions of reading, writing and putting numbers in column for the executive procedures;
- recovery of numerical facts;
- the algorithms of the written calculation.

Conversely, some children, the gifted ones, have a high intelligence, which if undetected can become a problem for the child because it can be confused with apathy or no interest in the subject. The research in this field identifies in the school, in the family and in the peer group, the three elements involved in the development of individual potential talent (Monks & Ferguson, 1983), and that can make a difference, so that giftedness does not become ground for new situations of marginality (Sorrentino & Pinnelli, 2017).

Students with intellectual giftedness, according to the perceptions of some Italian teachers, do not always coincide with the vision of ”excellence” that emerges from school legislation. If the latter promotes excellence through competitions of various kinds, for some well-known Italian pedagogues (Becchi, 1963), (Frabboni, Guerra, & Scurati, 1999), (Baldacci, 2002) excellence should be achieved through a teaching-oriented development of the talents of each. These students must be supported to have an opportunity to develop their mathematical potential. The schools where there are gifted pupils should adopt innovative teaching strategies that promote and enhance the different excellences before the asynchrony in the development, as evidenced by research (Colangelo & Davis, 1997), can generate situations of fragility that impact on the health of the person and on academic performance (Reis & McCoach, 2000).

The current status in Italy: prevalence of the difficulties in mathematics

Both the number of developmental dyscalculia diagnoses and the difficulties in learning mathematics seem to increase every year with an incredibly fast rate. The increasing number of diagnoses of dyscalculia in recent years may be related also to teachers' increased awareness of the disorder.

On the MIUR website (Ministero dell'Istruzione dell'Università e della Ricerca) a statistical study was published concerning students with Specific Learning Disorders (DSA) in the Italian private and public schools (June 2019). Data refer to the 2017/2018 school year. Pupils with dyscalculia (Figure 1) have increased, compared to the 2013/2014 school year, from 33.000 to almost 87.000 (+ 160.5%). Overall, in the 2017/2018

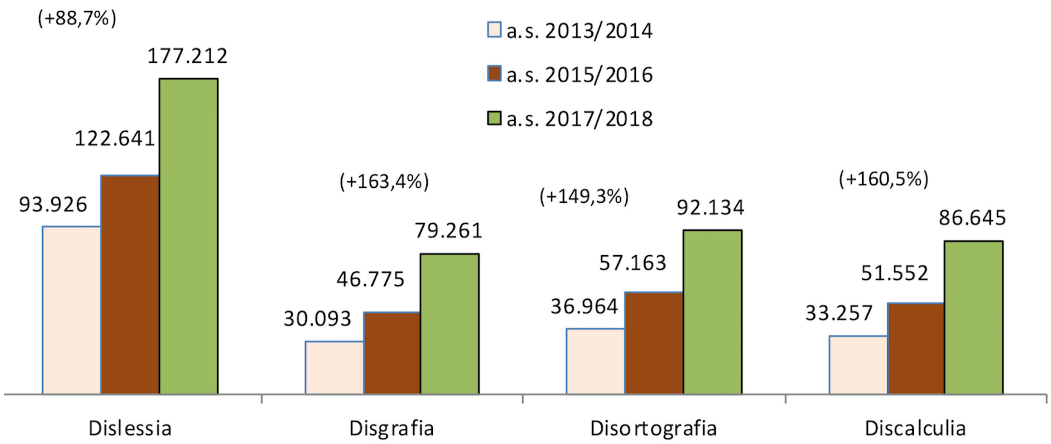


Figure 1: Students with DSA by type of disorder - comparison of school years 2013/14 - 2015/16 - 2017/18 (primary, middle and high school are considered).

School order	Dyslexia	Dysgraphia	Dysorthographia	Dyscalculia	Total pupils with DSA
Kindergarten*	-	-	-	-	1,717
Primary school*	34,491	16,861	20,579	11,473	53,379
Middle School	59,777	29,823	36,177	31,395	96,176
High School	82,944	32,577	35,378	43,777	124,837
Total	177,212	79,261	92,134	86,645	276,109

Table 1: Pupils with DSA by type of disorder - school year 2017/2018; the total number of pupils with DSA may not coincide with the sum of pupils by type of disorder since some pupils may have more types of disorders. * for kindergarten and for the first and second year of primary school: these are students at risk of learning difficulties, identified following specific tests at the competent healthcare facilities.

school year (table 1), 177,212 students had dyslexia (learning to read), 79,261 dysgraphia (learning to write), 92,134 dysorthography (disturbance in using the linguistic code) and 86.645 dyscalculia. The increase in the number of certifications recorded over the past 3 years is considerable. The role of teachers and educators becomes fundamental, in terms of sensitivity and competence, in intercepting the signals that children transmit, regarding their weaknesses and their emotional fragility, within the world of numbers, for the early identification of an eventual developmental dyscalculia. The author of this thesis collaborates with many schools of all grades. On the occasion of each meeting with primary school children, they are always asked to “draw mathematics". In one of these meetings a 8-year-old girl, attending the third grade of the primary school, told what was happening in the classroom in math hours: "Virginia (fancy name, dyscalculic potential) is a friend of mine, she has a clogged mind, she doesn't understand anything about numbers and then how does she understand math if she writes numbers like broken chairs?". Figures 2 and 3 present two drawings with which she tried to explain what happened during her math time with her friend. This creative girl illustrated a situation, difficult to represent, with incredible simplicity and her drawings, more explanatory than many words, were a request for help that this research did not want to ignore.

Benefits of using digital tools in teaching and learning

There are many claims found in the literature for the usefulness of digital tools in encouraging learning: they offer distinctive benefits for teachers and learners, and for the research. For teachers they represent customizable, personalized, and sharable instruments enabling them to give their individualized attention to the learner and to partake in solving teaching challenges together with other colleagues (Butterworth, Varma, & Laurillard, 2011). In training and educational settings it is suggested that digital tools can reduce training time and instructor load, for example affording opportunities for drill and practice (which is a form of instruction where learners rehearse sets of material following the same pattern), thereby enhancing knowledge acquisition and retention (Mitchell & Savill-Smith, 2004). Moreover they could be useful in forging better scholarly-related attitudes (Shirvani, 2010), and facilitating social skills through collaborative interactions (Spence & Feng, 2010), (Hasselbring & Glaser, 2000), (Durkin, Whitehouse, Jaquet, Ziatas, & Walker, 2010), (Griffiths, 2002), (Morton et al., 1988). From the point of view of the student, technological tools offer opportunities to carry out tedious tasks (i.e. drill and practice) in an alternative fun context, thereby enhancing knowledge acquisition and retention (Brownfield & Vik, 1983), (Ricci, 1994).

Technological tools are also valuable instruments in encouraging alternative ways of studying and thinking (Gee, 2003). Virtual environments enable “private“ feedback that is extremely valuable for learners who strive to carry on, and repeatedly suffer defeats in normal classroom contexts (Butterworth et al., 2011).

In addition to advantages for teaching and learning, technological tools can provide insights into the students’ cognitive processes and difficulties, which makes them a fruitful setting for research on how learning takes place (Peltenburg, Van Den Heuvel-Panhuizen, & Doig, 2009). This is possible because many of these tools can potentially register intermediate responses, track the learner’s choices, the progression from one task/level to another, the sequence of actions that lead to a specific error, the time taken to answer to a specific problem, etc., thereby providing the opportunity to characterize the student’s strategies, and to better identify the basis of their numerical deficits. Moreover, the pedagogy design and its effects are captured digitally; this enables a more accurate description and reproducibility of the technological intervention as compared to human interventions, which can vary considerably from one teacher to another (Butterworth et al., 2011).

Purpose of this study

The purpose of this work is to develop an ICT support tool for educators, teachers and clinicians, in order to identify, and subsequently reduce, difficulties in mathematics both from a cognitive and emotional point of view. Learning cannot be designed but environments and contexts can be set up to promote learning. In particular:

1. How can ICT contribute to identifying the student’s mathematical difficulties and strengths?

2. How can ICT contribute to detecting emotional difficulties or anxiety states which can interfere with children’s learning? Can these results provide clues about the “child/teachers” and “child/parents” relationship that may be useful in

pedagogical strategies?

3. Can ICT be used in the classroom to enhance children's mathematics abilities?

4. Can ICT be successfully incorporated in the home environment to enhance children's mathematics abilities and to establish educational alliances between school and families?

Each chapter of this work was dedicated to each of these questions in the order in which they are listed.
In the first chapter, a software is presented in detail to measure skills and difficulties in learning mathematics: the MathPro Test. The chapter also describes the standardization activity performed throughout Italy, which began with the administration of the test to 1,728 children from 22 schools. Chapter 1 describes how the profiling of 5 classes of two Varese schools (from the first grade to the fourth grade) took place, using this online software, able to detect weaknesses and strengths in mathematics. It was an experience with the students and their teachers in their classrooms. At the end of the administration, the reports generated by the software were evaluated. From this study, it is clear that the contribution of MathPro can be extremely useful to delineate the child's learning profile and to support teachers to identify the weaknesses and strengths of their students.
Chapter 2 deals with an experiment using skin conductance signals. By using a biotracker sensor, a pilot experiment was performed with one fourth-grade child. It was an experience with a child, her teacher and her mum. At the end of the session the reports generated by the software were analyzed. The results suggest that technology could reliably assess the emotional state of children and opens up a multitude of possibilities. If the results were replicated in future studies, these tools could be successfully used in the classroom as teaching aids and for evaluating the interaction between the child and the adults in the process of learning.
In chapter 3, a review is presented on 15 studies on the effectiveness of math edu-

Figure 2: My friend and me.

cational software. A random effects meta-analysis indicates that digital-based interventions generally improve mathematical performance, though there is a significant heterogeneity across studies. There is no evidence that videogames offer additional advantages with respect to drilling and tutoring approaches. Moreover, the effect size was not moderated when interventions were delivered in primary school or in preschool.

Finally, in chapter 4 a literature review is presented concerning the importance of a possible educational alliance between school and family. In particular it focuses on studies showing how children, even at home, can make use of educational videogames as math training. Parents and teachers should work together but the number of studies is too limited and further research is encouraged.

Mathematics teaching and learning in (and not 'with') technology, at school and at home, can contribute to define a holistic and multidimensional vision of each student. The results of this thesis should bring new insights for future pedagogical and educational decision.

Figure 3: My friend and our math teacher.

Chapter 1

The Mathematical Profiles

The aim of this chapter is to describe the standardization of the Mathematical Profile and Dyscalculia Test (MathPro Test) and to find out if the teachers' evaluation may be completed with the support of a profiling software to identify the weaknesses and strenghts of their students. 1728 children, in Grades 1-6 from 22 primary schools in Italy, participated to this experimentation. MathPro is an online computer-based battery consisting of 18 numerical tasks. A report is produced automatically after the completion of the test and the performance of the students is given in terms of percentile ranks. Implications for future educational decisions are discussed.

1.1 Introduction

1.1.1 Identify children's mathematics profiles as early as possible

Despite school curriculum reforms, technological innovations and research outcomes in education, one of the central problems for math education is certainly represented by learning difficulties.

As far as mathematical learning is concerned, first of all, it is necessary to reflect on the intrinsic and specific characteristics of the discipline. Mathematics is a cumulative discipline, so for example, if a child does not understand addition it will be very difficult for him/her to understand subtraction. Thus, gaps or moments of transitory difficulties result in some parts being left behind, especially when the student is pressing to complete the curriculum. Furthermore, it is a very composite discipline: quantities, numbers, transcoding skills, calculations (which are many), estimation processes, etc. Being mathematics a cumulative and composite discipline, over time the general domain resources, that are required to support learning, increase.

Some children meet difficulties because they are potential dyscalculics, others simply need 'specific strengthening pathways' because the difficulties in mathematics that are not identified, and promptly addressed, can lead to irreversible damage development (Taylor, Adelman, & Kaser-Boyd, 1983), 'learned impotence' or 'mathematics phobia. A timely and targeted intervention leads to a positive evolution of learning difficulties and consequently affects the overall emotional and cognitive development of children (Jackson, Paratore, Chard, & Garnick, 1999), (Byrne, Fielding-Barnsley, & Ashley, 2000), (Baker & Smith, 1999) (D. Morris, Tyner, & Perney, 2000), (Schneider, Roth, & Ennemoser, 2000), (Vadasy, Jenkins, & Pool, 2000). It, therefore, becomes a priority to identify and profile the conceptual weaknesses of the students as soon as possible, as indicated by the current international research lines that consider it of primary importance to perform early interventions to avoid the permanence of difficulties (Aubrey, Godfrey, & Dahl, 2006).

Hence the importance of diagnosis and intervention processes for difficulties in mathematics to avoid a series of educational failures in children, simply because their specific learning disorders were not identified early or because their reasoning in arithmetic was not strengthened. In fact, early detection can help the child to overcome the complexities that he/she is unable to face with his/her own strength and to lower the level of defeats he/she will have to suffer, preventing them from being perceived with such gravity as to become definitive renunciations. Many learning difficulties could be overcome by selecting specific teaching strategies, starting from the student's strengths.

At the same time, it is also important to identify gifted children, those who have a high intelligence. The abilities of children with a high cognitive potential emerge from the first years of life and it is essential to identify them to support and potentiate their abilities. In fact, failure to recognize a child with a high cognitive potential can have important repercussions on his cognitive and emotional development. Gifted children need a stimulating school environment that constantly nourishes their desire to learn and to know. The regular school curriculum is insufficient to meet the needs of the gifted child (Wolfle, 1986).

1.1.2 Mathematical Learning Difficulties (MLD)

MLD (Mathematical Learning Difficulty), as a construct, is very complex (M. Mazzocco & Räsänen, 2013), (Szücs, 2016), (Lewis & Fisher, 2016), (Fletcher, 2007), not well defined, and containing several cases:

- Children with specific learning disorders (dyslexia, dyscalculia, dysgraphia), language deficit, non-verbal skills deficit, motor coordination deficit, ADHD - Attention Deficit and Hyperactivity Deficit;
- Children coming from families with socio-economic, linguistic and cultural disadvantage;
- Children with developmental disability conditions (e.g. Down Syndrome, Fragile X Syndrome).

Indeed, the factors that influence students' learning and performance relate also to environmental aspects, besides neurobiological and intrinsic aspects. Furthermore, many children with specific learning disorders have often also a deficit of attention or a problem of motor organization, motor coordination or visual-spatial organization, which also has repercussions outside the close context of the mathematical field. As for the purely mathematical field, Lewis and Fisher (Lewis & Fisher, 2016) argue that, when talking about MLD, it is important to consider complex mathematical domains: the domain of arithmetic, algebra, geometry, calculus, etc. They also underline how important it is to evaluate spatial and geometric reasoning, the ability to generalize, abstraction, mathematical relations and patterns.

The used terminology is not homogeneous either and the picture is very complex.

For example, in public documents there are many words and expressions such as the following: dyscalculia, disability, specific learning disability, disorder, mathematical learning disability, low-level performance, inclusion (that means pay attention to the people who have special needs), deficit, difficulty, resistance to intervention. They are used without distinction, but the meanings are not the same and often they are interpreted in different ways by the different communities, or even different researchers from the same community. A recent study by (Lewis & Fisher, 2016) systematically analyzed the methodological criteria used in international studies over the last 40 years, to identify students with a Mathematical Learning Disability (MLD); the analysis focused on the presence of a wide variability in the criteria used to identify and classify cases of MLD, and highlighted how to check non cognitive factors to identify students who demonstrate low persistent math performance.

In such a complex reference area, the one presently used to frame the difficulties in mathematics is the DSM5, Diagnostic and Statistical Manual of Mental Disorders. This manual categorizes the specific learning disability as a single entity under the wider heading of neuro-developmental disorders, where substantially all the conditions that require special attention at school, from intellectual disability to language disorders, and attention deficit disorder to motor coordination problems, are included. The DSM5 also defines the specific learning disorder with impairment in mathematics (i.e. dyscalculia) but allows the possibility to broaden the concept. This was recognized by the Italian law 170 (2010), which established that the concept of dyscalculia might be considered within the spectrum of mathematical difficulties. The following example can be considered: the DSM5 was introduced to make a diagnosis of a specific disorder for the students who have difficulty in mathematical reasoning, and therefore in solving mathematical problems. Not all those who are struggling to solve problems, however, have a type of dyscalculia defined as having difficulty in processing the calculation quickly, automatically or quickly and efficiently recalling arithmetic facts.

Therefore, the DSM5, compared to previous editions, broadens the categories under which children with difficulties in mathematics can be included, ranging from those who have problems with *number sense* to those who have a difficulty in memorizing arithmetical facts, and from those who have a difficulty in making quick and accurate calculations, to those who have a difficulty in solving mathematical problems. There may be many who have specific difficulties in these areas and children who have many more extensive impairments for very different reasons, ranging from the physical or neurobiological ones to the environmental, educational, pedagogical or even cultural areas. It is clear that these labels (dyscalculia, low intelligence, etc.) do not help to understand how students of this type work in mathematics and that it is fundamental to know whether the student faces mathematical difficulties or mathematical disabilities. Surely the strengthening, and in particular the resistance to a strengthening intervention, is one of the factors that is often used to distinguish between a difficulty and a disorder. Strengthening helps in overcoming difficulties while disturbance, despite strengthening, tends not to be solved. But before implementing any intervention, it is necessary to make assumptions that are aligned with the true difficulties of the students. Unfortunately, the complexity described in this section means that there are more and more children with various difficulties in mathematics, therefore with extremely varied profiles which are complex to identify.

This complexity is also evident in school practice and particularly when teachers have to evaluate the mathematical competence of their students. The concept of competence is linked to the knowledge and skills that an individual puts in place enabling relationships with people or situations and, somehow, challenging his/her personal ability to provide answers and then to evaluate his/her results in a satisfactory way (Pellerey, 2006). Knowledge is connected to knowing things: each person elaborates, assimilates and creates an awareness of the concept independently (D'Amore, Fandiño Pinilla, Marazzani, Santi, & Sbaragli, 2009). Skills instead, are typically associated with the 'know-how'. From the point of view of mathematics education, 'know-how' without the 'knowledge' does not mean much, because mathematics is a building and application discipline. ‘Learning mathematics is more than acquiring an array of facts’ (Kilpatrick & Quinn, 2009).

Many difficulties are connected to the complexity of the evaluation process. Since Scriven, in 1967 (Scriven, 1967) , coined the term ’assessment’, providing a distinction between formative and summative functions, learning assessment has become part of the teaching and learning process. Based on assessments, the teacher should adapt the teaching process to find an individual mean to steer the student down the right path towards learning (Tornar, 2001). Evaluation should be a real teaching tool. Yet, often the teacher tends to set educational tasks which do not ensure the use of the student’s cognitive and motivational resources which are typical of competent behavior, but instead the teacher replaces the students (Brousseau, 1986), (D’Amore & Fandiño Pinilla, 2001) by explaining what to do and what is expected of them in great detail. The student is placed in a position where he/she learns what the expectations of the teacher are, whether these be expressed or implied. In addition, when the teacher has to evaluate the student’s mathematical skills, he/she is required to depict and define the child’s multi-dimensional capacity by using a one-dimensional measurement (usually a grade or a number).

In this, already complex context, we must add the influence of bonding and relationship formed between teachers and pupils. In 1961, Aiken (Aiken Jr & Dreger, 1961) was one of the first researchers to speculate that affective factors interact deeply with cognitive processes and, consequently, the quality of the learning process. In mathematical competence, too, this component, understood as a desire to respond to a solicitation that guides and supports the student to work in a certain way, has a significant role (Fandiño Pinilla, 2003). Furthermore, the teacher’s emotions and beliefs play a crucial role in learning and teaching (Coppola, Di Martino, Pacelli, Sabena, et al., 2012), (Tirosh, 2009). These aspects were investigated by Di Martino and Zan (Di Martino & Zan, 2015) who also proposed a model that well describes the attitude of teachers in mathematics. It is divided into three closely related parts which examine: emotional aspects, the teacher's vision of mathematics and perceived competence and his/her beliefs. Emotions and attitudes towards mathematics strongly influence teaching practices and the quality of teaching and, therefore, have an enormous impact on the learning process of students (Garvis, Fluckiger, & Twigg, 2012), (Raths, 2001). The student himself/herself also plays an important role in the building of knowledge and skills, and his/her success will also be due to his/her own intention to commit, to contextualize and re-contextualize the information he/she confronts (Pellerey, 2006).

1.1.3 Purpose of this study

The goal of learning mathematics often seems to be: "do well in mathematics". In this context, we often speak about the performance of students and not about their learning. Attention should instead be shifted. In the routine, daily effort at school, even to achieve educational goals, or the highest level, such as in the Invalsi (Istituto Nazionale per la Valutazione del Sistema educativo di Istruzione e di formazione) tests, there is an overload that distracts the child from the process, in order to get to the result. Teacher evaluation is often based on content. This is also a problem for pupils who become interested only in the grade they manage to get.

The aim of this chapter is to discuss and evaluate whether an education in mathematics which uses digital tools and progresses according to each individual's needs based on the results obtained in his/her assessments, could allow for a transition from a one-dimensional to a holistic and multidimensional evaluation.

Multidimensional evaluation can also be achieved using paper-and-pencil tests, without digital tools. However, digital tools provide unbiased administration to all individuals taking the test and provide reports within a few minutes.

In order to verify this possibility, this study describes the standardization of an online test with numerical tasks for students at risk of experiencing mathematical difficulties or dyscalculia. In Italy, this is, to our knowledge, the first computerized battery, to evaluate mathematical learning profiles in order to use a holistic and a multidimensional assessment of the strong and weak cognitive aspects of a 6-11 year old student.

It could potentially become the first standardized computer-based diagnostic test of mathematical learning difficulties (MLD).

The author of this thesis participated in the preliminary meetings with the teachers and representatives of the schools involved in the experimentation. Afterwards, she took part in the actual implementation following the administration of the tests to detect the difficulties/excellence in mathematics in some schools ('Istituto Comprensivo G. Adamoli' - Besozzo and 'Istituto Comprensivo Carducci' - Gavirate). Then, she proceeded to collect the evaluation forms, filled by the maths teachers and the Italian language teachers, in order to compare their opinions with the computer data generated by the profiling software.

Finally, the author of this thesis participated in the sessions in which results were described to the teachers of all the schools involved in this experimentation in the province of Varese. She also conducted specific interviews to study the perception of teachers concerning the multidimensional assessment measures of the mathematical abilities of their students.

1.2 The Mathematical Profile & Dyscalculia Test (Math-Pro)

1.2.1 The four-pronged model

The lack of consensus in the identification of the main characteristics of individuals with MLD, the comorbidity and heterogeneity that characterize MLD individuals (Bartelet, Ansari, Vaessen, & Blomert, 2014); (Watson & Gable, 2013); (Szücs, 2016) required an attempt to re-organize the main advanced hypotheses in psychology and neuroscience to study the "mathematical learning profiles" of the students. Starting from an idea which requires a vision wider than the difficulties in mathematics, (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014) have developed a codification and conceptualization of all the possible areas, domains (as they are defined in neuropsychology), which can contribute to a learning difficulty in mathematics. From a very extensive review of the literature, they have collected the difficulties and ways of dealing with these difficulties in mathematics in four areas (Figure 1.1) that classify mathematical competences in terms of (1) basic numerical, (2) visuo-spatial processes, (3) processes of memory and (4) reasoning processes (Karagiannakis, Baccaglini-Frank, & Roussos, 2017).

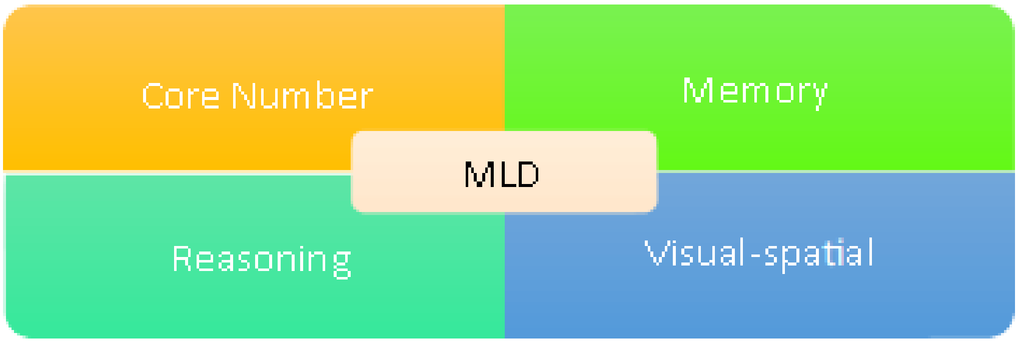


Figure 1.1: Domains of the four-pronged model (Karagiannakis et al., 2014)- MLD stands for all those cognitive difficulties that students may experience in mathematics.

1.2.2 The MathPro Test

The MathPro test is a standardized, online computer-based battery of numerical tasks that have been designed and tested to identify individual mathematical learning profiles of 1^{st} to 6^{th} grade students, therefore from 6 to 12 years of age. The test can also be used to identify students at risk of difficulties in mathematical learning, those at risk of dyscalculia or to detect children with exceptional mathematical skills.

The MathPro test is the result of the collaboration between two researchers, Giannis Karagiannakis from the National & Kapodistrian University of Athens (Greece) who had specialized in mathematics, and the child neuropsychologist Marie-Pascale Noël from the University of Louvain (Belgium). Both are experts in children's numerical development and math learning difficulties or dyscalculia.

The test, developed on the aforementioned four domains model (Figure 1.1), includes subtests that measure a wide range of abilities of pertinent mathematical skills in the following domains: Core number, Memory, Visual-spatial and reasoning.

Table 1.1 shows the classification model for MLD and the typical mathematical difficulties. The four domains model was preliminarily tested by (Karagiannakis et al., 2017) in a test involving 165 Greek students aged 10 to 12 years (mean age=11.26, 91 of whom were males). The students were subjected to a battery of mathematical exercises to be addressed via computer.

The MathPro test contains 18 mathematical tasks (Table 1.2) which are then further subdivided. The model of the four domains classified according to the domain they assess (Table 1.2) contains the following elements:

1. The Core Number domain refers to basic numerical skills: there is a comparison test of one-digit and multi-digit numbers regarding the numerical domain. As far as the visual-spatial tasks are concerned, there is the comparison of points and the subitizing test.

2. The memory tasks have been divided into two parts: the counting memory and the retrieval memory. The counting memory consists of enumeration, numbers dictation and, choosing the next and previous numbers with respect to a given one. The retrieval memory domain is instead evaluated through the addition fact retrieval, then very simple additions, and the multiplication fact retrieval.

3. The reasoning tasks have been divided into numerical and spatial. The numerical ones comprises mental calculations, word problems, calculation principles and numerical patterns. The visual-spatial domain consists of the number line from 0 to 100, the number line from 0 to 1000, squares, and building blocks.

It must be said that not all children undertook all these tasks but it depends on the attended class. The results of the experimentation with Greek children indicated strong evidence in support of the solidity of the four-pronged model (Karagiannakis et al., 2017).

Subtype	Specific systems involved	Mathematical difficulties[1]
1. Core number	Internal representation of quantity: • ANS • OTS • Numerosity-Coding • representation of symbols • Access deficit	Arithmetical domain: 1. Basic sense of numerosity (Butterworth, 2005), and estimating accurately a small number of objects e.g., 4–5 (subitizing) (Butterworth, 2010; Piazza, 2010) 2. Estimating approximately different quantities (Piazza et al., 2010) 3. Placing numbers on number lines, SNARC effect (Zorzi et al., 2002, 2005; Menon et al., 2000; Siegler and Opfer, 2003) 4. Managing Arabic symbols (Ansari et al., 2006; Rousselle and Noël, 2007) 5. Transcoding a number from one representation to another (analog-Arabic-verbal) (Wilson and Dehaene, 2007) 6. Grasping the basic counting principles (Gallistel and Gelman, 1992; Geary et al., 1996; Geary and Hoard, 2005) 7. Capturing the meaning of place value (including in decimal notation) (Russell and Ginsburg, 1984; Geary, 1993); 8. Capturing the meaning of the basic arithmetic operation symbols (+, −, ×, :).
2. Memory (retrieval and processing)	• Working memory[2] (WM) • Inhibition of irrelevant information from entering WM • Semantic memory	All mathematical domains: 1. Retrieving numerical facts (Geary, 1993, 2004; von Aster, 2000; Woodward and Montague, 2002) 2. Decoding—confusing terminology (numerator, denominator, isosceles, equilateral,...) (Geary, 1993; Hecht et al., 2001) 3. Transcoding verbal rules or orally presented tasks (Rourke and Finlayson, 1978; Rourke, 1993; Brysbaert et al., 1998; Andersson, 2007; Swanson et al., 2008) 4. Performing mental calculation accurately (Campbell, 1987a,b, 1991; Ashcraft, 1992; Andersson and Östergren, 2012) 5. Remembering and carrying out procedures as well as rules and formulas (Pellegrino and Goldman, 1987; Gerber et al., 1994) 6. (Arithmetic) problem solving (keeping track of steps) (Jitendra and Xin, 1997; Passolunghi and Siegel, 2001, 2004; Fuchs and Fuchs, 2002, 2005; Andersson, 2007; Swanson et al., 2008).
3. Reasoning	Various executive mechanisms: • Entailment • Inhibition (not connected to WM) • Updating relevant information, shifting from one operation-strategy to another • Updating and strategic planning • Decision-making	All mathematical domains: 1. Grasping mathematical concepts, ideas and relations (Schoenfeld, 1992; Geary, 1993) 2. Understanding multiple steps in complex procedures/algorithms (Russell and Ginsburg, 1984; Bryant et al., 2000; Geary, 2004) 3. Grasping basic logical principles (conditionality—"if... then..." statements—commutativity, inversion,...) (Núñez and Lakoff, 2005) 4. Problem solving (decision making) (Schoenfeld, 1992; Desoete and Roeyers, 2006).
4. Visual-Spatial	• Visuo-spatial (VS) WM[3], • Visuo-spatial reasoning/perception	Domains of written arithmetic, geometry, algebra, analytical geometry, calculus: (Geary, 1993, 2004; Rourke and Conway, 1997; Venneri et al., 2003; Mammarella et al., 2010) 1. Interpret and use spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, or geometrical figures) 2. Placing numbers on a number line (Cooper, 1984; Dehaene and Cohen, 1997) 3. Recognizing Arabic numerals and other mathematics symbols (confusion in similar symbols) (Venneri et al., 2003) 4. Written calculation, especially where position is important (e.g., borrowing/carrying) (Heathcote, 1994; Mammarella et al., 2010; Szucs et al., 2013) 5. Controlling irrelevant visuo-spatial information (Mammarella and Cornoldi, 2005; Mammarella et al., 2013) 6. Visualizing and analyzing geometric figures (or subparts of them), in particular visualizing rigid motions such as rotations (Thompson et al., 2013) 7. Interpreting graphs, understanding and interpreting when the math information are organized visual-spatially (tables)[4].

[1] These can also be read as "mathematical skills" if the model is being used to identify the student's stronger specific systems.

[2] In particular the phonological WM used in selecting verbal over spatial information as relevant (for e.g., De Smedt et al., 2010).

[3] There is increasing evidence showing that many of these difficulties may be related, but not limited, to deficits in VSWM (Heathcote, 1994; Cornoldi et al., 1999; Kyttälä et al., 2003; Mammarella et al., 2006, 2010).

[4] Difficulties of type 4.7 are well known in the mathematics education literature, but we are not aware of studies that relate these to basic cognitive abilities.

Table 1.1: Classification model for MLD, proposing 4 subtypes, possible specific systems, and typical mathematical difficulties - Karagiannakis et al. 2014.

Task	Subgroup	Domain
1. Single-digit numbers comparison	Numerical	CORE NUMBER
2. Multi-digit numbers comparison		
3. Dots Comparison	Spatial	
4. Subitizing		
5. Enumeration	Counting	MEMORY
6. Numbers dictation		
7. Next number		
8. Previous number		
9. Addition facts retrieval	Retrieval	
10. Multiplication facts retrieval		
11. Mental calculations	Numerical	REASONING
12. Word problems		
13. Calculation principles		
14. Numerical patterns		
15. Number lines 0-100	Spatial	
16. Number lines 0-1000		
17. Squares		
18. Building blocks		

Table 1.2: Grouping of the tasks of the MathPro Test.

1.3 The Italian standardization of the MathPro Test

1.3.1 Participants

The population tested in the MathPro test standardization study, throughout Italy, consists of 1,728 children from Grades 1-6, coming from 22 schools scattered around Italy (Table. 1.3).

Grade	Sex		Sum
	Boys	Girls	
1	104	90	194
2	109	132	241
3	174	180	354
4	154	185	339
5	146	158	304
6	136	160	296
Sum	823	905	1728

Table 1.3: Sample used for the standardization of MathPro in Italy.

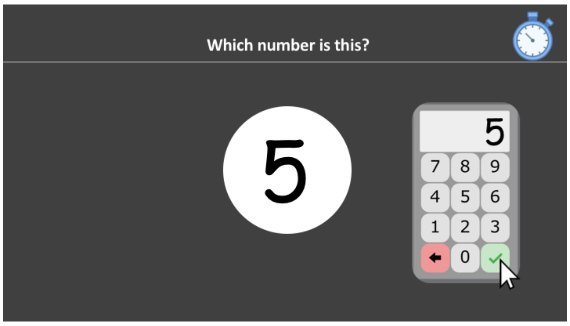


Figure 1.2: Numbers typing.

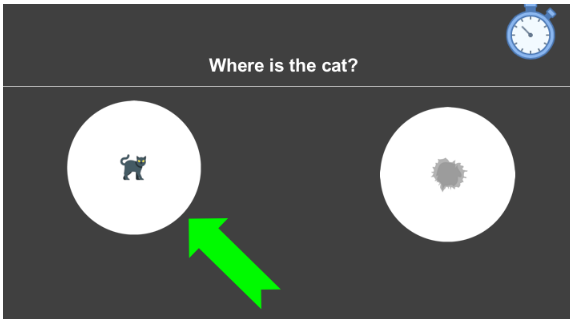


Figure 1.3: Find the cat.

1.3.2 The MathPro Battery

1.3.2.1 Reaction time Test on the ability to use the instrument

Target group: All grades

Before the start of the actual test, the children had to perform two prerequisite tests: 'Numbers typing' (Figure 1.2), selecting as fast as possible the number in the calculator next to it, and 'Find the cat' (Figure 1.3), in which they had to find and click on the cat that appeared in a circle. These first very simple two tests permitted the evaluation of their ability to use the instrument, the speed with which they were able to use the mouse and to calibrate the parameter based on their actual individual capacity. The parameters on which they were measured were the 'reaction time' parameter and the 'accuracy' parameter. Reaction time was computed with a millisecond precision.

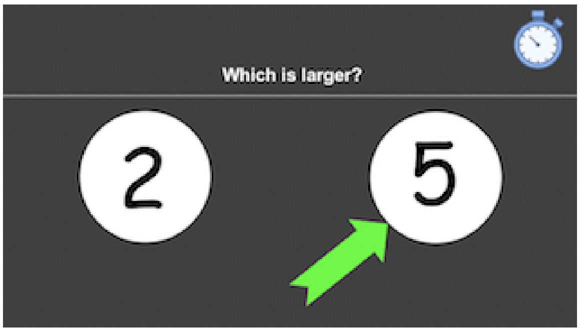


Figure 1.4: Single digit numbers comparison - Which is larger?

1.3.2.2 Single-digit numbers comparison and Multi-digit numbers comparison (Number magnitude Comparison)

Target group: All grades (different ending point)

The response time is calculated as a measurement of performance if the accuracy is large enough.

The first test to which all the children were subjected was the comparison test of single-digit numbers (Figure 1.4) in which two Arabic numbers were presented on the screen and the child had to click with the computer mouse on the larger number between the two, as quickly as possible. There were 36 comparison pairs in total. The reaction time was calculated as well by using the 'reaction time' parameter.

The comparison tests of multi-digit numbers followed; it was evaluated with the same 'reaction time' parameter: two-digit numbers (Figure 1.5), three-digit numbers (Figure 1.6), four-digits numbers (Figure 1.7) and decimal numbers (Figure 1.8). Grade 1 students were only presented with the single-digit numbers, Grade 2 students were also presented with 2 and 3-digit numbers, Grade 3 children were also presented with 4-digit numbers and all the items were presented to older students.

1.3.2.3 Dots Comparison (Dots Magnitude Comparison)

Target group: All grades

The accuracy is the measurement of performance.

In this task (Figure 1.9), the children were asked to select the image containing more

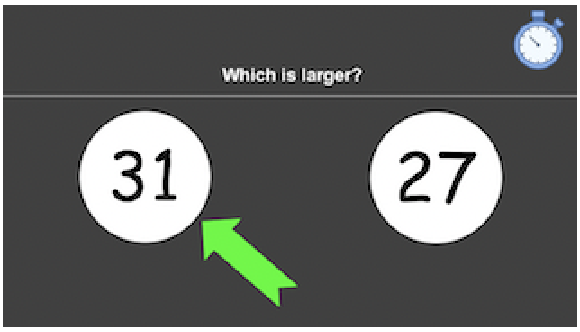


Figure 1.5: Two-digit number comparison - Which is larger?

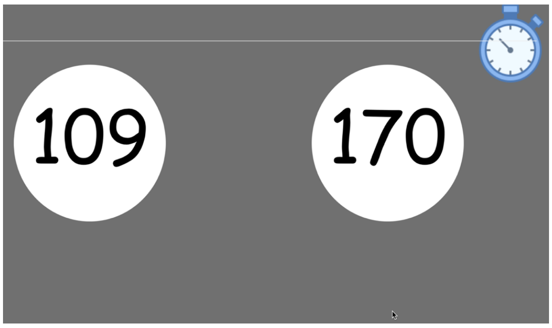


Figure 1.6: Three-digit number comparison - Which is larger?

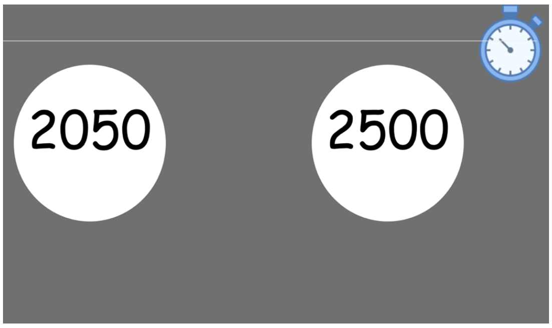


Figure 1.7: Four-digit number comparison - Which is larger?

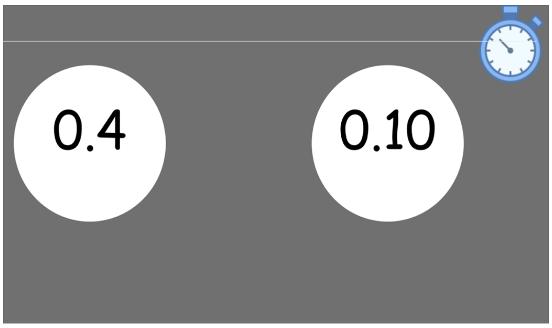


Figure 1.8: Decimal numbers comparison - Which is larger?

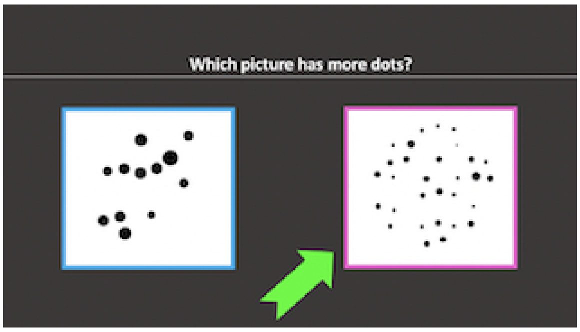


Figure 1.9: Dots comparison - Which picture has more dots?

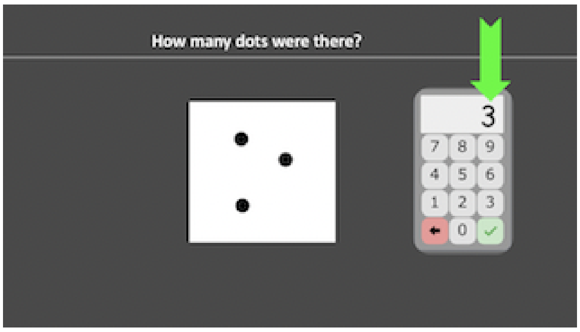


Figure 1.10: Subitizing - How many dots are there?

dots. Two collections of dots were presented on the computer screen very quickly and therefore the student did not have the time to count the dots but simply had to evaluate the larger set at a glance. The pairs were ordered by increasing ratio difficulty, that is, 2:3; 3:4; 4:5; 5:6; 6:7. Sets of black dots were created based on Gebuis and Reynvoet's work (2011). Controls were used for perimeter, area, density, etc.

1.3.2.4 Subitizing (Flashed dots)

Target group: All grades

The accuracy is the measurement of performance.

In the Subitizing test (Figure 1.10, Figure 1.11 and Figure 1.12), 1-6 dots appeared for 300 milliseconds on the computer screen and then disappeared covered by a grid. Then the child was asked to select the number of dots he could count by clicking on the computer. The images were presented very quickly, and even in this test, accuracy was the performance evaluation parameter.

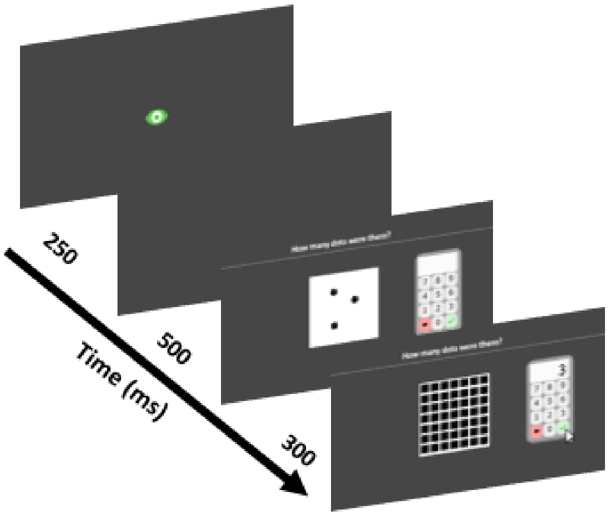


Figure 1.11: Subitizing.

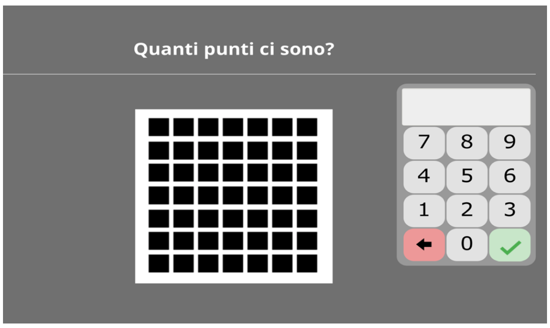


Figure 1.12: Subitizing - How many dots are there?

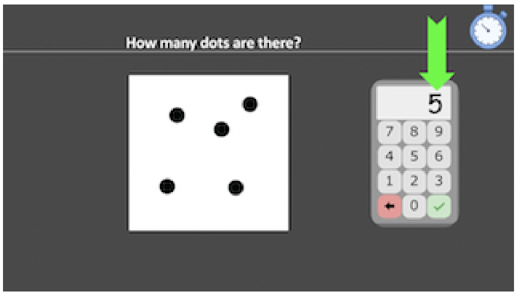


Figure 1.13: Enumeration - How many dots are there?

1.3.2.5 Enumeration (Fast counting)

Target group: All grades

The response time is calculated as a measurement of performance if the accuracy is large enough.

In this test (Figure 1.13), students were asked to indicate the total number of randomly arranged black dots (from 7 to 13) and to select the number of dots they had been able to count by clicking on the computer screen calculator.

1.3.2.6 Numbers dictation (Number-word to Arabic digits transcoding)

Target group: All grades

The performance is measured through the response time (starting from the beginning of the sound file and ending with the child's click on tick button to validate the answer) unless the accuracy is large enough.

This test concerned the child's memory, especially in counting Figure 1.14. The child listened to a number, through headphones, which was dictated by a voice through computer speakers. Then he had to select the number he had heard in a symbolic form on the calculator as quickly as possible. The items varying from single-digit to five-digit numbers. Stimuli were 30 items presented in order of increasing number of digits. The task stopped after three consecutive errors.

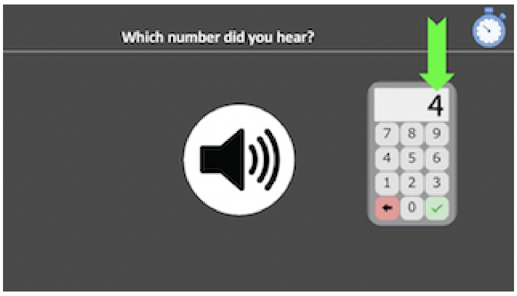


Figure 1.14: Numbers dictation - Which number have you heard?

Figure 1.15: Next number - Which number comes just after?

1.3.2.7 Next number (Subsequent consecutive number)

Target group: All grades (different ending point)

The response time is calculated as a measurement of performance unless the accuracy is large enough.

In this task (Figure 1.15), the child had to listen to a number on headphones and select the subsequent consecutive number as quickly as possible by clicking on the computer screen calculator. Eighteen items were presented: 6 single-digit, 6 two-digit and 6 three-digit numbers, presented in order of increasing number of digits. Only the first twelve trials were presented to grade 1 students.

1.3.2.8 Previous number (Preceding consecutive number)

Target group: All grades (different ending point)

The response time is calculated as a measurement of performance unless accuracy is large enough.

Here (Figure 1.16) children were asked to identify the previous consecutive number

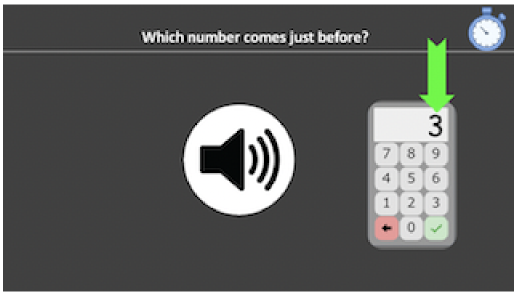


Figure 1.16: Previous number - Which number comes just before?

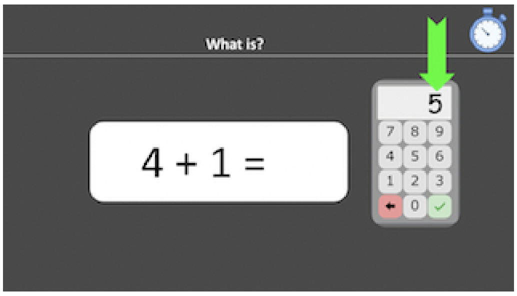


Figure 1.17: Addition facts retrieval - What is the result?

as quickly as possible by clicking on the computer screen calculator: *"Which number comes just before?"* The number ranged from single to three-digits and after three consecutive errors the test stopped. There were 18 items in total and only the first twelve items (1 and 2-digit numbers) were presented to grade 1 students.

1.3.2.9 Addition facts retrieval (Simple additions)

Target group: All grades

The response time is calculated as a measurement of performance unless accuracy is large enough.

This test (Figure 1.17) checked whether students could answer simple sums with addends from 2 to 9, the answer of which was always less than 11. There were twelve stimuli.

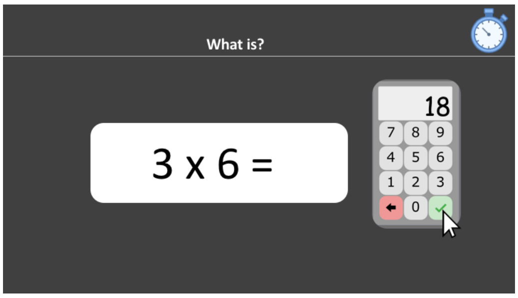


Figure 1.18: Multiplication facts retrieval - What is the result?

1.3.2.10 Multiplication facts retrieval (Simple multiplications)

Target group: Grade 3 - Grade 6

The response time is calculated as a measurement of performance unless accuracy is large enough.

As for addition, even in this task (Figure 1.18), there were simple multiplication calculations to make using single factors from 2 to 9. Children were asked to type the correct product as quickly as possible by clicking on the computer screen calculator. Fourteen single-digit multiplications were presented.

1.3.2.11 Mental calculations (Additions, Subtractions, Multiplications, Divisions)

Target group: Grade 2 - Grade 6

The response time is calculated as a measurement of performance unless accuracy is large enough.

In these tasks, addition, subtraction, multiplication and division operations (Figure 1.19, Figure 1.20 and Figure 1.21) with multi-digit numbers (up to three digits long) were performed. There were 24 stimuli ordered in terms of increasing difficulty. While in the second primary class only additions and subtractions were included, the complete battery was given in Year 3,4,5 and 6 classes.

1.3.2.12 Word problems

Target group: Grade 2 - Grade 6

The accuracy is calculated as a measurement of performance.

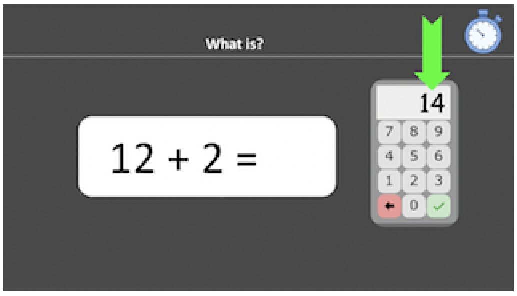


Figure 1.19: Mental Calculations - What is the result?

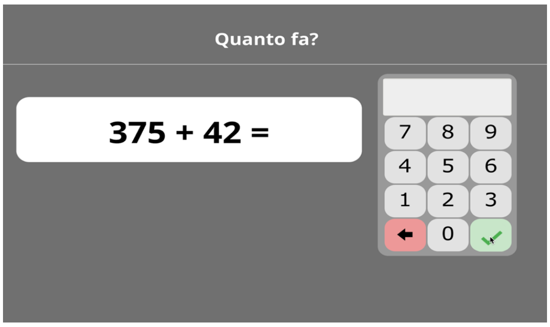


Figure 1.20: Mental Calculations - What is the result?

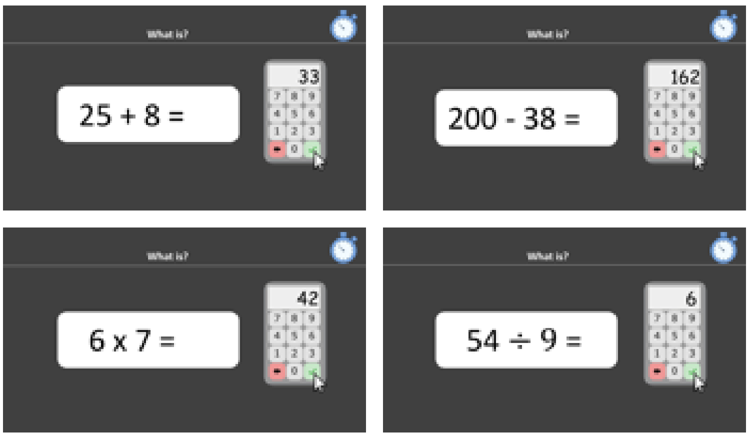


Figure 1.21: Mental Calculations - What is the result?

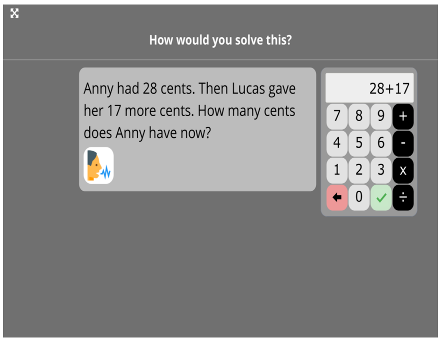


Figure 1.22: Word problems - How would you solve this?

In this task (Figure 1.22) the text of a problem was given, and the child was asked to write the operation necessary to solve it in the calculator, but not the final solution in order to evaluate the pupil's ability to reason (for example 28+17). In fact, the ability to calculate had already been measured in other tasks. Eighteen problems were selected ordered in increasing difficulty and only the first ten trials were presented to Grade 2 students. The task stopped after three consecutive errors.

1.3.2.13 Calculation principles (Solve the second based on the first)

Target group: Grade 3 - Grade 6

The accuracy is calculated as a measurement of performance.

In this test (Figure 1.23) a pair of numerical equivalents with two-digit numbers appeared on the screen, the first complete and correct, while the second one was missing a member. The student had to report on the screen keyboard the correct number of the missing member without calculating it, but with reference to the first problem. This part tended to explore property-based reasoning skills. Fifteen stimuli were presented and the task stopped after three consecutive errors.

1.3.2.14 Numerical patterns (Patterns' recognition - Find out the missing number)

Target group: All grades

The accuracy is calculated as a measurement of performance.

The task required to complete a sequence (Figure 1.24). In this task, the student was

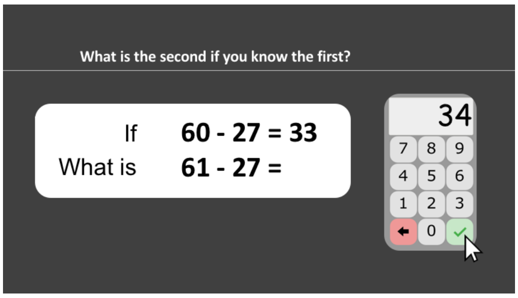


Figure 1.23: Calculation Principles - What is the second if you know the first?

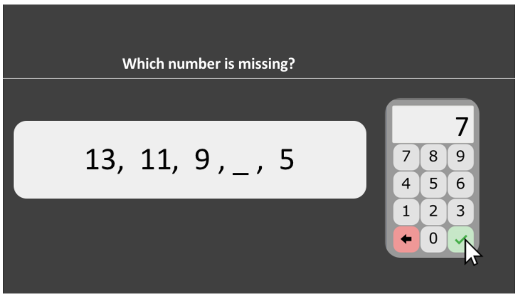


Figure 1.24: Numerical patterns - Which number is missing?

presented with sequences of numbers in which one was missing, in the central position. The student had to find the missing number by inferring the regularity used to generate the sequence and then type it on the keyboard. Eighteen stimuli were presented and the task stopped after three consecutive errors.

1.3.2.15 Number lines 0-100

Target group: All grades

The accuracy is calculated as a measurement of performance.

This task was a line test of numbers from 0 to 100 (Figure 1.25). Students were asked to place the number presented in the center of the computer screen, within the number line, by marking it with the mouse. Twenty-two stimuli were presented.

1.3.2.16 Number lines 0-1000

Target group: Grade 3 - Grade 6

The accuracy is calculated as a measurement of performance.

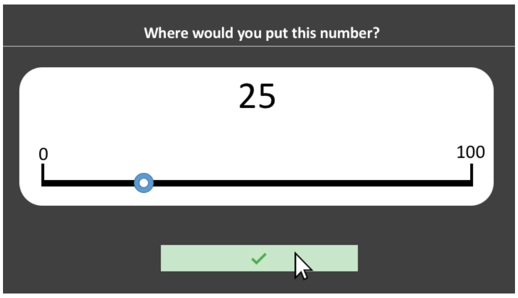


Figure 1.25: Number lines 0-100 - Where would you put this number?

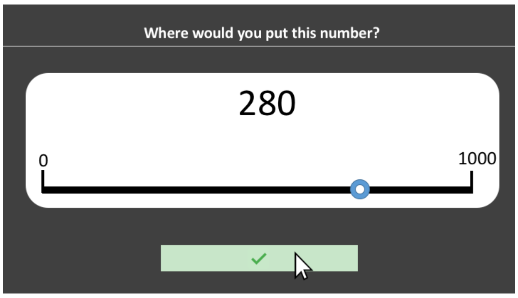


Figure 1.26: Number lines 0-1000 - Where would you put this number?

This task was for older children (Figure 1.26), from the third Grade, and, as in the previous delivery, they were asked to place a number on the number line presented in the center of the computer screen with 0 on the left end and 1000 at the right end.

1.3.2.17 Squares (2D Shapes)

Target group: All grades

The accuracy is calculated as a measurement of performance.

In this task, the student (Figure 1.27) had to derive the total number of squares from which each figure was composed, possibly by recomposing the squares with the small triangles that were used to form the total squares. The child was then asked to type the total number of whole squares by clicking on the computer screen calculator. This task aimed at examining the ability to decompose and reconstruct two-dimensional figures and all their components.

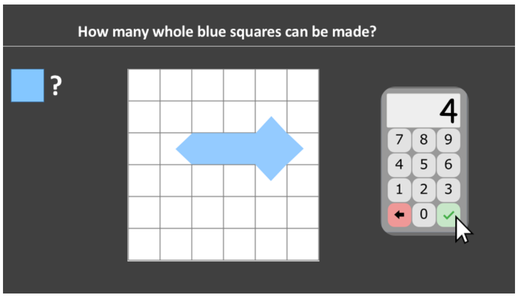


Figure 1.27: 2D Shapes (Squares) - How many whole blue squares can you do?

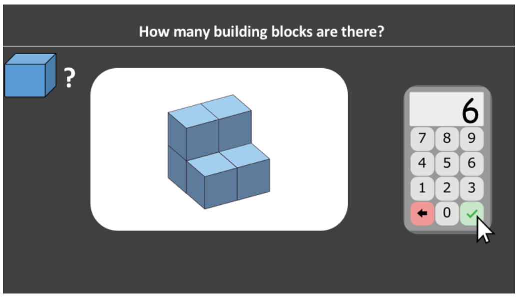


Figure 1.28: 3D Shapes - How many Building Blocks are there?

1.3.2.18 Building blocks (3D Shapes)

Target group: All grades

The accuracy is calculated as a measurement of performance.

In this task (Figure 1.28) students were asked to count all the cubes necessary to compose the three-dimensional figure. They also had to take into account any possible hidden cube.

The time required to complete the MathPro test ranges from 40 minutes to an hour depending on the class the student belongs to. The software then generates the profiles that teachers can print out or examine on the screen.

1.3.3 Statistical analyses

A statistical analysis of the results produced the Pearson correlation coefficients for the experimental battery, a Principal Component Analysis (PCA), and a Confirmatory

Figure 1.29: Students at work.

Factor Analysis (CFA).

Statistical analyses were performed using IBM SPSS 21 and AMOS 21. The Principle Components Analysis of the tasks of the MathPro Test is shown in Table 1.4, while Table 1.5 presents the Pearson's correlation coefficients between the value of the tasks of the experimental battery (p values: $^{***}p < .001$; $^{**}p < .01$; $^{*}p < .05$). Most of the coefficients were statistically significant (Karagiannakis et al., 2017). The different battery tests show how MathPro is structured. The statistical analysis of the general sample data confirms the reliability of the tests. The results, moreover, are consistent with the literature on the development of particular skills: the plots show that children evolve whatever they process, in terms of speed or in terms of accuracy. Given the significance of the results, the battery can be administered to derive students' mathematical learning profiles that are reliable.

1.3.4 The Results of the Italian experimentation and standardization

The results of the standardization for the Italian population are analyzed in this section. An example of students at work is shown in Figure 1.29.

	Components			
	Reasoning	Facts retrieval	Core number	Number lines
Mental calculations	.77			
Equations	.74			
Word problems	.70			
Number lines 0-1000	.70			
Maths terms	.59			
Calculations principles	.59			.47
Multiplication facts retrieval		.88		
Addition facts retrieval		.86		
Dots magnitude comparison			.80	
Subitizing-Enumeration			.79	
Number magnitude comparison			.66	
Ordinality				.84
Number lines 0-100				.64
Eigenvalues	4.57	2.14	1.09	1.03
% of variance	35.12	16.47	8.41	7.93

Table 1.4: The principle components Analysis (varimax) of the tasks of the MathPro test - Karagiannakis et al. 2017

	2	3	4	5	6	7	8	9	10	11	12	13
1. Subitizing-Enumeration	.64***	.49***	.48***	.47***	.20*	.14	.09	.29***	.23**	.17*	.33***	.29***
2. Number magnitude comparison	–	.41***	.55***	.49***	.21*	.21**	.18*	.34***	.30***	.19*	.48***	.30***
3. Dots magnitude comparison		–	.30***	.30***	–.00	.00	.16	.03	–.01	–.12	.11	.04
4. Addition facts retrieval			–	.92***	.10	.17*	.18*	.41***	.25**	.27**	.43***	.25**
5. Multiplication facts retrieval				–	.17*	.18*	.21*	.42***	.25**	.27**	.40***	.18*
6. Number lines 0-100					–	.33***	.22**	.28***	.33***	.20*	.25**	.28***
7. Ordinality						–	.03	.19*	.28***	.17*	.22**	.24**
8. Number lines 0-1000							–	.41***	.21**	.45***	.45***	.40***
9. Maths terms								–	.46***	.47***	.54***	.42***
10. Calculation principles									–	.36***	.46***	.48***
11. Mental calculations										–	.66***	.56***
12. Equations											–	.55***
13. Word problems												–

*$p < .05$; **$p < .01$; ***$p < .001$.

Table 1.5: Pearson's correlation coefficients between the tasks of the MathPro test - Karagiannakis et al. 2017

Figure 1.30: Achievement levels.The grey zone corresponds to the AVERAGE performance. The green zone corresponds to HIGH or EXCELLENT performance. The yellow zone corresponds to LIMITED or POOR performance.

1.3.5 Some emerging mathematical profiles

The results of the tests allow to describe some profiles, to discover the difficulties that may arise and to give indications to the teachers. The purpose of this tool is to gather the most complete and thorough information on the students, starting from their performance in math. One of the strengths of the MathPro platform is that it provides not only quantitative but also qualitative individual profiles. The report obtained for each student provides a view where, for each block of questions, the performance for the single task is evaluated with a score and for each task the pupil's performance is returned as a percentile so as to be able to see how it conforms to the norm. The scores can indicate (Figure 1.30) a really poor performance, a limited performance, a wide range of individual variability in the standard, a high performance and a really excellent one. Each individual profile is returned in a plot (Figure 1.31 -1.37) indicating the student's percentile result in each individual task.
For each block of questions, the percentile in which the student's performance is compared to the students of the Italian sample of the same grade is given. In particular, the bars describe the following possible situations:

- below the 15th percentile: very low performance, warning signal;
- below the 30th percentile: low performance;
- above the 30th percentile: average performance;
- above the 70th percentile: high performance;
- above the 85th percentile: excellent performance.

The tasks are grouped and presented according to the different domains immediately highlighting the performance in both numerical and spatial basic skills, the areas of counting and retrieval memory, numerical and spatial reasoning. The restitution also visually offers a harmonic or non-harmonic snapshot of what the trends are. Each bar indicates the child's performance in the individual test. Each bar represents a task with bars of the same color representing a similar subdomain. There are usually similar results in similar subdomains while possible discrepancies can be grasped in different domains. Not being able to exemplify all the obtained profiles, some typical examples are illustrated.

CASE 1 - Dyscalculia

These are two profiles of fourth-grade pupils with diagnoses of dyscalculia (Figure 1.31, Figure 1.32). The students are in the same class and the evaluations of their math teacher are generally moderately good. The two profiles resulting from the test are different, even though the diagnosis is the same: dyscalculia. These results support the hypothesis that MLD can have multiple origins (Dowker, 2005), (Jordan, Hanich, & Kaplan, 2003), (M. M. Mazzocco & Myers, 2003), (Szücs, 2016). For dyscalculics, the Italian law 170 of 2010 foresees the use of a calculator in mathematical procedures at school, but a calculator does not solve the problem when reasoning and not merely calculation is required. It must be clear what dyscalculia means. A diagnosis of dyscalculia is applied to very different cases; in fact, it is a label indicating a category of situations. The two examples presented in this section are a clear demonstration: there are strong and evident differences even in presence of the same diagnosis. The ability to reason is scarce for the first pupil (Figure 1.31) but the second pupil (Figure 1.32) demonstrates a very good attitude in the reasoning spatial tasks. The first profile is an example of extremely low performance over the whole spectrum (except in Subitizing). Tasks requiring reasoning are those closest to what is performed at school, so the teacher's evaluation is very often aligned with the student's performance in this area, but this is not these students' case. It is not unusual for a student with serious difficulties in mathematics to fail in tasks in this area because they require more skills and much more complex reasoning than other tasks. For the first student the mnestic features fall within the high level in the subitizing task (consisting of a number of dots varying from 2 to 6 presented on the screen and the student was asked to indicate the total number very quickly), this is not the case for the second student but there is the pre-attentive mechanism of recognition of small amounts. However, where large numbers were requested, there was a drop in accuracy for both students. In general, the idea is that these two tasks evaluate two early distinct mechanisms of sensitivity and numerosity. As it happens, the students fail when they should be able to sustain these mechanisms by acquiring the symbolic system, comparing and indicating which number is bigger. Numerical skills, however, seem not to be very solid, given the fact that the children had not acquired any scholastic competence in verbal sequencing and the memorization of calculation. When they are asked to reason they fall into a state of anxiety. Their abilities to make comparison between multi digit numbers in a symbolic format and numerical reasoning, are extremely weak. The visual-spatial aspect is very poor only for the first student. It would be useful to retrieve more qualitative data for these profiles to provide much more information to complete the picture. The DSM5 reports "specific disorder with impaired mathematical skills" within which very different profiles can be found. The concept of dyscalculia is inserted by the DSM5 in the notes claiming that it is a component of this great family of difficulties in mathematics, so it is effectively a superposition between two concepts. The certification says "dyscalculia", but it is much more. It is a difficulty in mathematical reasoning and problem-solving. What disorients parents and teachers is the fact that, even following the indication to use the calculator as compensating tool, the child does not improve because in fact this is not the problem. Given the diagnosis of dyscalculia, therefore, one should ideally not simply use a label, but obtain a functional profile that gives the teacher indications to understand what weakens instrumental learning and what resources are available to work with.

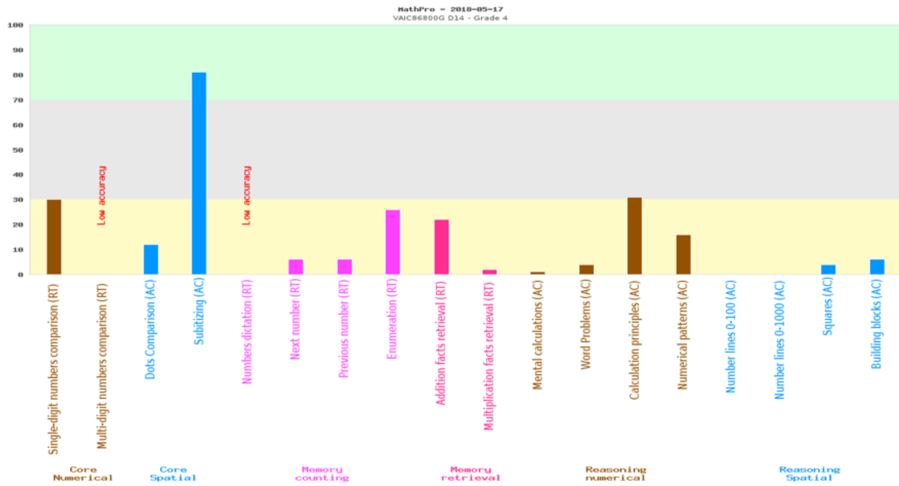


Figure 1.31: Grade 4; LD: dyscalculia; teacher evaluation in maths: moderate; teacher evaluation in literacy: moderate. Gender M

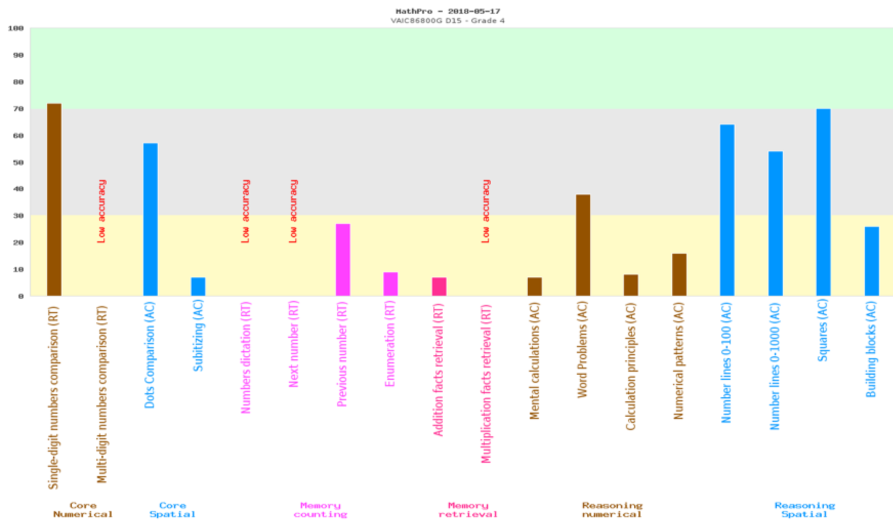


Figure 1.32: Grade 4; LD: dyscalculia; teacher evaluation in maths: moderate; teacher evaluation in literacy: moderate. Gender: M

CASE 2 - Twins

Figure 1.33 and Figure 1.34 show the profiles of two twins who had no certification reporting learning difficulties. They were in the same second class and were evaluated by the teacher as good in math and in verbal performance. Their profiles are very different. The results below the 50th percentile indicate a limited ability in mathematical activities. In these profiles the difference in results is relevant. Each profile is not a low profile but a conflicting one with points of excellence. For both students the system indicates ‘low accuracy’ in the task regarding Enumeration. When the accuracy is below a certain threshold that varies according to the task, or when the student makes too many errors, the system indicates low accuracy because there is no point in evaluating the action time in these cases. The children received a good grade by their mathematics teacher, but these results require a reflection on their failures. These results must be looked into by the teacher and researcher to understand why they are so divergent. In particular, the teacher should reflect on the indicators used to evaluate his students and the researcher on the MathPro features. During these meetings, it became clear the importance of mutual stimulation between the teachers and the research group.

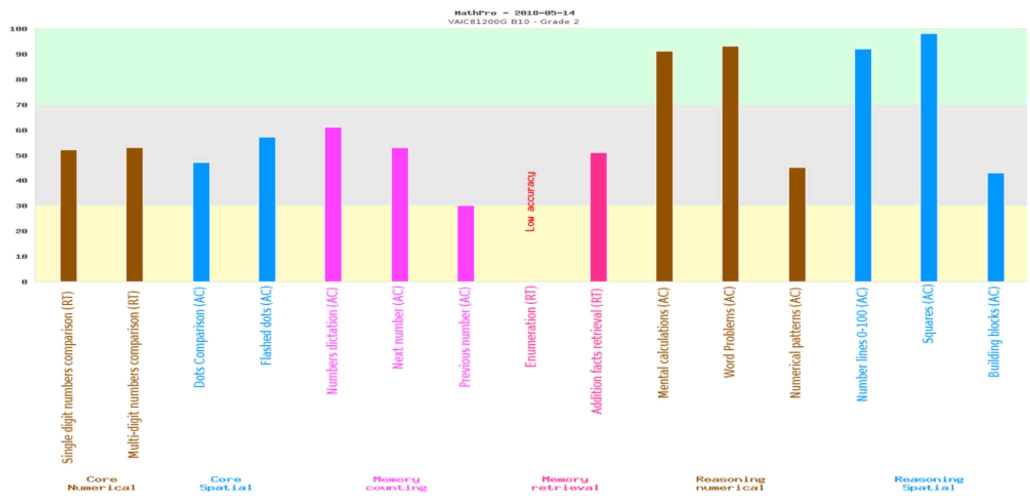


Figure 1.33: Grade 2; LD: no; teacher evaluation in maths: good; teacher evaluation in literacy: good. Gender: F.

1.3.6 Qualitative analysis

1.3.6.1 Individual Report

MathPro returns an individual 20-page report for each child who has been tested. For each block of tasks, the percentile in which the student’s performance is placed is indicated. Moreover, the software also returns the errors committed in all tasks and

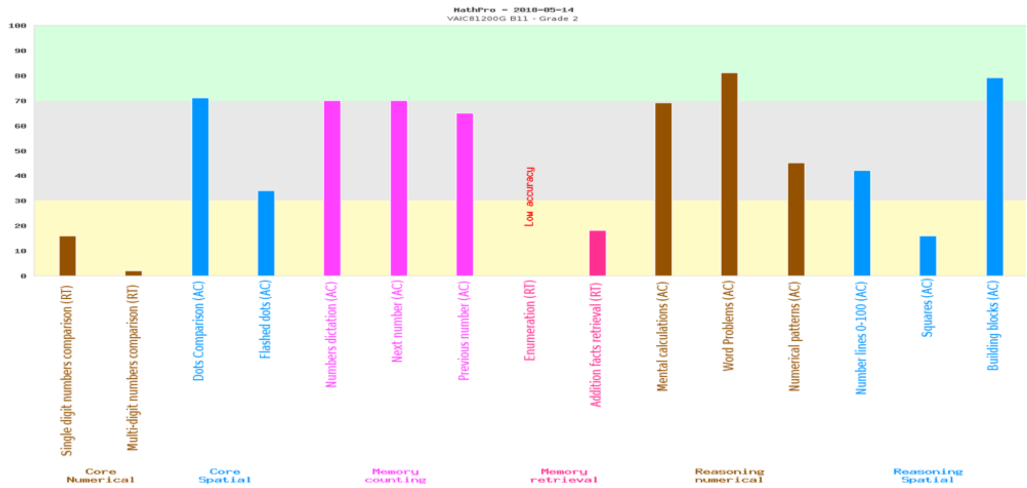


Figure 1.34: Grade 2; LD: no; teacher evaluation in maths: good; teacher evaluation in literacy: good. Gender: F

this is the main source of information to understand the qualitative profile. Clearly in all tasks with a dichotomous answer, ”is it bigger on the left or on the right?”, the error says nothing, in the sense that you have a ’yes’ or a ’no’. Another fundamental aspect of this tool is that, with this trick of giving the possibility to respond by typing on the calculator, it is able to present many tasks asking for the production of an answer, something that usually does not happen with other softwares. In fact, there are digitized tools that clearly, by requesting a response on the computer, allow the student to say ’yes’, ’no’ or ’choose the correct answer between two answers’ and so on. This greatly limits the possibility of observing the child’s resources or his way of thinking or the process underlying the production of an answer, features that MathPro allows.

1.3.6.2 ’Bravo in Maths’: Student Profile

It is important to devote time to the analysis of an individual profile. In fact, by studying the qualitative aspect of the test result in depth, we can understand and interpret each mistake a student makes. This is a profile of a Year 3 child performing in a good way with falls that need to be assessed (Figure 1.35). All results were good with some area of excellence but for two performances, Calculation principles and Squares (therefore two-dimensional spatial abilities) the results were very poor. In the individ-

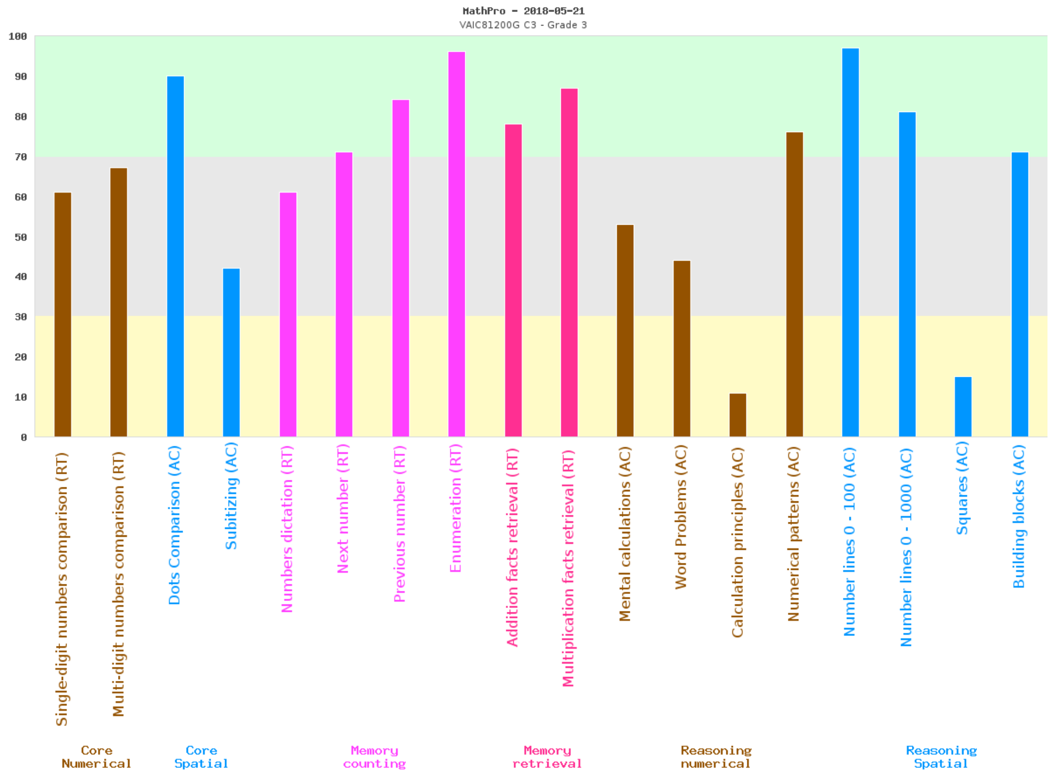


Figure 1.35: Gender: male; age:8; grade: 3; date of birth 2009-12-01; LD: no; date of test: 2018-05-21; testing language: Italian; teacher evaluation in maths: good; teacher evaluation in literacy: good.

ual report (attachment 1), the description of the task, the screen presented for the task, the performance allocated in percentile form and, obviously, the qualitative data are registered for each task. For example, for the Multi-digit numbers comparison task, the qualitative data do not appear, but they indicate which item was wrong and which number was chosen:

- Multi-digit numbers' comparison: the child had to compare 411 and 289 and he made a mistake when indicating the larger of the two. As far as speed was concerned, instead of analyzing the first digit, the pupil was guided by 8 and 9, the higher numbers of the three digits (these are effects that have been experimentally studied and that are misleading when a student is under time pressure). It is necessary to reflect on the fact that often the error made by the pupil, when under pressure, follows a certain kind of reasoning which means that in any case, he has applied a strategy that may not be always effective but the fact remains that he has used some underlying logic. Obviously, as only two numbers are involved, he may have also responded randomly.

- Dots comparison: dichotomous answer, excellent performance resulting in a percentile grade of 90.

- Subitizing: on average.

- Numbers dictation: his score was in the norm. Despite being an average performance, he made 4 errors which, in fact, are errors based on more complex stimuli. When the stimulus was 30,600 he wrote 300,600 (one 0 more), instead of 52,000 he wrote 5,200 (one 0 less), instead of 10,040 he wrote 1,040 (one central 0 less), and instead of 80,400 he wrote 800,300 (this was the most striking error because there is also a lexical error with the replacement of 4 with 3, but this might also have been a digitization error). Sometimes the child removed a zero but since there is always at least one more zero in the number, his errors are less likely to be digitization errors, rather, they could be oversights, or fragility in controlling the syntactic aspects. These are actually syntactically very complex stimuli and errors of this type are normal.

- Next number and also Previous number: performed very well.

- Enumeration: excellent.

- Addition facts retrieval: high performance, 11 correct over 12. There is only one error '4 + 3 = 6' which could be a typing or a distraction error.

- Multiplication facts retrieval: excellent, '4 x 9 = 3' is clearly a typo, the 6 key was probably pressed briefly; '4 x 7 = 12', is wrong.

- Multiplication facts retrieval: excellent.

- Mental calculations: on average, but there are errors. '375 + 42 = 416', instead of 417, a calculation error, but the procedure is in the perfect range (416 – 417 = -1). '35 - 7 = 48' instead of 28. Here the child may have calculated subtraction correctly but then added it; '48 - 16 = 42', again the tens were wrong, the child was probably struggling to keep the loan in memory. '200 - 38 = 162' the range is there but then the student made mistakes. '7 x 9 = 64' is close and above all it is the product of '8 x 8'. This is a small mistake which can mean that the child has memorized the tables, but he chose the wrong one, as he had not learned them by heart. '23 x 3 = 96' instead of 69 could be a digitization error. '38 x 3 = 412' instead of 114. The child may have multiplied '3 x 8 = 24' then wrote 4 in the hundreds column therefore writing the answer back to front.

- There are many errors concerning spatial visualization, perhaps it would be worth repeating the test without the visuals to avoid misleading the pupil.

- Arithmetic problems: on average.

The child always mistook which operation to use. Instead of adding 2 digits he divided them. Instead of subtracting them, he divided them. However, the child was only in third grade. Doubtless, these errors are also due, in part, to his limited experience. His final result was in the norm but all the difficult exercises were, in fact, wrong.
When the MathPro staff was returning his profile analysis, his teachers reported that the child was afflicted with a serious eye problem which had been discovered after he took the MathPro test in May. During the summer, in fact, the child had told his parents (nobody had ever noticed anything), that he couldn't see well from one eye. He had always thought it was normal to see with one eye. An eye examination showed that he had had the problem from birth. With a lens on the eye, writing and stacking have also improved. The teachers also suspected he was dysgraphic because of his poor handwriting. The DSA manager of the school had the child undergo a dyslexia test, as was the routine for all pupils in the third grade, but no anomaly was found. The child

recovered from the calculation deficiencies he had fallen into in his third year, and he improved in the use of the ruler when he began wearing glasses in September 2018. His poor calligraphy, however, has remained.

MathPro detected a problem, the errors were not random but guided by something. In depth attention given to the analysis of the individual profile and knowledge of the general situation of the child revealed that the child had performed only one of the 15 Calculation principles correctly. Observing the errors, we noted that he repeated the given result. This was probably because they were operations that varied only on one single item. He was required to scan the numbers carefully in a standardized way because he had to break down and reconstruct the equation. Clearly, he was unable to find the correct answer, but he was able to find a response strategy: ”the software tell me not to calculate, to use the previous answer, and then I give the result“.

- Numerical pattern: very good. Some mistakes but he is above average for his age. When spatial aspects were not exclusively tested, he was excellent and was very good at numerical estimates.

- Number lines: in this qualitative number task the child’s performance was very good because the task required the student to make an estimate of the correct number. The child had to place number 21 on a ’more or less’ space continuum. Unless by incredible fortune, no one ever places the mouse exactly on 21 (maybe on 21.2 or on 20.9). What the software did was to show the difference between the target number and the given number. An example asking for number 8 revealed the child positioned the mouse on 7.67 which was an extremely good result. Only when the proportional deviation was larger than 10% an error was considered. Data are provided but it is necessary to check to understand whether the qualitative data are informative or not.

- Squares: errors that make sense, a lot of sense. In fact, the task required the student to indicate how many small squares could be formed from those in black.

The child always indicated the squares that, partially or totally, were blackened. In fact, it was as if he considered the space occupied by the squares of the superimposed figure, even when some small squares were only partially blackened. So, again, he uses logic in his answers, even when incorrect, which, moreover, can be ascribed to his visual defect.

- Building blocks: good. In three-dimension tasks, where in reality space is free from the matrix.

This analysis allows us to understand how qualitative data become essential to give meaning to the quantitative profile. Sometimes there are unexplained downturns, but we can deduce that the student did not understand the delivery, so he made a systematic error (for instance always the Previous number instead of the Next number), or that some errors, random or not, can be qualitatively informative. In fact, from this profile the difficulty in the visual-spatial area of the child emerged and certain inexplicable failures understood.

1.3.7 Comparison of Year 1student profiles belonging to the same class

The plots in Figure 1.36 and in Figure 1.37 show two profiles of Year 1 pupils in the same class, therefore with the same teachers but each with very different performances and evaluations. In first grade, it might make sense to use the test to understand the diversity of skills developed by children from their previous experiences such as those gained from the family and kindergarten. In first grade, there are huge differences that, over time, will be healed and made uniform. It would, therefore, be useful to administer the test to whole classes, to classes of the same level or even to whole schools and analyze the obtained data. The system would be able to group student profiles with mathematically similar performances in delivery and provide information on the trends towards mathematical competence in different domains.

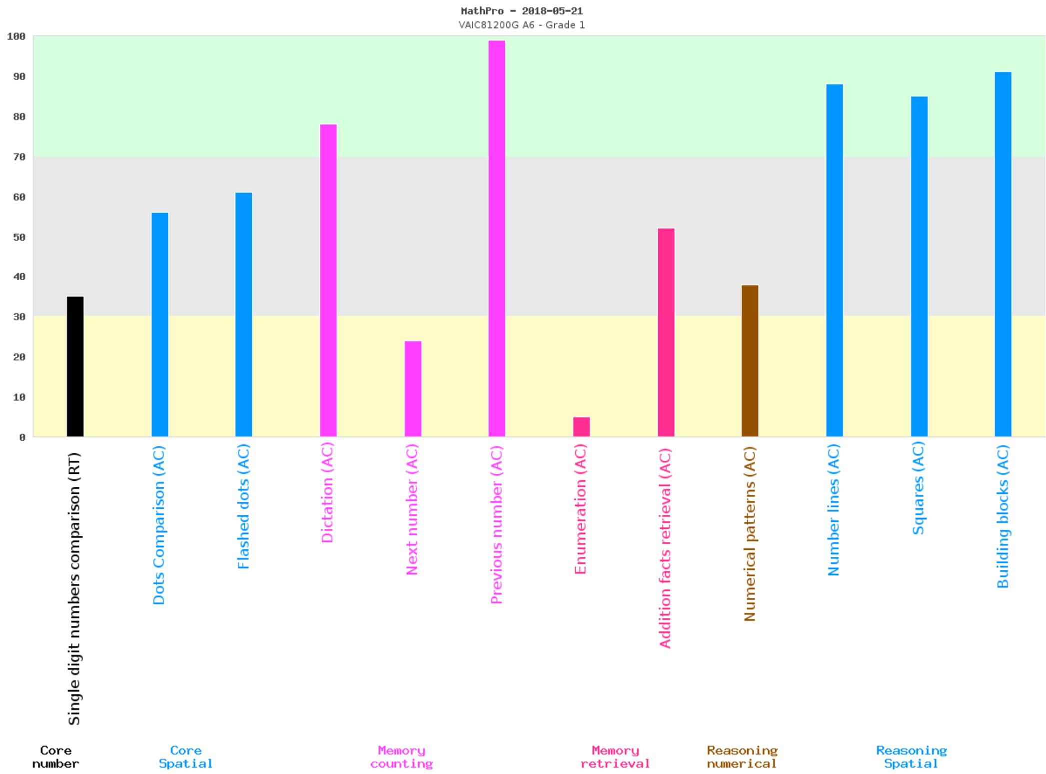


Figure 1.36: Grade 1; teacher evaluation in maths: very good; teacher evaluation in literacy: good.

1.4 Teacher's attitude

1.4.1 Traditional school and technological school

The author of this thesis took part in the actual implementation following the administration of the test to detect difficulties/excellences in mathematics in two schools that from now on will be indicated as: 'Traditional school' and 'Technological school'.

Traditional school - Classes: First, Second and Third of the Primary School

In this school children were not able to search for the website to connect to and to enter their login credentials. The teachers had to help them and position them at the beginning of the test. The children at the end of the tests were very tired, in particular the pupils of the first and second class.

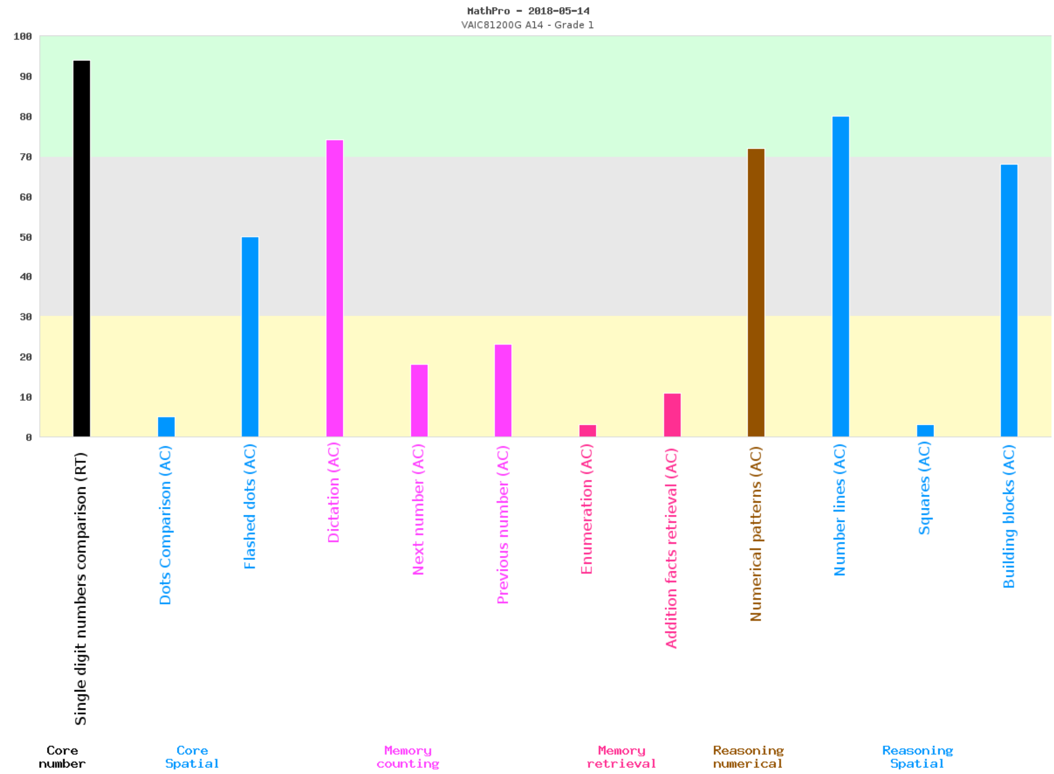


Figure 1.37: Grade 1; teacher evaluation in maths: poor; teacher evaluation in literacy: moderate.

In the First-Grade class (Figure 1.38), the tasks in which the students had most of the

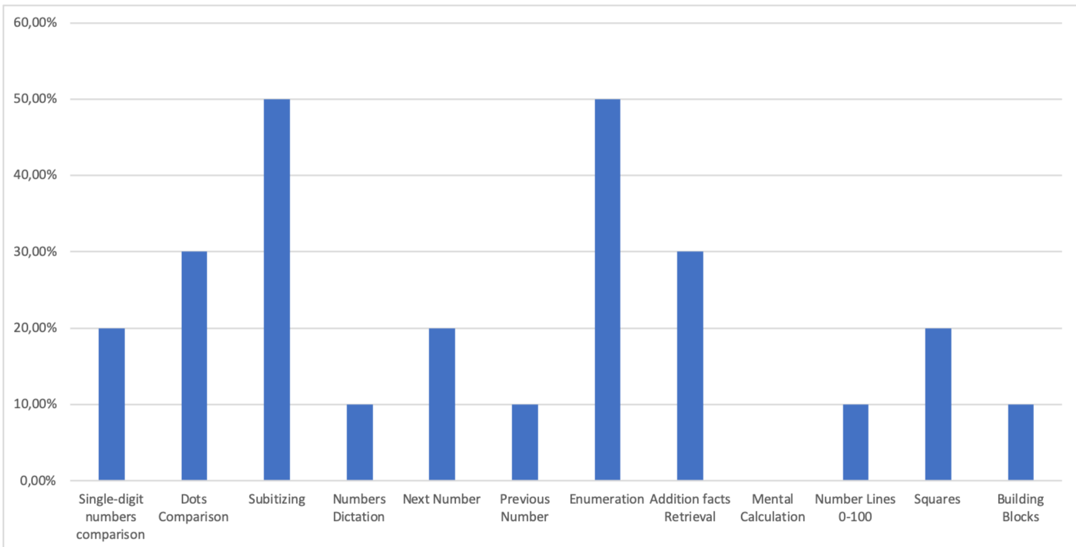


Figure 1.38: Traditional school, First-Grade class - MathPro tasks: percentage of failures.

difficulties were the Subitizing and the Enumeration tasks while the task they did best was the Mental calculation:

- Subitizing: 50% of failures;
- Enumeration: 50% of failures;
- Mental calculation 0% of failures.

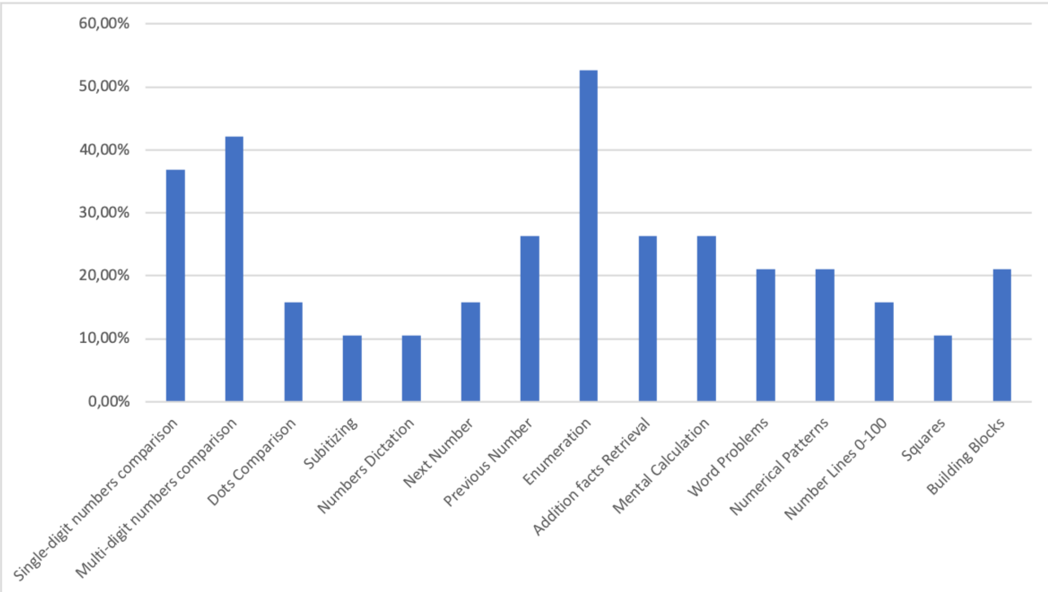


Figure 1.39: Traditional school, Second-Grade class - MathPro tasks: percentage of failures.

In the Second-Grade class (Figure 1.39) the task in which the students had most of the difficulties was the Enumeration. The tasks they did best were Subitizing, Numbers Dictation and Square:

- Enumeration: 52,63% of failures;
- Subitizing: 10,53% of failures;
- Numbers Dictation: 10,53% of failures;
- Square: 10,53% of failures.

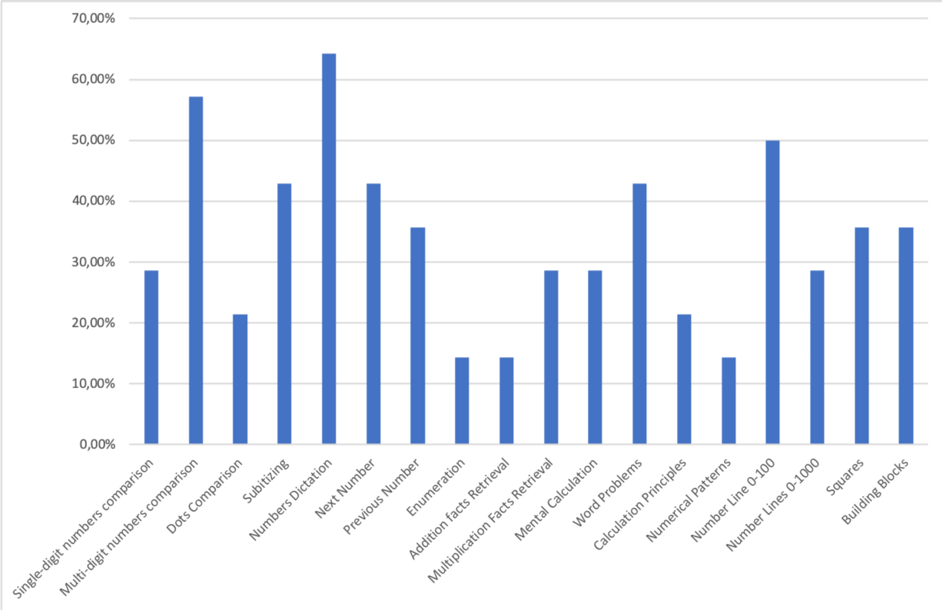


Figure 1.40: Traditional school, Third-Grade class - MathPro tasks: percentage of failures.

The students of the Third-Grade class (Figure 1.40) had had the most difficulties in the Numbers dictation. The tasks in which they did best were Enumeration, Addition facts retrieval and Numerical patterns:

- Numbers dictation: 64,29% of failures;
- Enumeration: 14,29% of failures;
- Addition facts retrieval: 14,29% of failures;
- Numerical patterns: 14,29% of failures.

Technological school – Classes: Second and Fourth of the Primary School

In this school the children were very good at using the PC: the teachers had indicated on the LIM (Interactive Multimedia whiteboard) the website to connect to and the access credentials of each student. The children connected and started working. In these classes, the children apparently worked with less effort, even the younger ones, and certainly with more enthusiasm. The test was faced as if it were a game, with a lot of enthusiasm and maybe this made them feel less committed to solving the tasks.

In the Second-Grade class (Figure 1.41),the task in which the students had the most difficulties was the Multi-Digit numbers comparison while the tasks they did best were Numbers dictation and Next number:

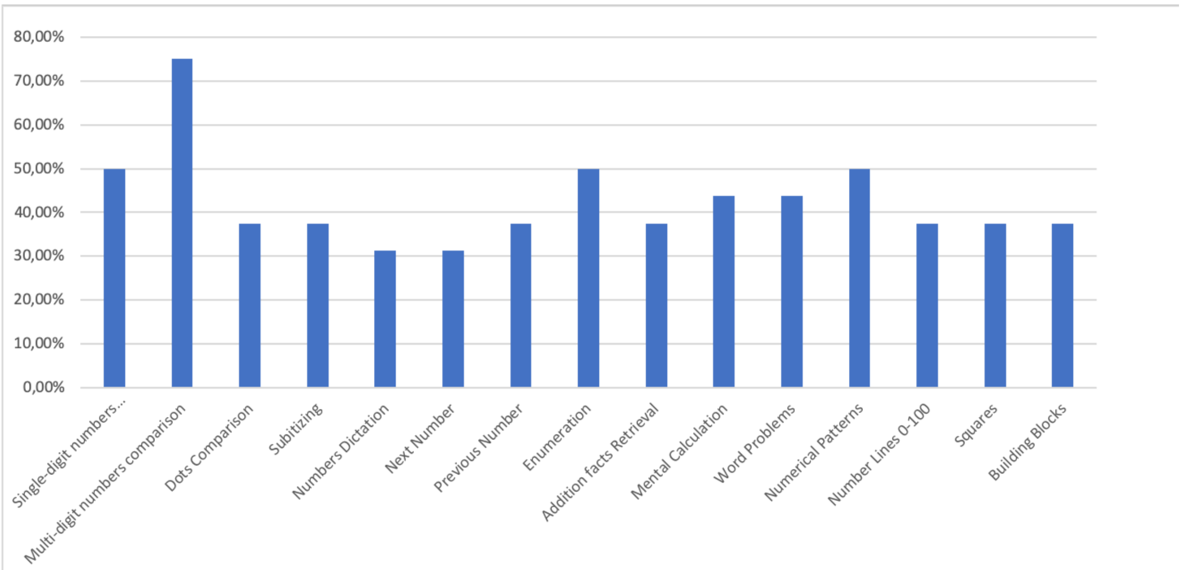


Figure 1.41: Technological school, Second-Grade class - MathPro tasks: percentage of failures.

- Multi-digit Numbers Comparison: 75% of failures;
- Numbers Dictation: 31,25% of failures;
- Next Number: 31,25% of failures.

In the Fourth-Grade class (Figure 1.42), the task in which the students had had the most difficulties was Multi-Digit numbers comparison while the tasks they did best were Dots Comparison and Subitizing tasks:

- Multi-digit Numbers Comparison: 95,65% of failures;
- Dots Comparison: 30,43% of failures;

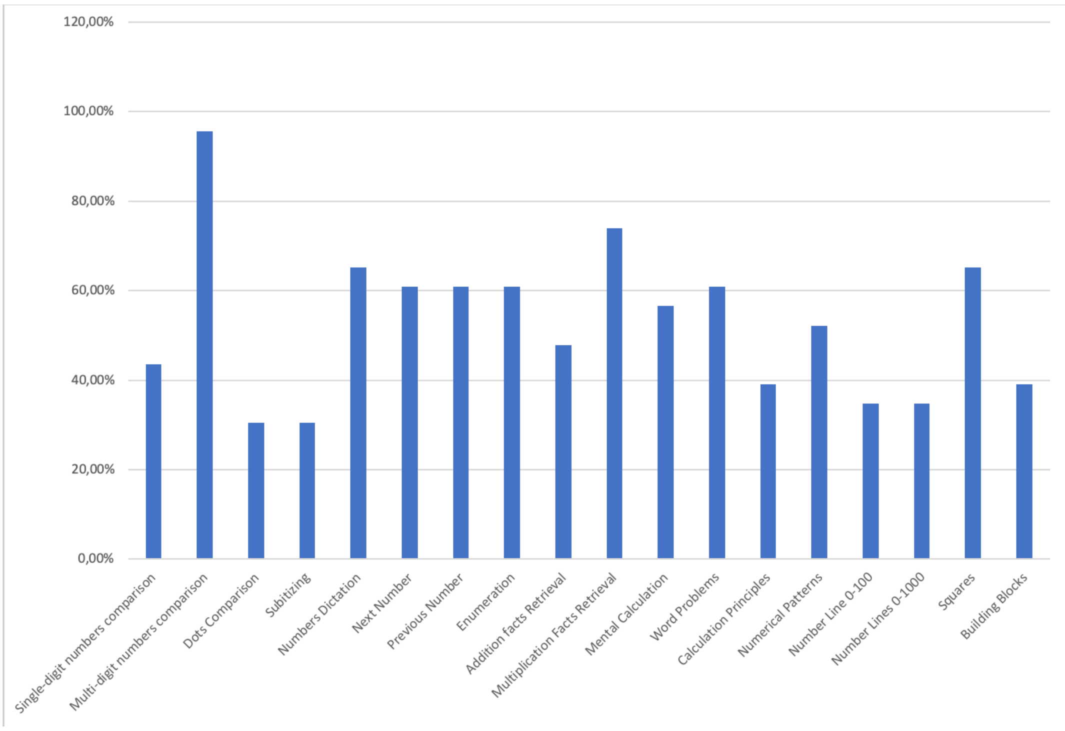


Figure 1.42: Technological school, Fourth-Grade class - MathPro tasks: percentage of failures.

- Subitizing: 30,43% of failures.

Subitizing is the name given to the fast enumeration. In this task the stimulus were given very quickly, although it does not require recourse to either memory or symbolic aspects, there was still an attentive request. To respond quickly the student needed to see a small number of dots on the screen, then to do subitizing. The speed of subitizing increases with age (Chi & Klahr, 1975) but these results show how the accuracy in the Subitizing task increases with the attended class only in the Technological school. The Dots Magnitude Comparison task has created problems to many children, in particular of first and second grades. In literature (Halberda & Feigenson, 2008); (Piazza et al., 2010) it is thought that being competent in this type of task depends on an innate ability that belongs to an 'approximate number system' (ANS). The ANS regards the ability to estimate and is one of the components that allows to perform subitizing (to recognize small quantities without counting). The results in this task were in line with the literature, because this ability showed progressive improvement: over the years the performance in terms of accuracy increases with the attended class of the students. In fact, the problem did not appear in the third and fourth class.

Problems also occurred with the Previous Number task which are clearly verbal sequences, texting flexibility, and speed in going forward and backward in the counting sequence. Also, the comparison of the multi-digit numbers was a task with the highest verbal demand: in fact, multi-digit number requires an understanding of the positional system. Some research groups suggest that the processing of multi-digit numbers is fully developed towards the end of Primary School (Chandler & Kamii, 2009); (Fosnot & Dolk, 2001). A fact that surprised us is that in the technological school the fourth grade students had still many difficulties with the Multi-digit comparison task (95,65 percent of failures). The Enumeration task allowed for the measurement of several abilities. In fact, to respond quickly a student obviously needed to see a small number of dots on the screen, then to do subitizing followed by the recovery of additive facts. The ability in this task increased with the attended class: in fact, for the second-grade pupils it was the most difficult task while for the third-grade pupils it was the best one (traditional school). The Addition facts retrieval task is a typical task of recovery of additive facts: at the beginning of the school it has not been yet consolidated in the child's memory so children must count on their fingers or, at least, make some kind of calculation. Then, on the contrary, this is no longer necessary: it is a real recovery of facts.

Building Blocks was, on average, the easiest task for the children. This task has not been learned formally, because it is not similar to geometry, so one would expect some knowledge of spatial reasoning and it was so for most students. Students made the effort in automating calculations and reasoning in the Building Blocks test, perhaps because they like to play with 3D bricks. There are a lot of games with 3D blocks: they became involved in what they were interested in. It is clear that their performances may depend very much on the type of task.

1.4.2 Teachers' opinion and qualitative evaluation of the battery

During a meeting, the teachers expressed their comments about the MathPro results and the tests that had created more or less difficulty. The teacher who used technology in her classroom had more doubts about the reliability of the results. The 'more traditional' teacher was more interested in how the exercises were presented to the students. This was their way to justify their pupils' bad results. Of course, every teacher expresses positions with different shades, but one constant is the belief of being able to properly assess their own pupils and the lack of openness to the comparison with the results of Math Pro when they differ from those defined in class.
We report in the following the teachers' evaluations about methodological limitations and unreliability of the results.
Methodological limitations

- A 45-minute test for first and second grade children is too long;
- The Mental Calculations task (figure 1.20) was presented horizontally while students are more familiar with calculation 'in column'.

Unreliability of the results

- Very often the profiles generated by MathPro did not match with the teachers' good assessments while there was no surprise with poor profiles for which there was a good agreement between the teachers' evaluation and the MathPro results.
- The MathPro system sometimes indicates 'low accuracy' in performance without giving a percentile;
- Sometimes it is difficult to give an interpretation to a performance because some profiles are flat and do not help.

Future directions

- Teachers would like to work in their classrooms with MathPro in order to test it over time;

- Some first-grade pupils were able to write a 5-digit number correctly and this was a great surprise for teachers. Using the test, it would be possible to identify gifted children and students that may appear listless, but who in fact are bored.

Surely the time needed to perform the test is too long for a small child, for the first-grade students in particular, because they are not used to keep the concentration for such long periods. In the technological school, however, the problem seemed minor, perhaps because of the students' ability of a more autonomous use of technology and their habit of frequently go to a 'computer room'. In fact, children used to work on the computer did not have to also add the lack of familiarity with the instrument. Sometimes the system indicates "low accuracy" in performance which cannot be given a percentile, because the performance on the set of items is evaluated on the response time and thus on the speed of execution. In the case of a too low accuracy, when it is below a certain threshold that varies according to the task, or when the student makes too many errors, the system indicates low accuracy because there is no point in evaluating the action time in these cases. The software does not provide a result, but it indicates that the task has not been performed in an adequate way. It is difficult to give an interpretation to a performance in these cases. Moreover , it is difficult to distinguish cases in which the student was very demotivated, from the ones in which he was not able to answer any question correctly (he just went on regardless), or in which, on the contrary, there was a commitment in his attempts to respond well to the test but he failed. Some children became involved only in what they were interested in and therefore their performances depended very much on the type of task. Some profiles do not reveal much, but they suggest that these students should be taken aside and perhaps interviewed to understand their mathematical profile better. These profiles require further studies. For example, to see whether task requests made in different ways could improve students' performances. If so, to intervene by changing or varying the way they are presented, following which, assessment should be made of the still remaining numerical difficulties.

1.4.3 Survey

After the administration of the test, in the province of Varese, we have gathered the opinions of math teachers who had taken part in the MathPro standardization. We sent them a questionnaire, to fill in an anonymous way, to try to understand how the teachers 'evaluate' their students' performances in mathematics. The teachers involved (indicated with a progressive number) were 12 (4 from middle school and 8 from primary school) and 10 of them have been teaching for over 5 years.

Question

Quali sono, secondo lei, le principali caratteristiche di uno studente bravo in matematica? - What are, in your opinion, the main characteristics of a good student in math?

Answers

1. Ottime capacità logiche e capacità di calcolo - Excellent logical capability and computing capability.

2. Lo studente bravo è: capace di trovare strategie; capisce, anche se fatica a memorizzare; applica ciò che ha capito; spiega ad altri ciò che impara; chiede aiuto - The good student is able to find strategies; he understands, even if he memorizes with many difficulties; he is able to apply what he has understood; he is able to explain what he learns; he asks for help.

3. Intuizione, logica - Intuition, logic.

4. Logica, impegno, costanza ed interesse - Logic, commitment, perseverance and interest.

5. Generalmente possiede motivazioni intrinseche all'apprendimento della disciplina e buoni livelli di autonomia e di organizzazione del proprio lavoro. Possiede una buona autostima - Generally, he has intrinsic motivations to learn the discipline and good levels of autonomy and organization of his work. He has a good self-esteem.

6. Buona memoria di lavoro, riconoscimento intuitivo della numerosità, buon orientamento spaziale, proprietà lessicale - Good working memory, intuitive recognition of numerosity, good spatial orientation, lexical properties.

7. Capacità di ragionamento logico - Logical reasoning capacity.

8. Capacità logica, capacità di problem solving e velocità di calcolo - Logical Capacity, problem solving skills and speed of calculation.

9. La disponibilità a mettersi in gioco, l'essere motivato e l'approccio positivo a questa disciplina - The willingness to get involved, being motivated and the positive approach to the discipline.

10. Intuito, metodo - Intuition, method.

11. Gran lavoratore - Hard worker.

12. Intuizione, memoria, logica - Intuition, memory, logic.

Question

Come valuta se uno studente ha/non ha le caratteristiche che ha indicato nella domanda precedente? - How do you assess if a student has (or has not) the features indicated in the previous question?

Answers

1. Dal lavoro in classe, lavoro a casa, verifiche - From class works, homework, tests.

2. Comunque positivamente, potrebbe non aver ricevuto in merito istruzioni - In any case positively, the student may not have received any instruction about the work he/she has to do.

3. Velocità di apprendimento, risoluzione di problemi - Time taken by a student to learn, problem solving.

4. Velocità con cui risolve un problema, le domande " curiose" che fa durante la lezione, la capacità di risolvere un problema in modo alternativo, interesse per quello che fa, gioia e apertura nell"apprendere cose nuove - The time a student takes to solve a problem, the "curious" questions he asks during the lesson, the ability to solve a problem in an alternative way, interest in what he does, the joy and openness to "learn new things ".

5. Mediante osservazioni sistematiche e griglie di osservazione/valutazione - By systematic observations and observation/evaluation grids.

6. Attraverso prove standardizzate - By standardized tests.

7. Quando non riesce a svolgere un problema o un esercizio che non richieda la semplice applicazione di una regola - When a student fails to solve a problem, or an exercise, that not requires the simple application of a rule.

8. Osservazione di risoluzione dei problemi matematici o dei problemi che si presentano nella quotidianità scolastica - Resolution and observation of mathematical problems or problems that arise in everyday school life.

9. Tenendo conto dei singoli e dei loro punti di forza e di debolezza - Taking into account the individual and their strengths and weaknesses.

10. Metodo: se ha o meno costanza nell'eseguire gli esercizi; intuito se riesce ad anticipare le risposte e trovare le sue strategie per arrivarci - Method: whether or not a student has perseverance in performing the exercises. Intention: if a student can anticipate the answers and find his strategies to get there.

11. Si vede - A teacher understands it.

12. Risoluzione dei problemi, calcoli, risposte logiche - Troubleshooting, calculations, logical answers.

Question

Su quali delle caratteristiche elencate pensa di poter intervenire al suo livello scolare? - On which of the listed features do you think you could operate?

Answers

1. Potenziare le capacità logico matematiche e di calcolo - Reinforce the logical and mathematical calculation capabilities.

2. Didatticamente penso di intervenire sulle caratteristiche elencate - Educationally I think on all them.

3. Risoluzione di problem - Problem solving.

4. Modo di proporre la lezione (per renderla più coinvolgente Un maggiore legame alla realtà (senza dimenticare che nella matematica le nozioni base vanno imparate per poterle applicare), proporre metodi alternativi alla risoluzione di problemi - To make lessons more engaging, more linked to reality (not forgetting that in mathematics the basics must be learned in order to apply them), to propose alternative approaches to problem solving.

5. Su tutte, mediante una metodologia attiva che favorisce lo sviluppo dell'apprendimento - On all of them, by means of an active methodology that allows the development of learning.

6. Maggiormente nel riconoscimento della quantità, nell'orientamento spaziale, nella proprietà lessicale - Most in quantities recognition, spatial orientation and in the lexical properties.

7. Proposta graduale di esercizi o problemi con elaborazione/riflessione sui procedimenti in classe - Proposal of exercises or problems with processing/reflection on the procedures in the classroom.

8. Problem solving, velocità di calcolo - Problem solving, speed of calculation.

9. Applicando una didattica esperenziale capace di coinvolgere tutti gli alunni e rispettando i tempi di ognuno - Applying a teaching method capable of involving all pupils and respecting their time.

10. Intuito e metodo - Intuition and method.

11. Aiutarli ad avere più fiducia nelle proprie capacità - Help students having more confidence in their abilities.

12. Tutte - All of them.

1.4.4 Discussion

Teachers often have a too evaluative attitude. In their answers about the reasons why a student is good at math it never appears the idea that the pupil has traveled very motivating school routes, he/she is good because of innate characteristics. Being this the situation, the teachers try to recover the 'disadvantaged students'. What struck us most is that some answers are one the opposite of the other. A teacher answered: 'Good working memory' while a colleague indicates '...even if he memorizes with many difficulties'. Mason, 2006 (J. Mason, 2006) identified two interpretations of intelligence:

- it can improve thanks to the personal commitment;
- it is a stable attribute, it is not subject to change.

A teacher's answer would not seem connected to the question: 'a good boy in mathematics has good self-esteem', as if self-esteem would imply the ability to master the subject. Her answer may be connected to the first interpretation of the intelligence. When we asked them to explain how to assess the characteristics of good students in mathematics, the given explanations were very different. The results of the investigation pointed out that teachers tend to be more oriented to the implementation of quick schemes of evaluation than on the analysis of errors. Failure to achieve the goals is assessed by standardized tests or the experience or the speed at which the student learns or performs the test properly. Mathematics is often presented by the teachers as a discipline in which there is always one and only one solution. If the result is not correct the teacher marks the error and the child learns that it is an element to avoid, an obstacle that cannot have positive consequences. Instead, it is necessary to shift the focus on the learning processes that the student activates, on his efforts, on his problems and errors as an opportunity to build new knowledge (Di Martino & Zan, 2013). One teacher said that a teacher can understand if a student is 'bravo' in math without any explanation. This may be a case where the student must understand what his/her teacher wants from him/her. Analyzing the answers to these questions, it emerges an interesting observation: for three teachers the evaluation represents the push towards educational changes. A methodological change in teaching seems to be a mode of action to work with their students with enhancement activities. They speculate, therefore, that one of the causes of low performance of their pupils can be attributed to their math teaching that should be reviewed. It is also interesting to note that many teachers stress the need to motivate their students not only in reasoning but to have more self-confidence.

The individual reports generated with MathPro may be, for a teacher, an important source of information to better understand the needs of his/her students. This battery of numerical tasks can improve the quality of the teachers' evaluation, can allow teachers to develop ways and tools to compare the results of all the students of the class to better understand their potential strengths and weaknesses and finally to work on educational methods and teaching environments able to respond to the needs of each single student.

1.5 Conclusion

The research relating the pedagogical content knowledge of mathematics and teachers' beliefs on the mathematics, pertain to different domains (Graeber & Tirosh, 2008) but they interact with each other (J. Mason, 2001), (L. Mason, Boldrin, & Zurlo, 2006), (Philipp, 2007), (Handal, 2003). There are several studies to outline the teaching content and pedagogical skills of teachers (L. S. Shulman, 1986), (L. Shulman, 1987), (Askew & Askew, 1997), (Ball & Feiman-Nemser, 1988), (Ball, 1991), (Ball, Thames, Phelps, et al., 2008), (Marks, 1990), (Fennema & Franke, 1992), (An, Kulm, & Wu, 2004), (B. Wilson & Cole, 1991). When investigating on the teachers' skills we also need to deepen the convictions and conceptions of mathematics teachers (Thompson, 1984), (Thompson, 1992), (Cooney, 1985), (Ernest, 1989) . The teacher often has a too evaluative attitude while he should dwell more on the students' answers to really understand not only if they are 'right answers' but their meaning (Leinhardt & Glaser, 1993)). From the analysis of the mathematical profiles of the students, it is clear that we cannot stop at the summative assessment made by the mathematics teacher: "the student is good/not good at mathematics". Even in the presence of children with positive, or even very good math grades, the contribution of new technologies that delineate the child's learning profile and help them in and out of school can be extremely useful. The profile of the boy with a vision problem in one eye (see 1.3.6.2 Bravo in maths) is certainly an extremely interesting case whose solution was simpler than other cases with difficulties dictated by cognitive factors. In this case, a 'secondary' factor had influenced the child's learning and caused him problems. Unfortunately, we are not able to say that we can do the same when other factors are not so easily treatable, but these factors can certainly be identified. The test, in fact, made it possible to highlight the strengths and weaknesses that the teachers have not been able to detect. Teachers expressed the appraisal "good in mathematics" without breaking down the evaluation into areas demonstrating the variety of reached skills and gained competences, each of which is influenced by the other. This "flattened" assessment, probably made on the basis of his average performance, did not allow the detection of the child's mathematical learning profile and therefore to better adapt the teaching to him, in this case the learning situation with correct lens. Therefore, it is essential to go deeper into several learning profiles because they can provide a wealth of information upon which one can build the appropriate educational intervention. The more the evaluation is flattened, for example when it is on a single dimension, the less we can make good working hypotheses for that student or class.

The evaluation that the teacher can give may change as a result of the knowledge of the profiles emerging from a software of profiling as was the case analyzed in this work. The assessment of mathematical skills should be objective and multidimensional because the skills involved in mathematical learning processes are multiple. This should happen not only with students with difficulties in mathematics or with disability certifications, which, we have seen, are not able to tell us how a child's mind works and above all how we must intervene to strengthen the domains in which there are weaknesses.

The profiling software can also be used for action research: to study the response of different profiles to educational intervention, to prevent false positives in the diagnosis of dyscalculia, to create and test educational material, to reduce learning difficulties and to adapt the training offer to the students' mathematical learning profiles.

Overall, MathPro looks very promising. If on the one hand, it is hard to recognize the strategies adopted by students to solve the tasks that are proposed by MathPro, on the other, it is an innovative and almost automatic method of guiding and monitoring the evaluation of children's numerical development. This battery also contributes to the evaluation of the mathematical abilities of all children throughout each grade of primary education. MathPro is currently available in English, French, Greek, Italian, and Maltese, a feature which will enable intercultural comparisons of the development of numerical abilities of children from different nationalities and walks of life and therefore from school systems with very different features.

Furthermore, this software enables longitudinal studies. It would be enough to interview a small number of student, on several occasions to evaluate the progress of each of them over time. Monitoring the learning path of some children for some years would undoubtedly offer valuable feedback on which to reflect for the future development of MathPro.

Chapter 2

Technology and emotions

The purpose of this chapter is to complete the profile obtained in the first chapter, which highlighted the student's mathematical and cognitive abilities, with the emotional part. It is possible to recognize emotions using facial expressions, audio signals, body poses, gestures etc. but physiological signals are very useful in this field because they are spontaneous and not controllable. This chapter deals with an experiment using skin conductance signals for measuring physiological responses associated with mathematical task performance.

2.1 Introduction

The growth of children during kindergarten and school is crucial because they are 'building' their feelings and confidence in their abilities. In this crucial phase they need to be encouraged not to doubt their ability to succeed and it is therefore also important to analyze their emotions concerning the process of learning mathematics.
The profiling software analyzed in the first chapter of this work provides information about the 'functioning' of children in math but is not able to assess the emotional state of children when they do math.
Recent advanced researchers in neuroscience are highlighting the connections between emotions, social skills, and cognitive processes. In particular, neurobiological evidence suggests that the aspects of cognition that are most valued in schools, such as learning, attention, memory, decision-making and social functioning, are all deeply influenced

and included in the processes of emotion (Immordino-Yang & Damasio, 2007).

Researchers have recently connected high math anxiety to lower performance in math tasks, developmental dyscalculia, and lower self-efficacy towards math learning.

The concept of "number anxiety" was introduced by Dreger and Aiken (Dreger & Aiken Jr, 1957) and has received increasing attention in recent years. In 1972, Richardson and Suinn ((Suinn, 1972)) have defined mathematics anxiety as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ... ordinary life and academic situations" and published the first systematic instrument for assessing math anxiety: the Mathematics Anxiety Rating Scale (MARS). Highly math-anxious individuals tend to avoid math (Hembree, 1990) and being good at math is considered to be an inherently aptitude far more important than effort ((Geary, 1994)). An obvious consequence of the avoidance tendency is that highly math-anxious individuals are less exposed to math and have, apparently, a lower achievement as measured by standardized tests (Fennema, 1989). Eysenck and Calvo (Eysenck & Calvo, 1992) wrote about a model, called processing efficiency theory, where general anxiety effects were hypothesized to disrupt ongoing working memory processes because of fear of math. Working memory is very important for doing math because it allows individuals to remember and think about several things at the same time. It seems that the working memory, that should have been used for solving the math problem, is used by anxious people for the anxiety ((Ramirez, Gunderson, Levine, & Beilock, 2013), (Alfano, Beidel, & Turner, 2002) speculated that students with teachers that 'showed annoyance when students gave wrong answers' or 'typically did not respond to mistakes and misunderstandings with explanations ' may feel 'vulnerable to public displays of incompetence'. This may be an example of classroom methods that can generate math anxiety. Math-anxious teachers, and parents, have a significant influence on the children's math achievement (Beilock, Gunderson, Ramirez, & Levine, 2010) and this suggests the effectiveness to address math anxiety at the teacher level to improve students' math achievement (Betz, 1978), (Hendel &

Davis, 1978), (Vance & Watson, 1994), (Fen-ping, 2011). The construct of mathematics anxiety has been an important topic of study also because it is very important to address math anxiety at the earliest possible age (Wigfield & Meece, 1988) designing interventions to ameliorate these anxieties, which in turn may contribute to higher math achievement in the students. Motivational factors may become prevalent when students must choose their career, with few math courses (Kelly, 2008), (Ryan & Pintrich, 1997). Math anxiety negatively impacts children's math achievement and interventions designed to identify these anxieties may be important, also for young students, to increase positive attitudes about math (E. A. Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015).

2.1.1 Psychophysiological assessment

Mathematics anxiety refers to the individual's negative effect when engaging in numerical and mathematical tasks (E. Maloney, Ansari, & Fugelsang, 2011); (Rubinsten & Tannock, 2010); (Hoffman, 2010); (Kesici & Erdogan, 2010), (Aarnos & Perkkila, 2012). Early mathematical anxiety can include negative experiences with parents or teachers that could affect children due to their negative attitudes and beliefs towards mathematics (Bekdemir, 2010).

Biotechnology can help us understand the great influence of emotions on the rational mind. On one hand, biology is making progress in understanding the mysteries of the brain and feelings, while on the other, computer scientists are providing us with the opportunity to process data never known before. Reliable and solid technology is needed to detect the emotional signs in people in everyday conditions.

Over the years, many studies have been performed that have used different physiological measures such as brain waves, skin conductance, skin temperature, electrocardiogram, blood pressure, hormone levels, etc.

These psychophysiological parameters can be recorded in order to obtain objective measures of the emotional state of individuals.

Biometric sensors, however, are special in that they do not place restrictions on the wearer's behavior, increasingly reducing the boundary between electronic devices and bodies.

2.1.2 Purpose of this study

Skin conductance can be measured virtually continuously and provides information about affective and emotional hidden processes in individuals' choices (Bechara, Damasio, Tranel, & Damasio, 2005). Emotional excitations represent the intensity of emotions (Breese & Ball, 1999), (Lang, 1995). Galvanic Skin Response (GSR) is one of the most widely used response systems in the history of psychophysiology and has been applied to examine attention, emotion and, in general, brain mechanisms. Our Autonomic Nervous System (ANS) plays a significant role in emotion and motivation. Emotions arouse the ANS by altering various physiological measures: heart rate, blood pressure, respiratory rate, galvanic skin response and muscle activity (Mark W et al., 2001). GSR is a sensitive peripheral index of the sympathetic division of the ANS (Cacioppo, Tassinary, & Berntson, 2007). In fact, when a person becomes more or less stressed, his sweat glands increase or decrease the production of sweat and, consequently, his skin conductance increases, or decreases proportionally. GSR, often also called electrodermal activity (EDA) (Boucsein, 2012) is an autonomic variable, therefore, it cannot be controlled by the user.

The aim of this chapter is to investigate, in a pilot study, whether GSR can give us clues about the physiological changes associated with mathematical tasks of increasing difficulty and how the relationship 'child/teacher' and 'child/parent' influences the process of learning. The final aim will be to use these data as another cue in the pedagogical strategy. In our study we are interested in recording the skin-conductance parameter using a bio-tracker. The high cost of this equipment did not allow us to test them on a whole class.

2.2 Skin Sensor

A bio-tracker, based on the conductance of the wearer's skin can measure:

- Emotional Strength;
- Difficulty;
- Stress Index;
- Cognitive Load.

The psychological significance of the galvanic skin response signal as a reliable indicator of stress and frustration has been used in many studies (Prendinger, Mori, & Ishizuka, 2005).

By using a bio-tracker sensor, an experiment was carried out in a private room where the illumination, sounds, and room temperature were controlled to maintain uniformity.

Each session of this type has 'the baseline problem'. It refers to the problem of finding a condition against which physiological change can be compared: the baseline. An obvious choice is a 'rest' period where the subject can be assumed to have no particular emotion. In our experiment, we used an initial relaxation period where the subject listened to the sound of fish at the bottom of the sea.

2.3 Session: June 13, 2019 4.30 pm

Participants

Little girl:

- She was born on April 23, 2010
- She had just finished the 4th grade (an early school entrant)
- Montessori method school

- She did not attend the nursery school or even the kindergarten
- Mathematical grade: 10
- Science grade: 10
- Italian grade: 10

Teacher:

- She taught Mathematics, Science, and Italian in the class attended by the child
- She has been the child's teacher since the child attended the first grade (4 years).

Tasks

The session aimed to evaluate the stress and the cognitive load of the child, during two tests/interrogation sessions, comparing two variables (Difficulty and Interviewer).

Difficulty variable

The interviewer was the teacher and had 10 minutes for all questions:

- Relaxation phase with the sound of fish at the bottom of the sea as background;
- Teacher: 5 minutes of questions related to Mathematics topics (difficulty level: low for a girl who finished fourth grade): table 5, 1, 10 (all requested but in no particular order) - arithmetic operations by 10;
- Relaxation phase with the sound of fish at the bottom of the sea as background;
- Teacher: 5 minutes of questions related to topics in mathematics (difficulty level: high for a girl who finished fourth grade): tables 7, 8, 9 (all required but in no particular order) - operations over 10;
- Relaxation break.

Interviewer variable

1. The interviewer was the teacher who had 5 minutes for all questions:

    - relaxation phase with the sound of fish at the bottom of the sea as background;
    - questions related to non-Mathematical topics: 'Who belongs to the vertebrate class?', 'Indicate three mammals', present and future of the verbs to be and to have;
    - Math topics: table 6 (all in random order, including zero); table 11 (all in random order, including zero); the square (angles, sides, perimeter); the rectangle (angles, sides, perimeter); the triangles (sides, perimeter); the rhombus (sides, perimeter).

2. The interviewer was the child's mother and had 5 minutes for all questions:

    - Same questions asked by the teacher (non-Mathematics - Mathematics) because the comparison was teacher/mother, so the questions had to be the same.

Method: Tools and setting

The little girl was wearing a bio-tracker on the tip of her left index finger (Figure 2.1), a microsurgical skin sensor with a sampling frequency of 120 Hz. Therefore, wearable technology was non-invasive to explore the unconscious dimension (Figure 2.2).

Method: Stress Index/Cognitive Load

A direct indicator of difficulty and cognitive load (Figure 2.3) was derived from the measurement of physiological activity during the execution of any cognitive task.

2.3.1 First session results

Average Galvanic Skin Response values

The Galvanic Skin Response (GSR) is one of several electrodermal responses. It is an indicator of skin conductance (SC), and increases linearly with a person's level of

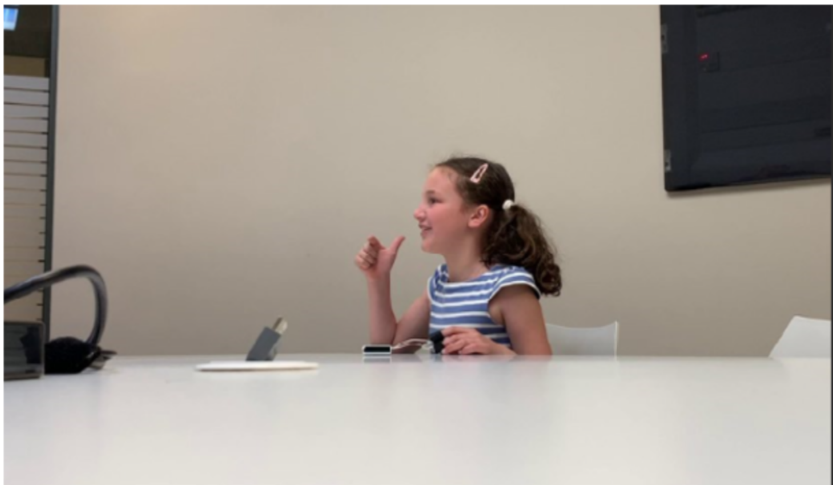


Figure 2.1: The little girl.

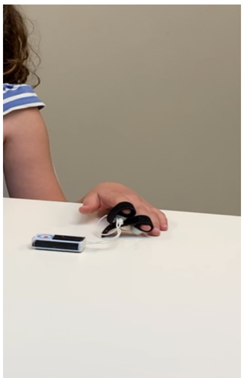


Figure 2.2: The Bio-tracker.

overall arousal. The output of the GSR amplifier is the skin's conductance expressed in units called microSiemens (μS). Figure 2.4 shows how the "difficult" task elicits on average a greater physiological activation, which reflects a higher cognitive load.

GSR Peaks

Figure 2.5 shows that there is also a difference in terms of peaks:

- Maximum simple task peak: 7.28 μS;
- Maximum task difficult peak: 7.99 μS.

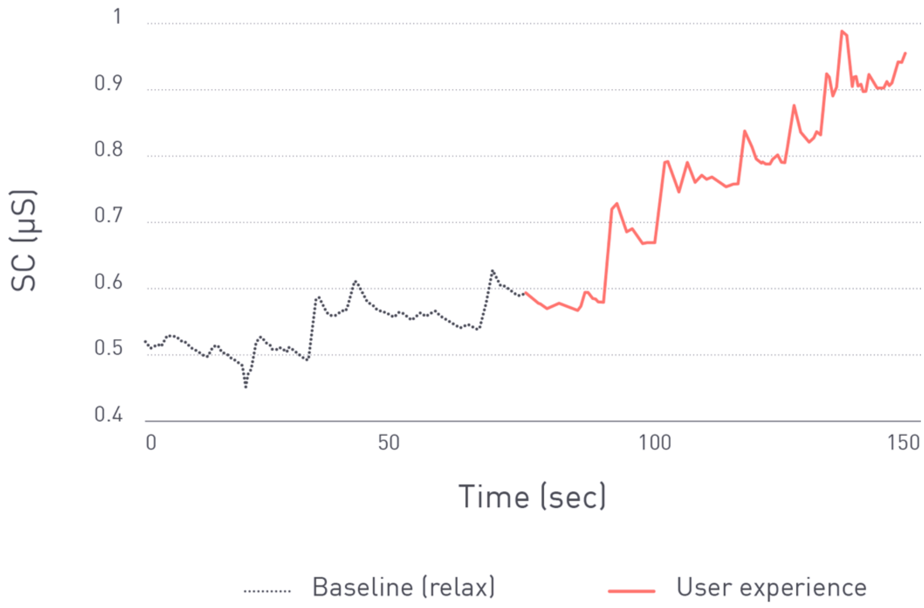


Figure 2.3: Stress Index/Cognitive Load of the little girl.

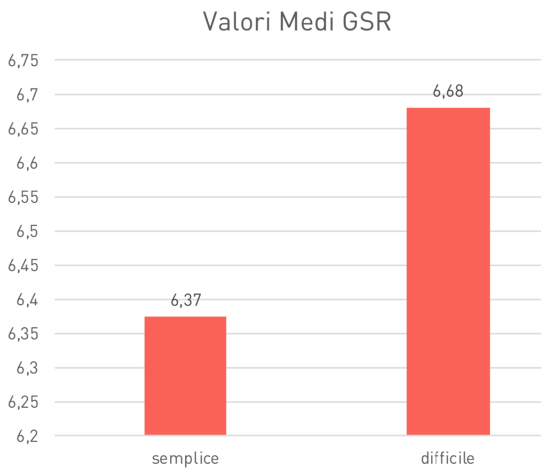


Figure 2.4: Average GSR values.

’Simple task’ activation pattern

The following values emerge from the measurement of physiological activity:

- Average: 6,3 μS;
- Maximum peak: 7,28 μS.

Figure 2.6 shows:

1. The initial ’warm-up’ trend, when the child began to reason and answer;

Figure 2.5: GSR Peaks.

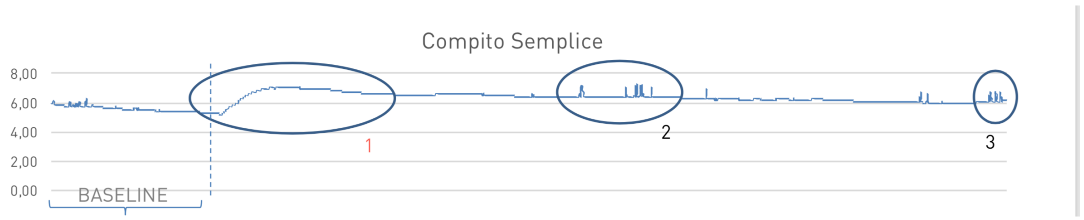


Figure 2.6: 'Simple task' activation pattern.

2. The teacher changed type of questions ... 'Now we will ... subtractions by ten'. The child reasoned for a second and then said: 'Yes I understand' and started making calculations;

3. The little girl was happy to have completed the task.

'Difficult task' activation pattern

The following values emerge from the measurement of physiological activity:

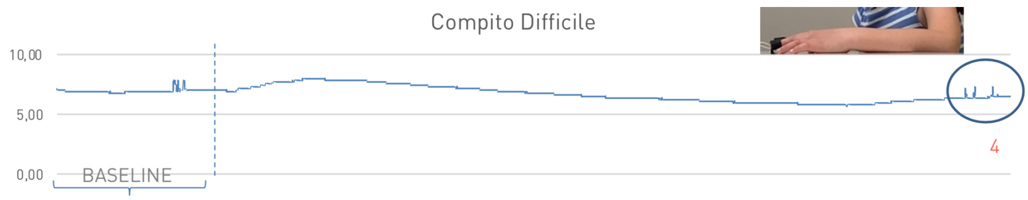


Figure 2.7: 'Difficult task' activation pattern.

- Average 6.68 μS;
- Maximum peak 7.99 μS.

Figure 2.7 shows that the trend is stable and sustained: the child was stimulated but had no great difficulty in responding.
Region (4) shows a moment of stress related to the following question: '34 + 8 =?'. The average activation value was larger than the first protocol.

2.3.2 Second session results

Average values GSR

From the measurement of physiological activity, it emerged that (Figure 2.8):

- during the task with the teacher the child was (on average) more relaxed;
- during the task with the mother, she was on average more agitated even in moments of tranquility and silence.

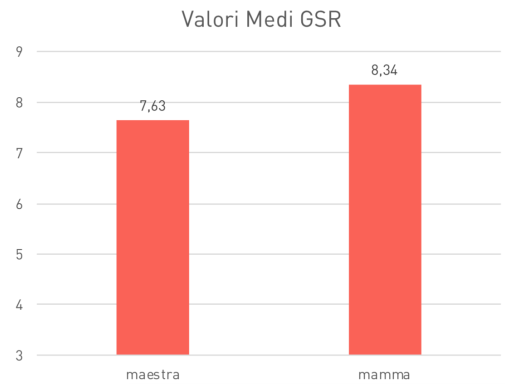


Figure 2.8: Average values GSR.

GSR Peaks

Figure 2.9 shows a difference also in terms of peaks:

- Maximum peak with teacher 8.92 μS;

Figure 2.9: GSR peaks.

- Maximum peak with the mother 9.86 μS.

Activation pattern with the teacher

The following values emerge from the measurement of physiological activity:

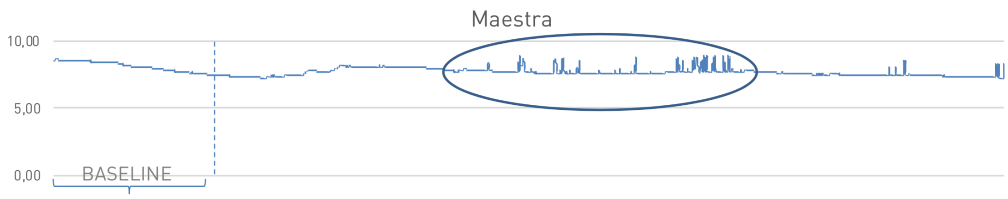


Figure 2.10: Activation pattern with the teacher.

- Average 7,63 μS;
- Maximum peak 8,92 μS.

Figure 2.10 shows how the rhythm remains constant, also thanks to the continuity concerning the previous tasks and the agitation when the little girl was faced with questions about verbs.

Activation pattern with the mother

The following values and critical issues emerge from the measurement of physiological activity (Figure 2.11):

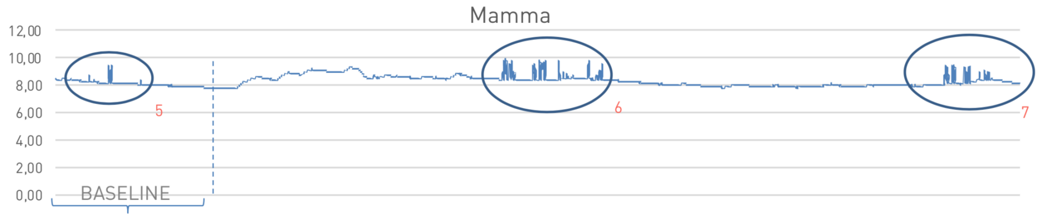


Figure 2.11: Activation pattern with the mother.

- Average 8.34 μS;
- Maximum peak 9.86 μS;
- (5) Agitation in front of the mother, even in the relaxation phase;
- (6) 'Present indicative ...' the child began to show signs of agitation;
- (7) 'Mammals ... birds ... amphibians...?'

2.4 Conclusion and future directions

Technology could reliably assess the emotional state of children that would help in the choice of the pedagogical strategies to be undertaken.

The results also reveal the emotional strain the mother puts her children under when wanting them to reach expected goals. In this regard, there are several studies that show how parents' expectations greatly affect children's performance: (Lefevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010) found that parents' numeracy expectations predicted home numeracy practices and outcomes. To be more precise, higher parental expectations on their child's achievement in numeracy had a positive influence on the parents' behavior toward the child. This may result in increased numeracy practice and, therefore, higher achievement by the child (Georgiou, 1999); (Phillipson & Phillipson, 2007).

Young children have poor introspection and the experiment of using skin conductance as a window on emotional reactions related to math can be important. The application of this methodology on one participant does not represent a relevant contribution to the literature. This experiment was carried out with only one fourth-grade child because the high cost of this equipment did not allow us to test them on a whole class. Future research should be carried out in this direction. When students work at a computer, stay in almost permanent manual contact with a mouse or with a keyboard. The incorporation of a sensor, such as galvanic skin sensor (GSR), in the input devices, would be able to determine students' physiological conditions in a non-invasive way.

We think that attention to emotions should enter school classrooms and technology could be an ally of teachers to promote the well-being of children and encourage the complete flowering of human beings (Naranjo, Cheli, & Callegari, 2011).

In particular, in childhood, it might be important to shift attention from a culture that aims almost exclusively at a cognitive maturation, to the detriment of affective aspects, towards a more global vision of education and teaching.

In the perspective of improving pupils' school life, the theme of affective and emotional knowledge as a driving force for learning, and for the expansion of one's personality, should be at the center of the educational experience from the earliest childhood (Mancini, 2011).

The integration of the emotional system in the learning processes is indispensable for the path of integrated development of the self.

Helping children in the process of discovering emotional and behavioral self-regulation plays a crucial role in learning. In fact, developing emotional skills allows them to enhance problem-solving and constructive thinking skills (Gavazzi & Crugnola, 2011).

The synergy between information technology and psychophysiology opens up a multitude of scenarios that will leave more and more room for the possibility of using these tools in the classroom as teaching aids.

Chapter 3

Effectiveness of digital tools designed for children with developmental dyscalculia and at risk for developing numerical deficits: a meta-analysis

The purpose of this chapter is to meta-analyze empirical evidence about the effectiveness of digital-based interventions designed for students with dyscalculia in primary school and severely at risk for dyscalculia in preschool ages (Benavides-Varela et al., 2019-submitted). Furthermore, we investigated whether the school level of the participants and the software instructional approach were decisive modulated factors. A systematic search of randomized controlled studies published between 2003 and 2019 was performed. A total of 15 studies with 1073 participants met the study selection criterion.

3.1 Introduction

The digital tools can, as we have seen in the previous chapters, be of great help because they can offer an accurate mathematical and cognitive assessment that the teachers are not able to give and, above all, do not have the time to deepen having to work with several classes. Introducing a software in the teaching to generate the individual profiles of all the students, to understand which are the weaknesses and strengths of the students, could represent a tool of great support for the teachers, for the early identification of the difficulties in mathematics and to implement strengthening interventions. However, in addition to early detection, early intervention is also required.

A developmental learning disorder can be a very serious handicap for a child, especially if the affected skills, like the mathematical ones, are critical in modern societies (Duncan et al., 2007; S. J. Ritchie & Bates, 2013). Low numeracy affects various aspects of people's life. It negatively impacts school attainment, mental health and self-esteem in children (Fritz, Haase, & Räsänen, 2019). Moreover in adulthood, it reduces the range of working opportunities (Rivera-Batiz, 1992) and it compromises an individual's independence in activities of the everyday life (Arcara et al., 2017).

The seriousness of the mathematical difficulties can vary considerably. Most students struggle with mathematics more than with any other subject at school (OECD, 2016). About 25% show mathematical learning disabilities (Geary, 2011; Geary, Hoard, & Bailey, 2012), and approximately 3%-8% of children obtain a diagnosis of the developmental disorder called dyscalculia (Desoete, Roeyers, & De Clercq, 2004; G. Nelson & Powell, 2018; Shalev, Auerbach, Manor, & Gross-Tsur, 2000). The difficulties can also manifest at different points in a childs school career, not only in the learning of basic facts or in learning to apply previously acquired knowledge to solve numerical problems but also in the learning of preliminary mathematics skills such as counting, seriation (Van De Rijt & Van Luit, 1998), number sense, or subitizing, which can be traced before entering primary education.

Fortunately, many efforts have been made in the last decades to support students in primary or secondary school (for recent reviews see (Fritz et al., 2019; Chodura, Kuhn, & Holling, 2015; Dowker, 2017; Jitendra et al., 2018)), and in preschool for boosting children's first steps on mathematics learning (Aunio, 2019; Bryant et al., 2011; Outhwaite, Faulder, Gulliford, & Pitchford, 2019; Benavides-Varela et al., 2016; A. J. Wilson, Dehaene, Dubois, & Fayol, 2009). Moreover, recent advances in technology have further facilitated the advent of new intervention approaches based on digital tools (Räsänen, Laurillard, Käser, & von Aster, 2019). However, from the great number of emerging apps, programs, websites, etc. that are available to train mathematical skills, only a small portion has been subjected to formal evaluation among children with specific numerical deficits or developmental dyscalculia (Drigas, Pappas, & Lytras, 2016; Kroeger, Brown, & O'Brien, 2012). The distinction between the results obtained in typically achieving children and children with special needs is by no means trivial. Significant effects reported among typically performing children (Gouet, Silva, Guedes, & Peña, 2018; Li & Ma, 2010; López-Morteo & López, 2007; Rosas et al., 2003; Valle-Lisboa et al., 2016; Kulik, 1994), might not necessarily replicate among children that require specialized assistance. Children with dyscalculia are by no means precluded from video game play. However, their special characteristics may impact on the patterns of their play and what they gain from the activity. Children with dyscalculia may for instance face obstacles learning the rules of a mathematical game, mastering play options, and determining which actions are appropriate in response to different types of game feedback (Yuan, Folmer, & Harris, 2011). This affects the learning rates and implies that, for example, a given training with a specific pedagogical approach, overall duration, number and duration of each single session, etc. and that has been proven effective among typically developing children in a certain context (e.g. home vs school) might not be useful to the same extent for children with mathematical difficulties. The results of the same training need to be proven also among children that are at the lower end of the distribution in math achievement. Further, many games impose time pressures on decision making, present a large amount of complex and rapidly changing on-screen information, which might be detrimental for children struggling to perform in maths. The successful implementation of new instruments among students who generally find learning especially challenging requires a careful integration of teaching strategies, research-based principles (Ginsburg, Jamalian, & Creighan, 2013), and specific design recommendations (Brunda & Bhavithra, 2010; Cezarotto & Battaiola, 2016) that could set the scene for the children conceptual change and active engagement in the learning of math concepts (Seo & Woo, 2010).

To this aim, highly innovative intervention programs as well as adaptive computer videogames based on neuroscience research have been recently developed for the remediation of dyscalculia e.g.(Butterworth et al., 2011; Kucian et al., 2011a; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; A. J. Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). These specialized intervention programs might offer distinctive benefits for learners, teachers and researchers. They can contribute at reducing training time and instructor load (Mitchell & Savill-Smith, 2004) and at the same time enable individualized attention to the learner (Butterworth & Laurillard, 2010). From the point of view of the student, they constitute alternative ways of studying (Durkin, Boyle, Hunter, & Conti-Ramsden, 2015; Gee, 2003) and facilitate the link to abstract concepts in ways that are not generally possible using paper and pencil, e.g. "zooming into a 1–10 number line to discover decimal numbers" (Butterworth & Laurillard, 2010). Furthermore, virtual environments enable a private feedback that is extremely valuable for learners who struggle to carry on, and repeatedly suffer defeats in normal classroom contexts (Butterworth & Laurillard, 2010). In that sense, digital programs could be particularly useful for improving the students' self-esteem and their motivation (Dempsey, Rasmussen, & Lucassen, 1994; D. Ritchie & Dodge, 1992). Despite the proposed advantages of using digital tools for increasing mathematical performance, currently researchers found mixed results regarding their effects.

3.1.1 Review of the previous summative studies into teaching math via digital tools

Numerous meta-analytic studies have been published generally reporting the outcomes of teaching math via digital tools to typically achieving learners (Li & Ma, 2010; Kulik, 1994). Some studies have reported the effects on students with learning disabilities (Jitendra et al., 2018; Kroesbergen & Van Luit, 2003; Li & Ma, 2010; Seo & Bryant, 2009) mental retardation (Kroesbergen & Van Luit, 2003; Mastropieri, Scruggs, & Shiah, 1991; Miller, Butler, & Kit-hung, 1998), and low performing in mathematics (Chodura et al., 2015; Kroesbergen & Van Luit, 2003). The findings of these studies provide mixed conclusions regarding the effectiveness of digital tools in mathematics education. In some of the studies, the authors concluded that computer-based tools were less effective than a teacher in assisting students with special needs (Kroesbergen & Van Luit, 2003), or that incorporating digital-based tools did not provide systematic effective changes to the learning process (Mastropieri et al., 1991; Seo & Bryant, 2009; Kulik, 1994). On the other hand, (Li & Ma, 2010) in their review found statistically significant positive effects of computer-technologies on mathematics achievements and larger effects on interventions for children with special needs compared to the effects on general education students (Li & Ma, 2010). Similarly, Jitendra and colleagues also carried out a meta-analysis including interventions for students with mathematical difficulties and learning difficulties in secondary school (Jitendra et al., 2018). This study reported that computer-based modules were more effective as compared with regular classroom instruction, but did not provide an additional advantage as compared to other instructional approaches (e.g. visual not-computerized modules). Noticeably, all these findings emerged from evaluations of special needs students presenting highly heterogeneous difficulties, including for instance students with low-IQ, various types of learning, physical, and emotional disabilities, ADHD, blindness, etc., besides those with specific mathematical difficulties. However, children with learning disabilities in general and with mathematical difficulties in particular, show different learning profiles. Indeed, developmental dyscalculia - one of the core school academic disabilities - may develop in children with normal IQ and in the absence of difficulties in other domains, skills or abilities (Butterworth, 2019). Focusing on interventions targeting children with specific difficulties in the domain of numbers may thus provide some important insights for effective interventions to these children.

To our knowledge only one summative study has focused on this specific group. The study by (Chodura et al., 2015) meta-analyzed 35 studies with the aim of evaluating the effectiveness of various types of interventions specifically designed for children with mathematical difficulties. The study found a overall positive effect of specialized interventions, but reported no significant differences between computer-based and face-to-face interventions. However, like previous studies, the study of Chodura and colleagues focused on interventions carried out from elementary school, which could not be generalized to the growing body of literature that evaluates early interventions for severely at risk children, before they enter elementary school. Moreover, because the study evaluated various types of interventions (not only digital-based ones), it did not investigate software characteristics thoroughly. For instance, one important distinction, which could influence the effectiveness of the digital intervention, is the software instructional approach: whether the intervention was based on adaptive and interactive videogames or on tutoring and drilling strategies. The first category refers to digital-based interventions that propose ludic activities and that require the student to indirectly apply numerical concepts or carry out numerical computations to win or to proceed to the next level of the game (Stultz, 2013). The second category includes programs that explicitly instruct mathematical concepts and repeatedly present information/exercises on a given topic to work on the students' specific areas of weakness.

3.1.2 Purpose of this study

While investigating the overall effectiveness and the parameters that may characterize successful interventions to help children's mathematical performance, the review research described above has some limitations. First, the summative studies have provided inconsistent conclusions concerning the effectiveness of incorporating digital tools on mathematics interventions and most of the reviews included primary investigations with students showing a wide range of learning difficulties. This makes it difficult to evaluate the interventions among children with specific deficits in mathematics. Second, most of the previous research has focused on children from elementary school (Chodura et al., 2015; Kroesbergen & Van Luit, 2003; Seo & Bryant, 2009). However, new theoretical and practical developments have pointed out the importance of early interventions that target children at risk already in preschool ages (Aunio, 2019; Bryant et al., 2011; Outhwaite et al., 2019; Benavides-Varela et al., 2016), calling for a systematic integration of the literature including also the studies carried out in these early stages. Third, research results are beginning to provide further insights into the benefits of incorporating video and interactive games to support math learning. To date, however, no study has directly assessed if videogame interventions moderate the general effects obtained on specialized interventions. In the context of this background, the primary goal of this study was to systematically review and meta-analyze peer-reviewed randomized controlled trials that focused on children with and at risk for developmental dyscalculia. Specifically, the present review sought to answer the following questions:

1. Do interventions with digital tools significantly impact mathematics achievement of students with and at risk for dyscalculia?

2. Do the effects vary by the school level in which the intervention is carried out (pre-school vs elementary school)?

3. Does the software instructional approach (videogames vs. tutorials /drilling) moderate the intervention outcomes?

3.2 Methods

The literature search used the PRISMA methodology suggested by Moher and colleagues (Moher, Liberati, Tetzlaff, & Altman, 2009) and implemented in previous studies to guarantee comprehensive and objective reporting of meta-results (Stanmore, Stubbs, Vancampfort, de Bruin, & Firth, 2017).

3.2.1 Literature search and inclusion criteria

A literature search was conducted until March 2019 by means of PsycINFO, Google Scholar, and Educational Resources Information Center (ERIC) databases. The search was restricted to the period from 2003 to 2019 since the majority of systematic and meta-analytic reviews focusing on digital-based interventions for children with learning difficulties had already included studies published between 1980 and 2002 (Kroesbergen

& Van Luit, 2003; Li & Ma, 2010; Seo & Bryant, 2009; Woodward & Carnine, 1993; Kulik, 1994). Eligible studies were group-designed randomized controlled trials which compared the effects of digital-based interventions to control conditions and which provided some statistical elements to support inferences at the population level. Single case interventions, observational reports, conference papers, dissertations, research foundation papers that are limited to describe software details and program development, intervention studies using tools that have not been directly evaluated in children with numerical difficulties or dyscalculia, and newsletter articles were not included in this review. Studies needed to report students' performance in any domain of mathematics as their dependent variable. Additionally, the studies needed to include statistical information (N, mean, SD) for their effect size calculation. Where only part of these data was reported, the corresponding authors of the respective articles were contacted to request the missing information. Studies could have been conducted in any country, but only English-language articles published in peer-reviewed journals were included. The reference sections of the eligible articles and previous reviews or book chapters were also scanned to locate other possible studies on this topic that could meet the initial criteria. The terms used to locate potentially relevant studies were the following: mathematics; dyscalculia; videogames; interventions; computer-assisted instruction; educational technology; mathematical learning; mathematics teaching; number sense; mathematics achievement; mathematical difficulties; randomized; controlled; control group; control condition. Eligible studies were group-designed randomized controlled trials which compared the effects of digital-based interventions to control conditions and which provided some statistical elements to support inferences at the population level.

3.2.2 Coding procedure

For the purpose of this review, a digital-based tool for low numeracy learners was defined as any software available on personal computers, Web, tablets or smartphones and that had been specifically designed to provide supplementary math learning opportunities to students with dyscalculia and severe numerical difficulties. Articles were screened for eligibility by three independent authors (SBV, BF and CZC). Disagreements were resolved through pairwise discussions until consensus was reached. A systematic coding form was used by SBV and BF to record relevant information from each study. The studies were coded taking into account the following characteristics:

1. Primary outcome – Mathematical performance: this was defined as the change in any ability to solve a problem in any domain of mathematics following a digital-tool intervention (or control condition). Sample sizes, means and standard deviations were extracted for the effect size calculation.

2. Potential moderators: data on factors that may influence the effect size of the interventions were also extracted from each article. In agreement with the aims of the present study, a categorical distinction was made while coding the type of program (i.e. tutorial and drill & practice vs videogames), and school level (high school/ primary school / preschool). Other specific characteristics of the intervention (length in weeks, number of sessions per week, session duration and total number of sessions during intervention), and topic (math facts fluency, fractions, subtractions, additions, etc.) were also extracted but not included in the moderator analysis due to high variability in the levels of each factor.

3.2.3 Statistical Analysis

The preliminary dataset included 15 studies. We summarized studies results considering effect size and variance based on the d_{ppc2} index suggested by (S. B. Morris, 2008). In the case of studies with repeated measures in both treatment and control groups (pretest-posttest-control design; PPC), d_{ppc2} allows to quantify the treatment effect size as the difference between mean pre-post change in the treatment group and the mean pre-post change in the control group, divided by the pooled pre-test standard deviation:

$$d_{ppc2} = c_p \frac{(M_{post,T} - M_{pre,T}) - (M_{post,C} - M_{pre,C})}{SD_{pooled,pre}}, \tag{3.1}$$

where $M_{pre,T}$, $M_{post,T}$ and $M_{pre,C}$, $M_{post,C}$ are respectively the pre, post mean scores of the treatment group and the pre, post mean scores of the control group. $SD_{pooled,pre}$ is the pooled pre-test standard deviation, computed considering only the pre-test standard deviation of the two groups, and c_p is a bias adjustment for small sample size. Compared to other indexes, d_{ppc2} offers betters results in terms of bias, precision, and robustness to heterogeneity of variance (S. B. Morris, 2008). The d_{ppc2} values were interpreted according to the criteria suggested by (Cohen, 1988): small effects from 0.2 to 0.5; medium effects from 0.5 to 0.8; large effects greater than 0.8. A complete description of the index is reported in the supplemental material.

When studies included multiple treatment groups or control groups such as the case of (Aunio & Mononen, 2018; Hassler Hallstedt, Klingberg, & Ghaderi, 2018; P. M. Nelson, Burns, Kanive, & Ysseldyke, 2013), we selected passive control group and computer based treatment group to obtain a better evaluation of the treatment effect. Whereas, if treatment groups were considered equivalent from a research point of view such as the case of (Baroody, Eiland, Purpura, & Reid, 2013) they were merged together to yield a combined mean and standard deviation (see chapter 25](Borenstein, Hedges, Higgins, & Rothstein, 2009)). We noticed that (Kucian et al., 2011a) and (Räsänen et al., 2009) included as control groups only children with age-appropriate calculation performance (not children with mathematical difficulties). This does not allow to obtain a meaningful interpretation of the d_{ppc2} index. Thus, the two studies were excluded from the meta-analysis and their results were discussed separately. The final dataset included 13 studies. Full description of the preparation of the dataset is reported in the supplemental material.

The d_{ppc2} estimated mean and variance was calculated using raw mean scores, standard deviation, and sample size of the treatment and control groups reported in the studies. In order to compute d_{ppc2} variance, the correlation between pre- and post-test scores is needed. However, only (Salminen, Koponen, Räsänen, & Aro, 2015) reported this correlation, with $r_{pre-post}$ ranging from .72 to .90. Thus, $r_{pre-post} = .70$ was used as reference value for all the studies and meta-analysis results were evaluated in a sensitivity analysis using $r_{pre-post} = .50$, which gives greater variance (i.e., more conservative results), and $r_{pre-post} = .90$, which gives smaller variance (i.e., less conservative results). Where studies reported more than a task, effect size and variance were firstly computed for each task separately and then combined into composite summary effect and variance, following recommendations to take into account dependence of the information (see chapters 23 and 24 in (Borenstein et al., 2009)). Different formula were applied if the multiple task were completed by the same subjects (such as the case of (Baroody, Eiland, Purpura, & Reid, 2012; Baroody et al., 2013; Hassler Hallstedt et al., 2018; Käser et al., 2013a; Salminen et al., 2015)) or if they were completed by different subjects (such as the case of(Burns, Kanive, & DeGrande, 2012)). In particular, to compute the composite summary variance, the correlation between the different tasks completed by the same subjects is needed. However, no study reported this correlation. Thus, $r_{tasks} = .50$ was used as reference value in all cases and meta-analysis results were evaluated in a sensitivity analysis using $r_{tasks} = .30$, which gives smaller variance (i.e., less conservative results), and $r_{tasks} = .70$, which gives greater variance (i.e., more conservative results).

The analysis was performed with R software (R Core Team, 2019). Firstly we ran a random-effects model meta-analysis using the restricted maximum likelihood method with the R package *Metafor* (Viechtbauer, 2010). Next, we explored the heterogeneity between studies through inspection of forest plot and evaluation of the Q-statistic (Hedges & Olkin, 2014). Q-statistic is distributed like the chi-square under the null hypothesis, with a significant chi value indicating the presence of heterogeneity across studies. Moreover, to estimate the magnitude of the heterogeneity we considered the I^2 index i.e., the proportion of observed variance that reflects real and not random difference between studies effect sizes;(Borenstein et al., 2009). High values of I^2 suggest that difference between results are related to real differences across studies (i.e., different constructs or different study design). On the contrary, low values of I^2 suggest that results across studies are similar and possible difference are related to random sampling.

3.2.3.1 Sensitivity analysis and evaluation of publication bias

To investigate robustness of the results, we ran two sensitivity analyses. First, we used the *leave-one-out* method to evaluate how results would change if studies were excluded one at time from the analysis. Substantial changes when a single study is removed are interpreted as lack of homogeneity and unreliable results (Viechtbauer & Cheung, 2010). Secondly, we evaluated results when different values are used for $r_{pre-post}$ and r_{tasks} coefficients. Moreover, publication bias was assessed using the funnel plot with the *trim and fill* method (Duval & Tweedie, 2000; Rothstein, Sutton, & Borenstein, 2005).

3.2.3.2 Effects of possible moderators

The role of possible moderators was examined using mixed-effects meta-regression models, the moderators were included as a fixed effects and were tested using Wald's chi-square (Viechtbauer, 2010). Considering the reduced number of studies and the unequal distribution among the different levels of the moderators, we conducted separately two analyses to evaluate the role of (1) software instructional approach, (2) school level. Results have to be interpreted with caution as it was not possible to evaluate all the moderators at the same time.

3.3 Results and Discussion

3.3.1 Search results

The search returned 161 results, reduced to 83 after duplicates were removed. Thirty-eight articles were further excluded after reviewing the titles and abstracts for eligibility. Full versions were retrieved for 35 articles, of which 13 articles were eligible for inclusion. Two additional eligible articles were identified from the reference lists of the full-texts. Thus, a total of 15 unique studies with independent samples were included in this review. The article screening process is detailed in Figure 3.1.

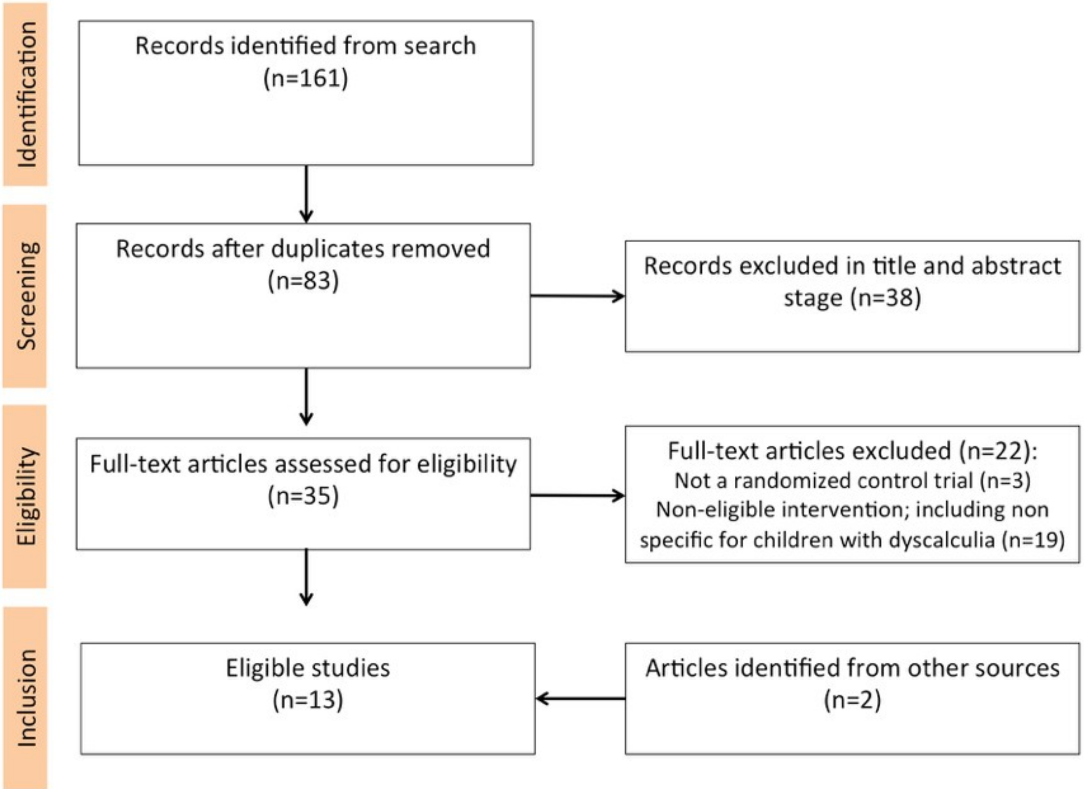


Figure 3.1: PRISMA flow diagram of systematic search and study selection.

3.3.2 Included studies, participant details and descriptive statistics

Descriptive statistics are reported in Table 3.1. Included studies were published between 2006 and 2018, most of them (10 out of 15) after 2012. Seven studies were conducted in the United States, three in Finland, two in Switzerland, and one each in Brazil, Sweden and Malaysia. Eligible outcome data was available from a total of 1073 participants across the 15 studies; 557 were assigned to the experimental intervention, 516 to control conditions. The sampled varied in size between 14 and 442 with most of the studies having less than 64 subjects. The mean age of the reported sample was 7.6 years (range =5.6–16.3 years) and 55.0% of the participants were male. The majority of studies were conducted with children enrolled in primary school (N= 10). Four studies were conducted with pre-school children, and one in high school. Looking at the control group, six studies included a passive control group, six studies included an active control group trained in mathematics, and only one study included an active control group trained in a different topic (i.e. spelling). One study compared the results of the children with difficulties in mathematics against an active normative group (typically achieving children), and one study compared it against a passive normative group. As stated in the methods section, the last two studies were excluded from the meta-analysis.

Interventions lasted on average 7.8 weeks (range = 1–20 weeks), with an average of 3.4 sessions per week, ranging from 10 to 90 min per session. Software packages were designed to tackle a variety of numerical difficulties including fact retrieval (N=3), reasoning strategy on calculation (N=2), problem solving skills (N=1), fractions (N=1), approximate and exact number comparison (N=1), number orientation and arithmetic sign identification (N=1), or multiple numerical skills at the time (N=6). N=7 software packages used tutorials or drill & practice approaches, N=8 used interactive videogames. Study details and intervention summaries are displayed in Table 3.2.

Table 3.1: Descriptive statistics of the studies included in the review

Study			**Participants**			**Treatment Group**		**Control Gro**
Authors	**Year**	**Country**	n	**Males/ Females**	**School level**	**Intervention**	$n_{treatment}$	**Control**
Aunio & Mononen	2018	Finland	14	5/9	Preschool	Video-Game	7	Passive
Baroody et al.	2012	United States	28	12/16	Preschool	Tutorial-Practice	15	Active Math
Baroody et al.	2013	United States	64	23/41	Primary School	Tutorial-Practice	43	Active Math
Burns et al.	2012	United States	442	NA	Primary School	Tutorial-Practice	216	Active Math
Castro et al.	2014	Brazil	26	16/10	Primary School	Video-Game	13	Active Math
Fuchs et al.	2006	United States	33	21/12	Primary School	Tutorial-Practice	16	Active Spelling
Hassler Hallstedt et al.	2018	Sweden	127	66/61	Primary School	Video-Game	75	Passive
Käser et al.	2013	Switzerland	41	14/27	Primary School	Video-Game	20	Passive
Kucian et al.	2011	Switzerland	32	13/19	Primary School	Video-Game	16	Active Normative
Leh & Jitendra	2013	United States	25	12/13	Primary School	Tutorial-Practice	13	Active Math
Mohd Syah et al	2016	Malasya	50	NA	Primary School	Video-Game	25	Passive
Nelson et al.	2013	United States	53	NA	Primary School	Tutorial-Practice	26	Passive
Räsänen et al.	2009	Finland	59	32/27	Preschool	Video-Game	30	Passive Normative
Salminen et al.	2015	Finland	21	9/12	Preschool	Video-Game	13	Passive
Stultz	2013	United States	58	36/22	High School	Tutorial-Practice	29	Active Math

Note: Total sample: 1073. NA: not available

3.3.3 Random-effects model Meta-analysis

3.3.3.1 Overall effects

The random effects meta-analysis showed a medium mean effect size, $d_{ppc2} = .55,\ 95\%CI\ (.19-.90),\ p = 0.002$, meaning that children in the treatment groups showed a greater improvement in mathematical ability than children in the control groups (Figure 3.2). It should be noted that despite the estimated effect size could be considered medium, the large confidence interval suggests that also small and large effect sizes could be considered consistent with the data.

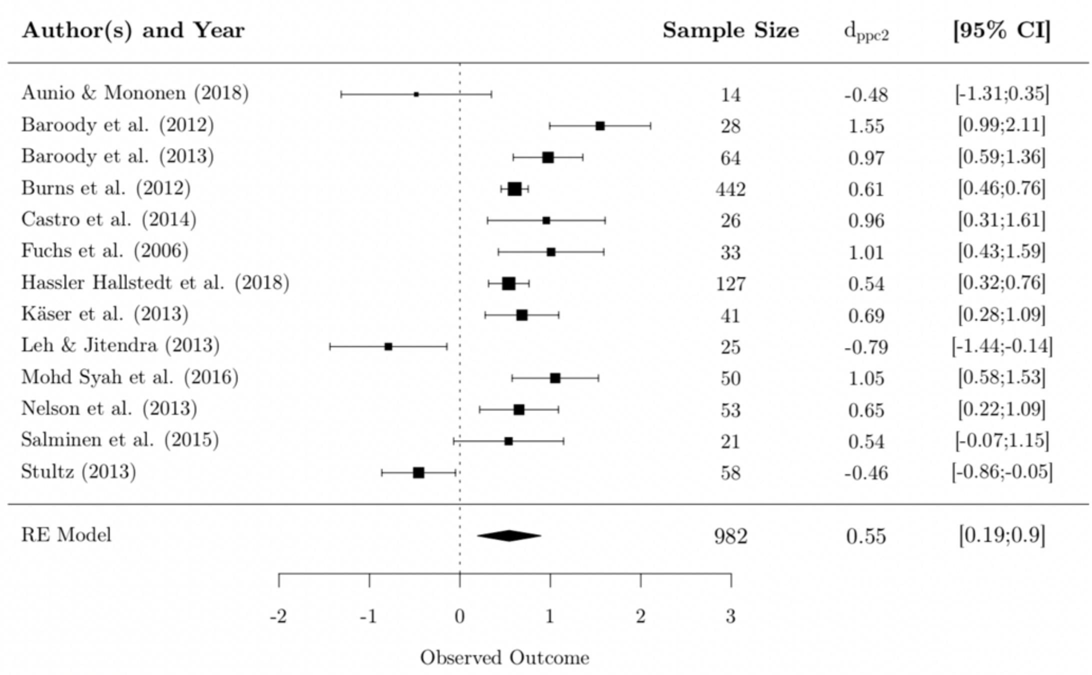


Figure 3.2: Forest plot. Each square represents the effect size of the study together with 95% confidence interval. The size of the symbol is proportional to the study's weight.

Table 3.2: Characteristics of the intervention for each study included in the re

Study		Intervention Characteristics			
Authors	**Year**	**Description**	**Total number of digital-based sessions**	**Duration of each session**	**Measured outcom**
Aunio & Mononen	2018	Multiple number early skills using "Lola's world"	15 sessions over 3 weeks	15 min	Total ENT evaluation Experimer Control
Baroody et al.	2012	Atructured add-0/1 rules	18 sessions over 9 weeks	30 min	Mental addition fluency. Pooled practiced and unpracticed (tra
Baroody et al.	2013	add-1 rule and near-doubles reasoning strategy	20 sessions over 10 weeks	30 min	Mental addition fluency structur Pooled ES including practiced ar (transfer) items
Burns et al.	2012	Fact retrieval using "Math Facts"	24 to 45 sessions over 8 to 15 weeks	10-15 min	Pooled ES including 3rd and
Castro et al.	2014	Multiple numerical skills using "Tom's Rescue"	10 sessions over 5 weeks	60 min	Mental math problems and writ operations using the Scholastic test of arithmetic
Fuchs et al.	2006	Fact retrieval using "Flash"	50 sessions over 18 weeks	10 min	Addition fact retriev
Hassler Hallstedt et al.	2018	Basic arithmetic (addition and subtraction facts up to 12), number knowledge and word problems using "Chasing Planets"	56 sessions over 20 weeks	20 min	Addition and subtraction
Käser et al.	2013	Multiple numerical skills using "Calcularis"	60 sessions over 12 weeks	20 min	Pooled ES including Subtraction line-10 tests
Kucian et al.	2011	Multiple numerical skills using "Rescue Calcularis"	25 sessions over 5 weeks	15 min	Accuracy of math problems (a subtraction)
Leh & Jitendra	2013	Problem solving skills	15 sessions over 6 weeks	50 min	Word problem solvi
Mohd Syah et al.	2016	Number orientation and arithmetic sign identification	5 sessions within a week	60 min	Pooled ES including Counting, Subtraction
Nelson et al.	2013	Fact retrieval using "Math Facts"	4 sessions within a week	15-20 min	Digits correct per mir
Räsänen et al.	2009	Approximate and exact number comparison using "Number Race" and "Graphogame-Math"	15 sessions over 3 weeks	10-15 min	Number comparison. Pooled E Number Race and Graphoga
Salminen et al.	2015	Multiple numerical skills using "Graphogame-Math"	12-15 sessions over 3 weeks	10-15 min	Pooled ES including Enumera counting, Number Sets, Basi
Stultz	2013	Multiplication and division using "BMCSB" for fractions	10 sessions	90 min	Multiplying and dividing f

3.3.4 Evaluating sensitivity of the analysis

The sensitivity analysis showed robustness of the results. The effect size did not vary considerably neither when results were computed excluding one study at time (i.e., *leave-one-out* method), nor when different values for $r_{pre-post}$ and r_{tasks} were used. In particular, the d_{ppc2} value varied between .47 and .65 ($mean = .55, SD = .05$) in the first case, and between .53 and .56 ($mean = .54, SD = .01$) in the second case (see Table 3.3 and 3.4). Thus, the effect size remained consistently between medium values. However, also heterogeneity was consistently high (I^2 ranging from 81.56% to 96.96%).

3.3.4.1 Evaluating publication bias

The funnel plot with the *trim and fill* method added no hypothetical missing study (Figure 3.3). Therefore, there was no evidence for publication bias. The included studies allow a representative analysis of the research questions.

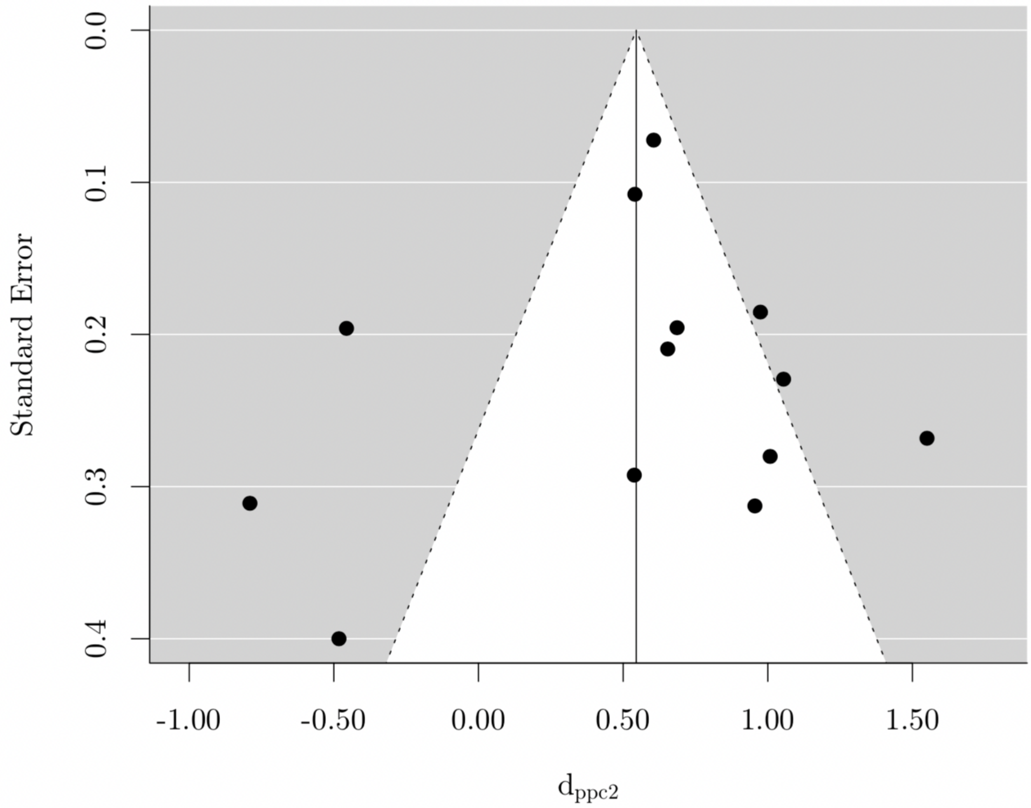


Figure 3.3: Funnel plot. Each black dot represents one study included in the meta-analysis.

Table 3.3: Sensitivity analysis using the *Leave-One-Out Method*

Author(s) and Year	d_{ppc2}	**95% CI**	I^2
Aunio & Mononen (2018)	0.61	[0.26 ; 0.96]	90.15
Baroody et al. (2012)	0.47	[0.13 ; 0.81]	89.25
Baroody et al. (2013)	0.51	[0.13 ; 0.88]	90.96
Burns et al. (2012)	0.54	[0.15 ; 0.93]	88.38
Castro et al. (2014)	0.51	[0.14 ; 0.89]	91.50
Fuchs et al. (2006)	0.51	[0.13 ; 0.88]	91.35
Hassler Hallstedt et al. (2018)	0.54	[0.15 ; 0.93]	89.89
Käser et al. (2013)	0.53	[0.15 ; 0.92]	91.44
Leh & Jitendra (2013)	0.65	[0.34 ; 0.95]	86.94
Mohd Syah et al. (2016)	0.50	[0.13 ; 0.88]	91.06
Nelson et al. (2013)	0.53	[0.15 ; 0.92]	91.53
Salminen et al. (2015)	0.54	[0.16 ; 0.93]	91.76
Stultz (2013)	0.64	[0.32 ; 0.96]	87.55
Mean	**0.55**		**90.14**

Note: Each line represents the results obtained excluding that study.

Table 3.4: Sensitivity analysis varing r_{tasks} and $r_{pre-post}$

		r_{tasks}		
		.3	**.5**	**.7**
$r_{pre-post}$	.5	0.56 [0.23 ; 0.90] (I^2 = 84.80)	0.56 [0.22 ; 0.90] (I^2 = 83.01)	0.55 [0.21 ; 0.89] (I^2 = 81.56)
	.7	0.55 [0.20 ; 0.90] (I^2 = 91.51)	0.55 [0.19 ; 0.90] (I^2 = 90.51)	0.54 [0.19 ; 0.89] (I^2 = 89.68)
	.9	0.53 [0.17;0.90] (I^2 = 96.96)	0.53 [0.16 ; 0.90] (I^2 = 96.60)	0.53 [0.16 ; 0.89] (I^2 = 96.30)

Note: d_{ppc2} [95%CI] and (I^2) are reported in each cell for given values of $r_{pre-post}$ and r_{tasks}. Total sample: $n = 982$.

3.3.4.2 Evaluating moderators

The tests of moderators were not significant: (1) software instructional approach $\chi^2(1) = .03, p = .86$, and (2) school level $\chi^2(1) = .006, p = .94$. In particular, to test the influence of school level we excluded the variable levels for which only one single study was available. Thus, high school students were not considered when testing age influence. This allowed to avoid unreliable results, as possible differences could be related to other study characteristics not considered. The full procedure is reported in the supplemental material.

3.4 Discussion

Several successful or promising educational technology applications have been developed in the last decades for mathematics intervention programs. To date, a few programs have been developed specifically for remediation purposes in individuals with dyscalculia, and even less have been subjected to evaluations in randomized control trials. Thus, an important question for teachers, parents, psychologists and other practitioners interested in implementing and pit such tools against other tools and methods is to what extent digital tools can be useful for students suffering dyscalculia.

The results of our work evidence a moderate but significantly positive effect of digital interventions on mathematics achievement (mean ES=+0.55). This finding partially confirms and extends the existing literature in various ways. First, it suggests that digital tools specifically designed for students with dyscalculia, compared to control conditions (e.g. face-to-face teaching, paper and pencil exercises, no intervention, interventions with non-specialized computer programs, etc.) resulted in higher mathematics achievement. This finding is in line with the positive results reported by (Li & Ma, 2010) when evaluating the effects of computer technologies among children with special needs compared to general education students. Moreover, the present study indicates that positive effects can be also observed in trials in which all participants have similar academic abilities (i.e. children struggling in maths both in the experimental and control conditions), as opposed to comparing the effects of the interventions in children with difficulties and typically achieving children of the same age, as Li and Ma did. Thus, not only low achieving children improve more than typically achieving children following digital interventions, but also they seem to gain more from digital tools than children with similar difficulties following other control interventions.

Other reviews and meta-analyses evaluating various types of mathematical interventions reported no additional benefits e.g. (Mastropieri et al., 1991; Seo & Bryant, 2009; Kulik, 1994) or no significant differences between interventions with and without digital tools e.g.(Chodura et al., 2015). Moreover, one study found that computer-assisted instruction was less effective than teacher instruction for students with special needs e.g.(Kroesbergen & Van Luit, 2003). These conclusions appear in disagreement with our findings. However, we note that in their evaluation these works pooled children with different special needs and learning disabilities. Our work, instead, focused solely on interventions specifically designed for helping dyscalculic children or severely at risk for developing it. It is possible that students with such a specific cognitive profile might obtain additional benefits from the technological tools. Thus, eventual inconsistencies between previous findings and ours might be partly ascribable to the learning profiles of the participants included in the studies. In addition, it should be considered that, besides group composition, our meta-analysis differs from previous ones in the time point in which the analysis was carried out. Primary studies included in the most recently published reviews and meta-analysis e.g. (Chodura et al., 2015; Kroesbergen & Van Luit, 2003; Li & Ma, 2010) were carried out 5 to 19 years ago. Continuing interest in digital-based interventions for children with mathematical difficulties is however evidenced in our review. Indeed, 10 out 15 studies identified were carried out in the last 5 years and were thus out of the scope of previous studies. The new data that have become available since the previous reviews might have thus influenced the final outcomes in the summative evaluations.

Another important factor that should be taken into consideration when interpreting the present results is the enormous variation found across studies, as indicated by the significant index of heterogeneity and the large confidence intervals. Previous studies focusing on digital-based interventions only e.g. (Seo & Bryant, 2009), or interventions for children with difficulties in mathematics e.g. (Chodura et al., 2015), also reported a high variation of the effect sizes. This variability across studies prevents strong conclusions and generalizations of the effectiveness of digital interventions for children with mathematical difficulties. Indeed, although a medium positive effect was found on average, some individual interventions resulted in small effect sizes or even negative ones (see the forest plot on Figure 3.2). This calls for an in-depth analysis and for a careful interpretation of the results. Besides group composition, which was carefully controlled for in the present analysis, other student characteristics, variations in the methodological and instructional features, and software characteristics might be possibly influencing the final outcomes.

Two such factors were quantitatively assessed in our study. The first one, school age, included interventions carried out on children from primary and preschool ages. Previous meta-analyses on mathematics evaluating digital-tools for improving mathematics reported on studies that were primarily conducted for students at the elementary e.g.(Seo & Bryant, 2009) or secondary level e.g. (Jitendra et al., 2018). There is no doubt that improving mathematics skills is important for these students. However, developing fundamental and basic conceptual understandings of mathematics is also crucial in the earliest phases of education. The recent advent of interventions in preschool ages -which had not been previously included in summative reports- respond to the growing need of assisting children when the first signs of mathematical difficulties emerge and also at a time in development when there are presumably greater chances for a substantial improvement. Studies indicate that children who enter kindergarten with low performance in numeracy skills continue to lag behind their peers in future school years and highlight the need of early interventions for low-performing children who are at risk for mathematical learning difficulties later at school e.g.(Duncan et al., 2007; Geary, 2011; Geary et al., 2012). In light of the new data available, one important question is whether digital-based interventions for kindergarteners severely at risk for dyscalculia yield better results than interventions carried out for older dyscalculic children. Contrary to our initial expectations, the findings of the present study suggest that school level has no significant moderating effects. This might be promising, on the one hand, for those children who receive help (somewhat) late. However, the current results were obtained with relatively few studies per school level. More primary studies are thus needed to definitely conclude on the effectiveness of digital-interventions for children who struggle with mathematics understanding and who receive assistance at different points in development.

Another factor that was considered in the moderator assessment of the present study relates to the characteristics of the educational software. In particular, we were interested in evaluating the outcomes obtained with videogame programs that have been proposed as an alternative to traditional tutorials or drilling approaches. Research results are beginning to provide insights into the benefits of incorporating video and interactive games to support math interventions among children with numerical deficits and developmental dyscalculia. Videogames incorporate various modern features such as high-resolution graphics, sound effects, changing backgrounds or settings to teach the material to the students (Stultz, 2013). They seek to link entertainment with educational goals, to stimulate the children's desire to win and, in turn, to encourage their interest in mastering specific mathematical skills through different strategies. Moreover videogames, unlike the tutorial and drilling tools, often proposed comprehensive interventions that trained various math skills and computations rather than focusing on a single skill at the time. This is considered a successful feature to help low achievers who often necessitate a full range of support not only to attenuate noticeable difficulties, but also to support fundamental weaknesses that might go unnoticed. Additionally, training basic tasks along with more complex ones could be particularly useful for supporting those learners who generally lack confidence, as it is often the case among dyscalculic children. The results of our study, however, showed no modulation of the outcome associated with this characteristic of the software. In other words, similar effects were obtained both with digital-based tools using tutoring and drilling approaches and with videogame implementations. Some other features of the programs and or experimental designs might be taken into consideration in future studies to better understand the potentialities of the various digital tools. For example, one possible factor to take into account could be the type of control group. One might expect larger effect sizes in studies comparing experimental versus passive control groups than in studies comparing experimental and active control interventions. In our study we noticed that most of the studies with videogame interventions used also a passive control group (5 out of 6), whereas most of the studies with tutorials and drill & practice approaches used an active control group (5 out of 6). Thus, larger effects could have been anticipated in the videogame interventions. Because of interdependence between the control and the software instructional approach, it was impossible to evaluate the relative role of these variables as moderators independently. However, it will be appropriate in the future, when more primary studies become available, to further investigate the effects of the chosen control group in the reported effectiveness of the interventions.

3.5 Conclusion

Altogether the findings indicate that digital tools positively impact mathematics achievement of students with or at risk for dyscalculia. Digital-based interventions can be thus conceived as a proper instrument to assist children with specific mathematical needs and to offer them additional opportunities to carry out mathematical tasks in an alternative technological context. Moreover, according to our data, digital tools improve numerical performance and understanding to a similar extent in dyscalculic children from primary school and in children deputed at risk from preschool ages. In other words, school level does not moderate the effects of the digital-based interventions. Finally, we found no evidence that videogames implementing a ludic approach offer additional advantages for children with specific mathematics deficits with respect to other digital tools implementing drilling and tutoring approaches. However, digital-learning materials will surely continue to expand in the forthcoming years, particularly with increased access and usability of new technologies by parents, children and adolescents. Software instructional approach used in those materials should be re-examined thoroughly when more data become available.

People interested in adopting these digital tools to help dyscalculic children should be also aware of the enormous variability found across studies. Ongoing research is required to establish other factors associated with intervention efficacy and eventual positive and negative consequences of digital tools for learning. This acquired knowledge should hopefully provide evidence-based criteria to select high-quality programs and to deliver appropriate interventions to the students with low-numeracy.

3.5.1 Limitations

It should be noted that these conclusions are based on 15 randomized-controlled trials, which met the criterion for this study. They cannot be extended to single-subject designs that often show more powerful results than those with a group design. Indeed, in single-subject designs the training is usually only stopped when the results are sufficiently high. Large effect sizes under such circumstances are thus expected (Kroesbergen & Van Luit, 2003). Moreover, the conclusions obtained from the moderator analyses are based on relatively small number of studies for each category. For example, only four preschool studies were included in the comparison of school level. Such limited number of studies obviously limits the practical implications of the results and the external validity of this study. More primary studies that examine digital-based interventions in mathematics for students with dyscalculia and at risk for developing it are warranted to extend and confirm the findings of this investigation.

Chapter 4

Implications for parental decisions: educational videogames at home

From the review in chapter 3, it emerges that digital tools can support teaching, even by assigning homework for dyscalculic children or those at risk. Learning, however, is not limited to the school environment. A number of studies show that parents, their expectations, and the amount of support they provide to their children, influence the young learners in their journey through acquiring new concepts, including mathematical ones. In order to create an integrative learning environment it is, therefore, necessary to establish new links for communication between parents and teachers. The meta-analysis of the previous Chapter tells us that educational software is as effective as face to face training in the school settings. An open question regards whether similar effects can be obtained if children were engaged in scientifically validated videogames to support mathematical learning at home, that is, under the supervision of parents and without the direct support of their teachers. In this chapter, a literature review was made. In particular it focuses on studies showing how children can make use of educational videogames as math training at home. The aim was to provide a summative overview of the current evidence, to date missing in the literature. The various studies will be described, and the effectiveness of the ludic instruments will be evaluated, discussed, and compared to the previous evidence in school interventions.

4.1 Introduction

Children from the first years of life are familiar with electronic devices such as tablets, cell phones, and computers, and use them, mainly, for playful purposes. The benefits that videogames can bring to children are numerous, particularly, in the strengthening of some brain functions (Eichenbaum, Bavelier, & Green, 2014).
Play creates a zone of proximal development for the child in which he/she always behaves beyond his/her average age, and better than his/her daily behavior (Vygotsky, Rieber, Hall, Carton, & Glick, 1997). Playing games involves consistence and concentration to achieve individual goals. It allows learning through making mistakes while having fun, activating a sort of trial and error which increases motivation and frequently also self-esteem in gamers (Hwa, 2018). Learning becomes a gradual process of acquiring skills and, above all, an active process, a way to explore the world and interact with others, not simply absorbing information passively.

4.1.1 Purpose of this study

Parents have to be the first to support children in upgrading school prerequisites since they can choose games and activities suitable for the child's age and level of development, even before primary school. Often the 'video game moment' becomes part of the daily life of the child and creates conflict with his/her parents regarding the time devoted to this activity as well as the choice of games. These moments could instead become an opportunity for children and their parents to share, in order to enhance learning playfully. Therefore, negotiation between parents and children is important to exploit the opportunity not only to supervise the games (rather than to prohibit them), but also to monitor the behavior of the children. Playing with children and choosing videogames with them can become a means to vent family tensions and encourage other qualities such as problem-solving skills (Voida & Greenberg, 2009). These moments could also represent an incentive in education to respect the rules, just as children must abide by the rules of the game, and to play they must obtain permission from their parents and respect the time limits imposed on them.

Hwa (Hwa, 2018), for instance, examined the differences that are recorded both in teaching and in learning mathematics, with or without the use of multimedia tools and digital games. His study highlights the benefits of these tools through simple lessons called DigiGem applied to children aged 7 to 9 years. The purpose is to make the learning of this subject more stimulating and interesting, in addition to studying the existence of a possible positive attitude towards learning of mathematical concepts. (Prensky, 2003) states that "digital natives", i.e. students born from 2000 onwards, need a different approach to learning that can attract their curious minds, confirming that traditional methods are no longer effective and need to be reviewed or replaced.

In this chapter, a literature review was made concerning the use of educational videogames at home as math training.

4.2 Methods

The literature search used the PRISMA methodology suggested by Moher and colleagues (Moher et al., 2009) and implemented in previous studies to guarantee comprehensive and objective reporting of meta-results (Stanmore et al., 2017).

4.2.1 Literature search and inclusion criteria

A literature search was performed by means of PsycINFO, Google Scholar, and Educational Resources Information Center (ERIC) databases, selecting the studies concerning the effects of educational videogames on the enhancement of mathematics in typically developing children. The studies conducted in parallel both at home with parents and at school with teachers were taken into account to verify their effects. The keywords used to locate potentially relevant studies were the following: preschool quality, home learning environment, numeracy skills, longitudinal study, cognitive development, interventions, computer-assisted instruction; educational technology, mathematics; videogame; education; self-competency; randomized controlled trial, early numeracy, activities within the family environment, numerical information learned at home, board games, home numeracy activities, basic number processing, calculation, kindergarteners, math achievement.

4.3 Results

4.3.1 Search results

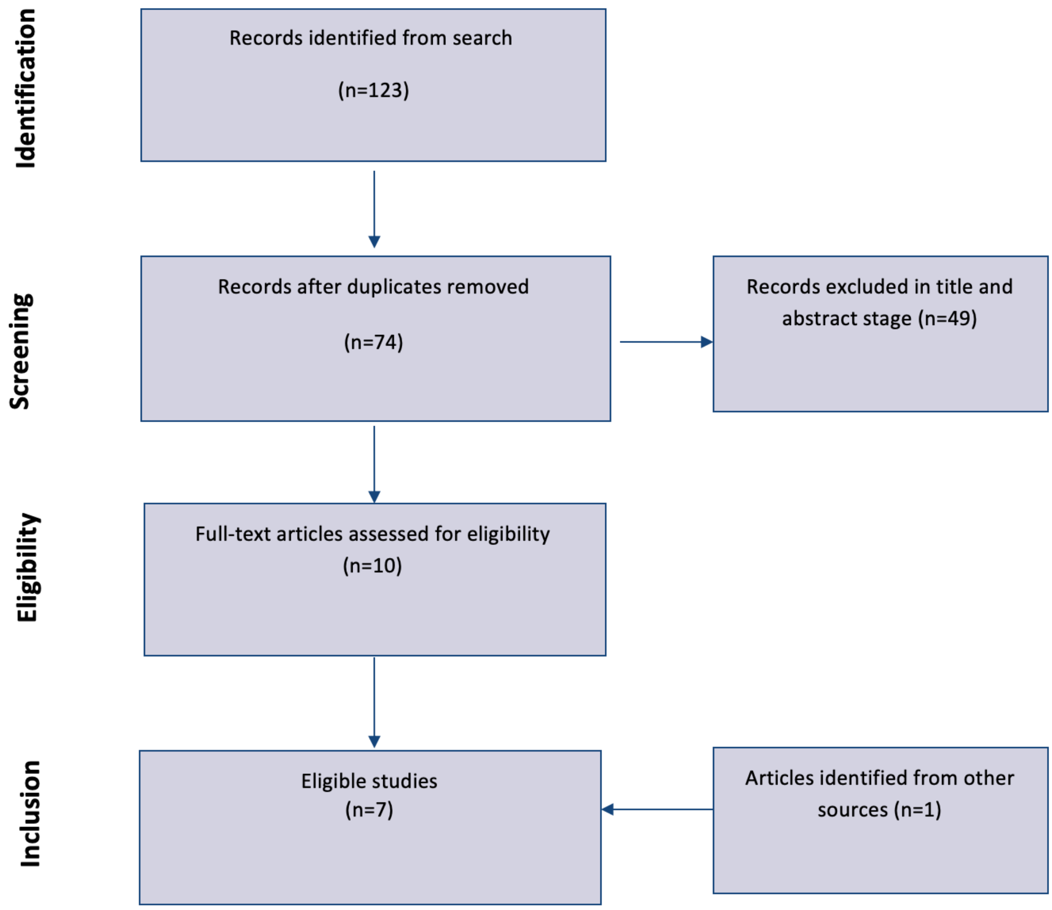


Figure 4.1: PRISMA flow diagram of systematic search and study selection.

The search returned 123 results, reduced to 74 after duplicates were removed.

Authors	Year	Country	Participants	School level	Type of intervention
Bakker et al.	2013	Netherlands	1005	Primary School	Mini Games
Bakker et al.	2014	Netherlands	719	Primary Schhol	Mini Games
Benavides et al.	2015	Italy	110	Primary School	Board games, no computer games
Berkowitz et al.	2015	United States	289	Primary School	App
Blevins-Knabe et al.study 1	1995		40	Pre-school	Paper and Pencil
Blevins-Knabe et al.study 2	1995		49	Kindergarten	Paper and Pencil
DE Florio et al.	2014	United States	178	Pre-school	Paper and Pencil/Books/Sw
Derboven et al.	2016		8	Primary School	Video Game
Huntsinger et al.	2015	United States	200	Preschool	Paper and pencil TEMA 2
Kasër et al.	2013	Switzerland	41	Primary School	Video Game
Kerawalla et al.	2001	England	77	Infant Schhol and Primary school	Computer
Kucian et al.	2011	Switzerland	32	Primary Schhol	Videogames
LeFevre et al.	2009	Canada	146	Kindergarten - Primary School	Paper and Pencil
Manolitsis et al.	2013	Greek	99	Kindergarten	Paper and Pencil
Mc Ewen et Dubé	2015	Canada	30	Primary School	Tablet
Missal et al.	2015		72	from 37 months to 69 months	Paper and pencil/Toys
Skwarchuk et al.	2013	United States	183	Preschool	Paper and pencil\song\ games
Yildiz et al.	2018	Belgium	128	Kindergarten	Tablet
Zhou et al.	2015	China	85	Preschool	Paper and pencil

Table 4.1: Characteristics of the selected studies.

Authors	Year	Country	Participants	School level	Type of intervention
Bakker et al.	2013	Netherlands	1005	Primary School	Mini Games
Bakker et al.	2014	Netherlands	719	Primary Schhol	Mini Games
Berkowitz et al.	2015	United States	289	Primary School	App
Kasér et al.	2013	Switzerland	41	Primary School	Video Game
Kucian et al.	2011	Switzerland	32	Primary Schhol	Videogames
Mc Ewen et Dubé	2015	Canada	30	Primary School	Tablet
Re et al.	2019	Italy	57	Primary School	Web app: 'I bambini contano'

Table 4.2: The studies included in the review.

Fifty-four articles were further excluded after reviewing the title and abstracts for eligibility. The research mainly includes studies conducted in the last years from 2013 to 2019. Table 4.1 presents a summary of the results of the studies that used mathematical games. There are also studies with board games and other games which were excluded to meet the criteria for the review. Finally, a total of 7 unique studies with independent samples were included in this review 4.2. The article screening process is detailed in Figure 4.1

Each study selected included a large sample of subjects and a control group in compliance with the criteria of validity and generalizability. Participants were randomly assigned to the two groups and were aged between 4 and 6 years and between 7 and 11 years, analyzing both preschool and primary school children.

Only 7 studies were analyzed as they met the criteria selected for the research were that studies carried out in two settings: both at school and at home, or just at home. Articles that referred exclusively to the studies performed in school were excluded instead. Some variables such as the type, duration of the session at home and at school, and prerequisites in relation to the population of origin, socio-economic status, age, and in some cases were evaluated and compared the level of parental education.

4.4 Discussion

Digital tools might be suitable in different learning contexts. They can be used not only to integrate the curricula in school-based instruction or tutoring but also present a significant opportunity out-of-school in everyday (e.g. museums, the playground) and home learning (Berkowitz et al., 2015) . There are a few studies that evaluate the effects of educational math games at home. Seven papers were analyzed.

The first examined study is that of (Berkowitz et al., 2015) performed on 289 primary school children. The aim was to evaluate the correlation between encouraging mathematical activities at home and a better outcome at school. The children and their parents had to use the Bedtime Learning Together (BLT) app on an IPAD. The problems to be solved for the children assigned to the experimental group concerned: counting, fluency, geometry, arithmetic, fractions, and probability. Each child in the experimental group was assigned to a group based on the time spent on the game each week: less than once, more than once and more than twice. The children assigned to the control group carried out instead reading activities, comprehension of the text, inferences, phonology, and pronunciation. The study also included data analysis based on the level of parents' anxiety for mathematics (E1: low anxiety, E2: high anxiety). The study showed that high quality of interaction between children and parents during the game improved the mathematical performance, even if it was poor. Furthermore, the intervention significantly increased the results in mathematics during the school year in the children of the experimental group, especially for pupils with parents who were anxious about mathematics.

A recent study by (Re, Benavides-Varela, Pedron, & Lucangeli, 2019-submitted) introduced a Web App called "I bambini contano" within a specialized training program for primary and secondary students with mathematical difficulties. This study implemented the Computer-Assisted Instruction (CAI) at home and the technological tool was designed to contribute to automatizing arithmetic facts. The students (n = 57) were randomly assigned to two groups: the experimental group followed the specialized training in a specialized center and also used the Web App at home in 30 sessions 15 minutes per day. The control group followed the training but did not use the Web App. Pre-to post-training measurements showed that children of the experimental group had an improvement significantly higher than the control group, in particular in arithmetic facts and written calculation. The study also highlighted that introducing the digital tool for automating arithmetic facts enabled a significant reduction in the number of face-to-face training sessions (48 versus 30 one-hour training sessions). Moreover, a follow-up evaluation indicated that the experimental intervention continued to be effective for up to two months after it ended (Re et al., 2019-submitted).

Bakker and coll. (Bakker, Van den Heuvel-Panhuizen, Van Borkulo, & Robitzsch,

2013) tested the intervention with multiplicative mini-games selected by the Rekenweb website on 1005 grade 2 primary school students. The intervention program lasted two periods of 10 weeks, the students played at school, at home without any attention at school and at home with supervision at school. These mini-games (16 in total) had to evaluate not only the aspects of the multiplicative mathematical abilities but also the insight in multiplicative number relations. There was a control group that played at school on mathematical games related to other math domains. The results were positive and students demonstrated remarkable improvements in the capacity for multiplicative reasoning after training compared to the control group and 'it appears that playing the mini-games at home with debriefing at school has the highest potential in promoting multiplicative reasoning ability' (Bakker et al., 2013)

Another study examined is that of (Bakker, van den Heuvel-Panhuizen, & Robitzsch, 2015) which focuses on the effect of videogames both on the automation of numerical facts and the relationship between numbers in three areas: knowledge, skills and mathematical intuition. In this study, the multiplicative reasoning skills were evaluated as an effect of videogames. Mini games were used under the following three conditions: at home, at school and finally at home and school. To evaluate these aspects, three tests were used: The Knowledge Test, Skills Test, and Insight Test, which respectively analyzed declarative, procedural and conceptual mathematical knowledge. What sets this study apart was that it was 'longitudinal': the level of learning with mini-games was measured at the end of the first, at the end of the second and at the end of the third year (from 2009 to 2012). The experiment was performed in 66 Dutch schools, although only 47 remained involved until the end. The children were asked to play the games for 4 periods, each of 10 weeks. At the end of each week a new game was presented, some of which were taken from the German site of mathematical games "Rekenweb" and modified to create more learning opportunities through exercise and experimentation of the properties of the operations and calculation strategies. The behavior of the children during the game in terms of time, effort and success as well as

the numbers of games was also assessed. The results showed that when mini-games were used first at home and then at school, they had an effect in intensifying multiplicative reasoning skills in the areas of intuition and skills, but not in mathematical knowledge, emphasizing their importance in promoting categorical (taxonomic) thinking rather than application knowledge. In fact, in the domestic setting, children have more time and can have more control over their learning. Furthermore, home-school conditions ensure that intervention at school motivates children to play further at home. Another game that has been designed to improve basic numerical skills in children with a diagnosis of developmental dyscalculia (DD) is called "Rescue Calcularis" (Kucian et al., 2011b). The game has been specifically developed to enhance spatial working memory, the understanding of the ordinality of numbers, the spatial representation of numbers, estimation, and arithmetical skills. In their study, Kucian et al. evaluated the effects of the game on 16 dyscalculic children from 8 to 10 years of age and compared their performance to that of 16 aged-matched control children without DD diagnosis. All children were evaluated by neuropsychological assessment, behavioral tests and functional magnetic resonance imaging (fMRI) before the intervention. Next, participants played at home every day 15 minutes 5 days a week with this videogame and after 5-week of training all the children underwent a second fMRI and behavioral tests. The authors reported a general benefit for all the children (with and without diagnosis of DD). Specifically, they show pre-to-post significant improvements in spatial representation of numbers and the number of correctly solved arithmetical problems. This is, to our knowledge, the first imaging study to examine the effects of a specific game-based intervention in children with developmental dyscalculia. As such, it provides important insights into the way in which educational software games may modulate brain functioning, and support learning efforts in affected children (Kucian et al., 2011b).
Following the previous achievements obtained with Rescue Calcularis, the same group of researchers evaluated the effects of a renewed computer-based training software, called Calcularis, among 2nd to 5th grade school children with difficulties in learning

mathematics. The software offers 10 different types of games and an adaptive algorithm to recognize the specific problems and personalize the learning of any child. It has the possibility to train a wide range of subjects, including ordinal number understanding, spatial number representation, basic arithmetic operations, among others. In this study, fifteen children completed an intensive training of 12 weeks, 20 minutes per day, 5 days a week and their performance was compared to that of a control group that began the training 6 weeks later. During the test sessions, all children underwent a series of mathematical performance assessments including arithmetic (additions and subtractions), number line, non-symbolic magnitude comparison and estimation. The results showed significant improvements in number line tasks and subtractions as well as a reduction in problem solving times in the experimental group as compared to the waiting control group (Käser et al., 2013b). Besides the positive effects, the interventions with “Rescue Calcularis” and “Calcularis” represent further examples of how CAI can be successfully integrated out-of-school in everyday contexts. In fact, in both studies children performed the training at home with exception of one mandatory training session per 6 weeks in the laboratory.

In the study of Dubé et al. (Dubé & Keenan, 2016) 30 children from 4 to 7 years of age were evaluated while playing videogames on the tablet to analyze the differences in cognitive abilities. These videogames included mathematical activities that did not directly present the learning content but highlighted it indirectly through some mini games. Through the ocular detection system, fixation times and the number of fixations were assessed to evaluate the involvement of children while they were engaged in mathematical activities on the tablet. The children recorded a raised motivation and fun in playing, more attention and improvements in working memory, except for children with poor working memory and difficulties in mathematics. The study proved effective in directing players’ attention to the intrinsic content of the game (operations with numbers and symbols and counting skills) demonstrating consistent interaction and improvement of cognitive abilities.

4.5 Conclusion

Playing videogames at home is important in enhancing mathematics, but the number of studies is too limited. Parents and teachers should work together, to ensure that what the children learn at home has positive repercussions on academic performance in school. In fact, a 2006 study by Marlene Kliman (Kliman, 2006) revealed how many parents did not believe that playing games could benefit the schoolwork. In choosing educative videogames, as a support to children with low-numeracy skills, parents need to make informed decisions and rely on practices that have substantive support from scientific research. Future research in this direction is necessary to evaluate the effectiveness of interventions with videogames performed exclusively at home.
In some respects, these are less attractive stimuli to researchers, because they do not always allow the same degree of control; nevertheless, they are very attractive stimuli to children, and they are pervasive in the everyday life of contemporary youth. Part of the psychological promise of videogames is that they could, in principle, be developed or adapted to address a very wide range of children's learning and skill needs (Boyle, Connolly, Hainey, & Boyle, 2012), (Weber, Ritterfeld, & Mathiak, 2006). Therefore, it would be useful to design videogames easily accessible, for example through web sites, implying the involvement of both areas, school and home, and their collaboration. Classroom and home learning environments, among others, are important contexts that need to be carefully considered in combination (Cozza & Oreshkina, 2013), (Gottfried & Gee, 2017), (Hampden-Thompson & Galindo, 2017), (Schütte, 2014).
Although most of the research to date on videogame play has focused on young people with typical development, recent work has shown that, if properly designed, videogames can also be used to compensate for learning difficulties in children with special educational needs (Conti-Ramsden, Mok, Pickles, & Durkin, 2013). The successful use of games in children who generally find learning especially challenging is by no means

trivial. It requires a careful identification of the game features that could set the scene for the children's active engagement and learning.

Interactions among home, classrooms, and children, influence children and allow them to learn basic skills, including math skills (Eccles & Roeser, 2011).

Conclusions

The following research questions guided this thesis:

1. How can ICT contribute to identifying the student's mathematical difficulties and strengths?

2. How can ICT contribute to detecting emotional difficulties or anxiety states which can interfere with children's learning? Can these results provide clues about the "child/teachers" and "child/parents" relationship that may be useful in pedagogical strategies?

3. Can ICT be used in the classroom to enhance children's mathematics abilities?

4. Can ICT be successfully incorporated in the home environment to enhance children's mathematics abilities and to establish educational alliances between school and families?

In the first chapter a profiling software is presented in detail to measure skills and difficulties in learning mathematics: the MathPro Test (Mathematical Profile and Dyscalculia Test) devised by Giannis Karagiannakis et al. The chapter describes also the standardization activity performed throughout Italy, which began with the administration of the test to 1,728 children from 22 schools. Not being able to exemplify all the obtained profiles, given also the complexity of the tests that have been administered, some typical examples are illustrated. The test is introduced after having defined the framework of difficulties in mathematics and described the existing studies regarding their identification and classification. Furthermore, the opinions of all math teachers, who had taken part in the MathPro standardization in the province of Varese, were collected with an anonymous questionnaire to try to understand how the teachers 'evaluate' their students' performances in mathematics. The evaluation that the teacher can give may change as a result of the knowledge of the profiles emerging from a profiling software such as MathPro.

The goal of the second chapter is to complete the profile obtained in the first chapter (which highlighted the student's mathematical skills/abilities and cognitive abilities) with the emotional part. This chapter proposes to recognize the children's emotions using the physiological signal generated by the skin conductance, measured wearing a bio-tracker on the tip of the left index finger. Only a pilot experiment was performed to demonstrate the feasibility and the potentiality of such a method because of the high cost of this equipment. The results can give clues about the "child/teachers" and "child/parents" relationship that can be used in pedagogical strategies.

Once the strengths and weaknesses in the learning processes of the children have been identified, it is necessary to outline hypotheses of intervention strategies, aligned with their profiles. Chapter three proposes a meta-analysis of 15 studies (with 1073 participants), about the effectiveness of digital-based interventions, designed for students with dyscalculia in primary school and severely at risk for dyscalculia in preschool ages. Finally, in chapter four, the effects of educational math games at home and the parents' contribution to Educational Technology are analyzed. The chapter presents a literature review. Interactions among home, classroom, and children, influence children themselves and allow them to learn basic skills, including math skills. In this chapter the literature review provides a summative overview of the current evidence to date missing in the literature.

This thesis is the result of a work performed across three main interactive components of the learning process: the educational, psychophysiological and psychological ones. Mathematics is pervasive in natural phenomena and in daily life. The school is the context where the learning of this complex discipline is formalized. The teaching and learning processes not only interact effectively with each other, but also with emotional factors, social skills and relationship, communication, classroom climate, behavior, teachers and family.

Piaget's extensive studies on children's affective, and cognitive or intellectual development showed that the outcome of the learning processes is the result of the integration of biological and environmental factors where family, school and teachers play a fundamental role. Teaching methods, family expectations, personality features (low self-esteem, insecurities, learned impotence, mathematics phobia, etc.) and cognitive factors (specific difficulties in math) have an effect on the child's attitude towards mathematics. A detailed analysis of each student in his/her physical, emotional and cognitive features would help in choosing the intervention strategies but it requires a large effort in terms of time that teachers and educators might not always have, especially when they have to follow a large number of students in class.

We have seen that technology can have an important role to define a holistic and multidimensional vision of each student. A standardized software, such as MathPro, can provide qualitative and quantitative individual profiles of math learning for each student, even in an early stage of their development. The evaluations are organized in groups based on several domains: Core Number (Numerical and Spatial), Memory (Counting and Retrieval) and Reasoning (Numerical and Spatial).

The reports generated by the software, at the end of the online test, allow to organize math teaching not only working on the development phases corresponding to the different ages, but also defining strengthening activities able to enhance the strengths of each student.

Nevertheless, a software as MathPro is not able to investigate the emotional sphere of the children undergoing the test. A difficulty in math learning and dyscalculia result in a series of relevant consequences for the child wellbeing, because the low self-esteem, the avoidance behaviors and the school failure will affect not only his/her results at school but also the quality of his/her life.

Emotions can act as a real compass for the individual and have a primary role in the determination of the cognitive activity (Piaget & Campbell, 2001). This indicates the importance to develop, since an early age, the Emotional Competence considered as a series of connected abilities that usually belong to three areas: expression, comprehension and emotional regulation (Denham & Dinn, 1998). Studies in the neuroscience field (Mercenaro, 2006) demonstrate that the emotion is strictly connected with the cognitive processes such as thinking, memory and learning. A poor emotional competence seems to be connected with the risk of a greater frequency of depressive disorders and aggressive behaviors resulting in poor learning (Mancini, 2011). The educational contexts should privilege a holistic and integrated vision able to recognize the existence and the importance of the relationship between these different domains which are really interconnected.

The synergy between information technology and psychophysiology offer tools to use in the classroom as teaching aids. Biometrical sensors, such as the Bio-trackers analyzed in this study, can be easily worn and are not very invasive. They are able to detect spontaneous and not controllable physiological signals. Psychophysiology could reliably assess the emotional state of children when they do math (and not only) to obtain criteria that would help in the choice of the pedagogical strategies to be undertaken.

Future directions

The individuation of accurate math learning profiles could allow to define individual and class intervention programs, that concentrate on the strengths of the students to compensate their weaknesses and to give them motivation. These activities can use digital tools which positively impact mathematics achievement of students with or at risk for dyscalculia, not only at school but also at home. Apart from the formal learning that happens in a class, each child meets experiences from his/her daily life in the family and in free time. This means that the times and the spaces of learning are much wider and contain every area of life and every time. The expression 'life-long and lifewide learning' refers exactly to this horizontal dimension and represents the overcoming of the places devoted to learning (traditionally the school) and the enhancement of the persons' experiences.

The sharing of activities and instruments between families and teachers can allow children to transform exclusively playful moments in pleasant informal learning moments by using at home educational math videogames. The number of studies is still limited and in this work a large-scale study was not possible. It would be important to evaluate the efficacy of activities with videogames performed only at home to understand the effect dimension. Future research in these directions is necessary and there must be more communication and planning between parents and teachers to allow children to engage in scientifically validated videogames.

As far as the future development of digital tools to define individual profiles, MathPro opens research possibilities that require feasibility studies, also from the longitudinal point of view, to be performed in class. Quantitative studies on students numbers large enough could allow multivaried analyses of the variances that would help in connecting phenomena with factors such as the scholar grade, the teacher evaluation, the gender, the performance on a given task or in other math domains such as:

- How do the mathematical abilities develop year to year (grade effect)?
- Are there any gender differences (gender effect)?
- Which mathematical skills most predict the students' performance on mathematics at school according to the school's teacher evaluation?
- Which mathematical profiles emerge from the population? Do children who were evaluated in maths by the school's teachers as "poor" or "very poor" constitute a distinguish group? If yes, do they appear with a homogenous profile?

The literature on the points listed above is vast, and we now have a better understand of these issues, but further studies would be important also considering the development of digital tools. Some questions are still open:

- When students work at a computer, they stay in almost permanent manual contact with a mouse or with a keyboard. The incorporation of a sensor, such as a galvanic skin sensor (GSR), in the input devices, would be able to determine students' physiological conditions in a non-invasive way. How could this combined profile better improve the design and implementation of educational activities?

- Could future developments of the MathPro allow to understand the strategies adopted by students to solve tasks?

- Could the artificial intelligence at the basis of MathPro allow for a platform, an online Intelligent Tutor System (ITS), for dedicated self-adapted recovery interventions administered by the software to single students depending on the found profiles?

Math teaching cannot be only the proposal of a discipline but it must be founded on the principle that educating means educating to the integration of the Self and ICT, as we have seen, can enter in class and at home and can represent a valid support to the educators. Designing holistic environments to effective teaching and learning for disadvantaged students to succeed in mathematics incorporating ICT may be a successful and effective solution.

Appendix

The following pages show the copy of the report generated with MathPro

Mathematical Profile & Dyscalculia Test

Participant's details

Name: VAIC81200G C3
ID: 13933
Gender: Male
Age: 8
Grade: Grade 3
Date of birth: 2009-12-01
Date of test: 2018-05-21
Testing language: Italian

What is the MathPro test

The Mathematical Profile & Dyscalculia Test (MathPro Test) is an online computer-based standardised battery of numerical tasks designed to sketch out individual mathematical profiles of 1st to 6th grade students as well as to identify students at risk of difficulties in mathematics or Dyscalculia. 1728 children in Grades 1–6 from 22 elementary schools scattered around Italy participated in the standardisation study of the MathPro test in Italian language.

The MathPro test is the fruit of the collaboration between two researchers, Giannis Karagiannakis from the National & Kapodistrian University of Athens (Greece) specialized in the field of mathematics and Marie-Pascale Noël from the University of Louvain (Belgium) specialized in child neuropsychology, both experts in children's numerical development and math learning difficulties or dyscalculia.

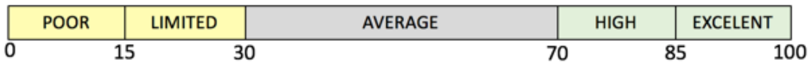
The test includes 18 tasks measuring a broad area of mathematical skills classified in the domains presented in the following table:

1. Dots Comparison 2. Subitizing	Spatial	CORE NUMBER
3. Single-digit numbers comparison 4. Multi-digit numbers comparison	Numerical	
5. Enumeration 6. Numbers dictation 7. Next number 8. Previous number	Counting	MEMORY
9. Addition facts retrieval 10. Multiplication facts retrieval	Retrieval	
11. Mental calculations 12. Word problems 13. Calculation principles 14. Numerical patterns	Numerical	REASONING
15. Number lines 0-100 16. Number lines 0-1000 17. Squares 18. Building blocks	Spatial	

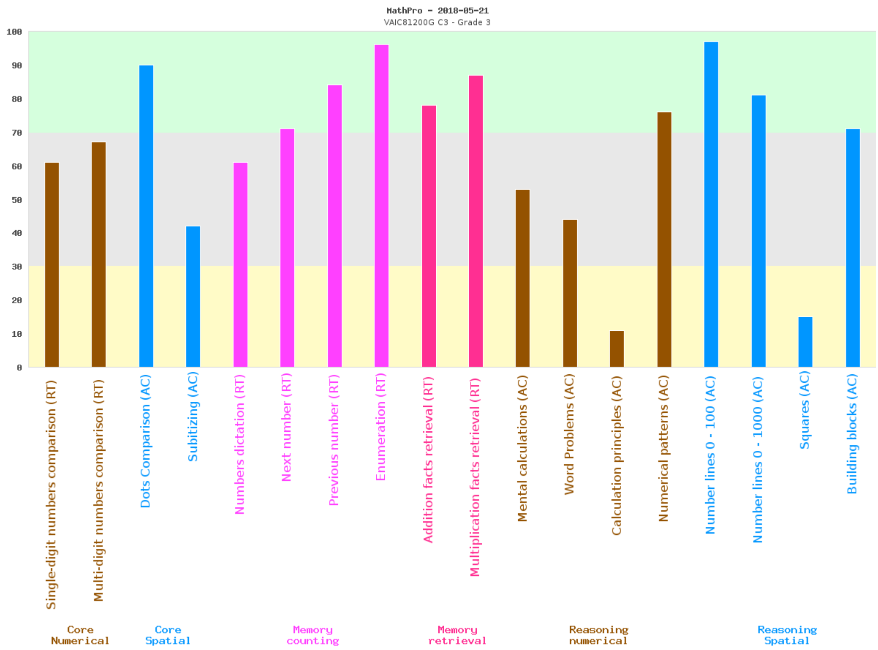
MEASUREMENTS

The RESPONSE TIME or/and the ACCURACY are calculated to measure the performance at each subtest. The RESPONSE TIME is adapted by the system to the personal response time of each student (this means that we subtract from response time, the time needed for the child to use the mouse and press on the digits pad of the computer screen). The performance of the students is given in terms of percentile rank. A percentile rank of a score is the percentage of the standardisation study sample scores that are at or below that score. For instance, if a child's performance at a test is at percentile 15, it means that 15 percent of the sample of same school grade children that have been tested with the battery perform below or at the same level as this child. Inversely, it also means that 85 percent of children in the test sample (i.e., 100-15) reached better performance than the child. Such a performance is thus rather poor. If a child's performance corresponds to percentile 50, it does not mean that he succeeded on half of the items but that he is performing as the average population, 50% of the test sample reaching below or same score and 50% of the test sample reaching higher performance.

ACHIEVEMENT LEVELS



I. Overview of the results



AC = ACCURACY

RT= RESPONSE TIME

The **grey zone** corresponds to **AVERAGE** performance.

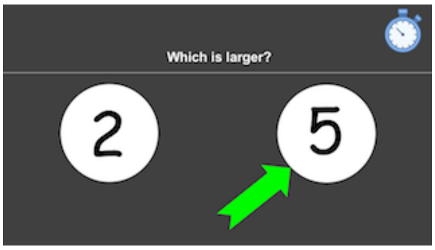
The **green zone** corresponds to **HIGH** or **EXCELLENT** performance.

The **yellow zone** corresponds to **LIMITED** or **POOR** performance.

II. Analysis of the results

Single digit numbers comparison (RT)

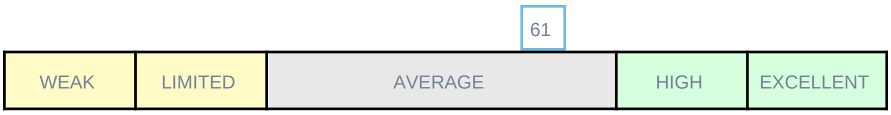
Two single Arabic numbers are presented and children have to decide as quickly as possible the larger numberby clicking on it with the computer mouse. This particular task intends to examine the speed to access Arabic number magnitude, or the speed with which the child access to the magnitude value of each digit and compare them. The **RESPONSE TIME** is calculated as a measure of performance if the ACCURACY is high enough as expected from the students of this grade.



RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **61th**, ranking the performance of the student at the **AVERAGE** level.

This means that **61%** of the same age students perform in this task equal or below.

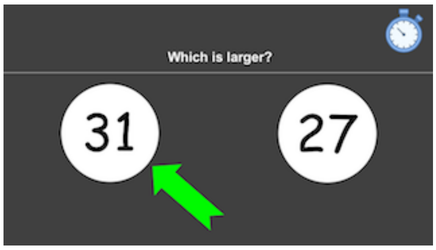


ACCURACY

The ACCURACY of the student in this task is 24/24 which is in the AVERAGE range compared to his peers in the same grade.

Multi-digit numbers comparison (RT)

This task is the same as the Single-digit numbers comparison but the number pairs consist of 2-digit and 3-digit numbers as well as decimal numbers. The Multi-digit numbers comparison intends to examine children's understanding of the positional value of 10-based numerical system. The performance is measured through the **RESPONSE TIME** if the ACCURACY is high enough.



RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **67th**, ranking the performance of the student at the **AVERAGE** level.

This means that **67%** of the same age students perform in this task equal or below.

67

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

ACCURACY

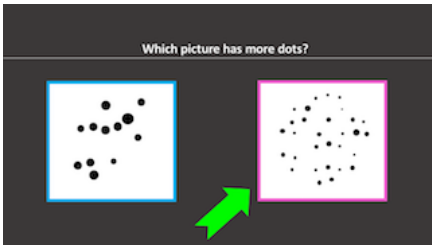
The ACCURACY of the student in this task is 8/9 which is in the AVERAGE range compared to his peers in the same grade.

Error analysis

Serial Number	Task	Correct answer	Student's answer
28	411 - 289	1	0

Dots comparison (AC)

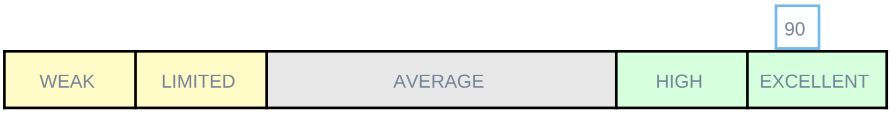
Two collections of dots are simultaneously presented on the computer screen for a very short time and children are asked to select the one that contained more dots by clicking on it with the computer mouse. The child is not allowed to count the dots but he is requested to answer on the basis of his approximate number sense. This task thus assesses the approximate number system aptitude. The **ACCURACY** is the measure of performance.



ACCURACY

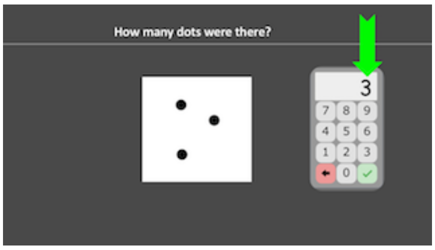
The ACCURACY is 22/30 which corresponds to the percentile rank of **90th**,ranking the performance of the student at the **EXCELLENT** level.

This means that **90%** of the same age students perform in this task equal or below.



Subitizing (AC)

Small arrays of black dots are very briefly presented on the computer screen and children are asked to indicate the corresponding numerosity by clicking on the computer screen calculator. This task assesses the subitizing skills, i.e., our ability to very quickly and very precisely detect the number of items of small collections such as 3 or 4. The **ACCURACY** is measure of performance.



ACCURACY

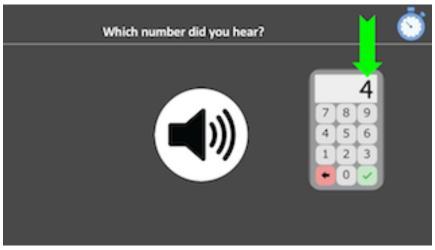
The ACCURACY is 15/20 which corresponds to the percentile rank of **42th**, ranking the performance of the student at the **AVERAGE** level.

This means that **42%** of the same age students perform in this task equal or below.

42

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Numbers dictation (RT)



Number-words are presented through computer speakers and children are asked to click on the screen calculator, the Arabic digits corresponding to the heard number. This task examines the *Number-word to Arabic digits transcoding skills.*It requires the comprehension of verbal numbers and the ability to master the positional system to produce Arabic numbers. The **RESPONSE TIME** is calculated as a measure of performance unless the ACCURACY is high enough as expected from the students of this grade. The task stops after three consecutive incorrect responses.

RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **61th**, ranking the performance of the student at the **AVERAGE** level.

This means that **61%** of the same age students perform in this task equal or below.

61

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

ACCURACY

The ACCURACY of the student in this task is 26/30 which is in the AVERAGE range compared to his peers in the same grade.

Error analysis

Serial Number	Task	Correct answer	Student's answer

25	30600	30600	300600
26	52000	52000	5200
29	10040	10040	1040
30	80400	80400	800300

Next number (RT)

Number words are presented through the computer speakers and children are asked to indicate as quickly as possible the number that comes just after by clicking on the corresponding digits on the computer screen calculator. This task examines the child's ease with numbers' ordinality. The **RESPONSE TIME** is calculated as a measure of performance unless the ACCURACY is high enough as expected from the students of this grade. The task stops after three consecutive incorrect responses.

RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **71th**, ranking the performance of the student at the **HIGH** level.

This means that **71%** of the same age students perform in this task equal or below.

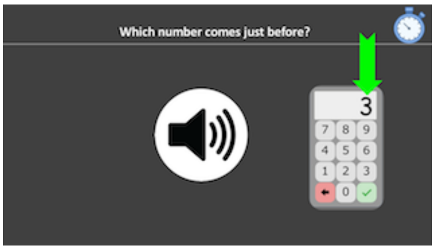
71

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

ACCURACY

The ACCURACY of the student in this task is 18/18 which is in the AVERAGE range compared to his peers in the same grade.

Previous number (RT)

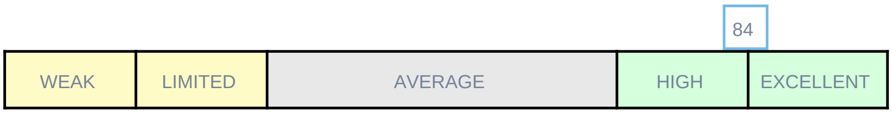


Number words are presented through the computer speakers and children are asked to indicate as quicklyas possible the number that comes just before by clicking on the corresponding digits on the computer screen calculator. This task examines the child's ease with numbers' ordinality in the descending order. The **RESPONSE TIME** is calculated as a measure of performance unless the ACCURACY is high enough as expected from the students of this grade. The task stops after three consecutive incorrect responses.

RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **84th**, ranking the performance of the student at the **HIGH** level.

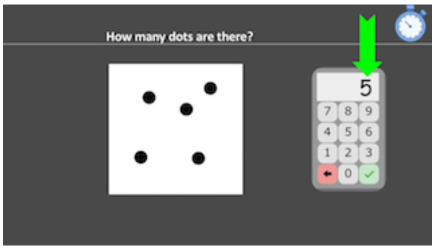
This means that **84%** of the same age students perform in this task equal or below.



ACCURACY

The ACCURACY of the student in this task is 18/18 which is in the AVERAGE range compared to his peers in the same grade.

Enumeration (RT)



Children are presented with arrays of black dots (arranged randomly) and are asked to count them and indicate the cardinal by clicking on the computer screen calculator. This task examines the counting skills. The performance is measured through the **RESPONSE TIME** unless the ACCURACY is high enough.

RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **96th**, ranking the performance of the student at the **EXCELLENT** level.

This means that **96%** of the same age students perform in this task equal or below.

96

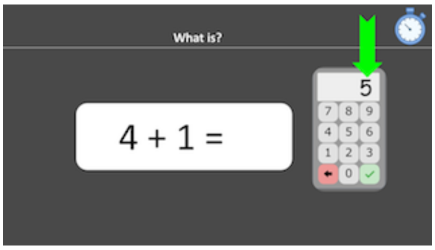
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

ACCURACY

The ACCURACY of the student in this task is 14/14 which is in the AVERAGE range compared to his peers in the same grade.

Addition facts retrieval (RT)

Single digit additions with operands from 2 to 9 and a sum equal or below 10 are presented in the Arabic code and children are asked to type as quickly as possible the correct sum by clicking on the computer screen calculator. This task intends to examine the simple addition facts retrieval skills. The **RESPONSE TIME** is calculated as a measure of performance when the ACCURACY is high enough as expected from the students of this grade.



RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **78th**, ranking the performance of the student at the **HIGH** level.

This means that **78%** of the same age students perform in this task equal or below.

78

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

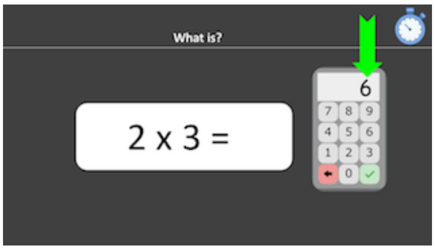
ACCURACY

The ACCURACY of the student in this task is 11/12 which is in the AVERAGE range compared to his peers in the same grade.

Error analysis

Serial Number	Task	Correct answer	Student's answer
1	4 + 3 =	7	6

Multiplication facts retrieval (RT)



Simple products with factors from 2 to 9 and at least one of them equal or below 5, are presented in the Arabic code and children are asked to type as quickly as possible the correct sum by clicking on the computer screen calculator. This task intends to examine the simple multiplication facts retrieval skills. The **RESPONSE TIME** is calculated as a measure of performance unless the ACCURACY is high enough as expected from the students of this grade.

RESPONSE TIME

The percentile rank for the RESPONSE TIME in this task is **87th**, ranking the performance of the student at the **EXCELLENT** level.

This means that **87%** of the same age students perform in this task equal or below.

				87
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

ACCURACY

The ACCURACY of the student in this task is 12/14 which is in the AVERAGE range compared to his peers in the same grade.

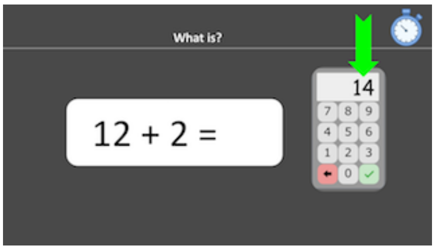
Error analysis

Serial Number	Task	Correct answer	Student's answer
5	4 x 9 =	36	3

14	4 x 7 =	28	12

Mental calculations (AC)

Addition, subtraction, multiplication and division operations with numbers up to three-digit long are presented to children who are asked to type in the screen calculator, as fast as they can, the correct answer. This task examines mental calculations skills.The **ACCURACY** is calculated as a measure of performance.



ACCURACY

The percentile rank for the ACCURACY (Addition: / , Subtraction: / , Multiplication: / , Division: /) is **53th** , ranking the performance of the student at the **AVERAGE** level.

This means that **53%** of the same age students perform in this task equal or below.

53

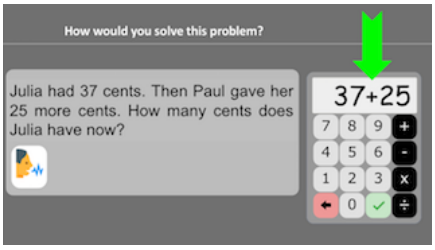
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Error analysis

Serial Number	Task	Correct answer	Student's answer
6	375 + 42 =	417	416
9	35 - 7 =	28	48
10	48 - 16 =	32	42
12	200 - 38 =	162	171
14	7 x 9 =	63	64

17	23 x 3 =	69	96
18	38 x 3 =	114	412
19	48 : 6 =	8	7
21	64 : 8 =	8	88

Word problems (AC)



Word problems are presented to the students who are asked to specify the calculation they would do to solve them by typing on the computer screen calculator the way each problem should be solved (for ex. 28+17) and not the final solution. This task assesses the skills of understanding, analysing, decision making, which are needed to solve a word problem. The **ACCURACY** is calculated as a measure of performance. The task stops after three consecutive incorrect responses.

ACCURACY

The ACCURACY is 6/18 which corresponds to the percentile rank of **44th**, ranking the performance of the student at the **AVERAGE** level.

This means that **44%** of the same age students perform in this task equal or below.

44

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

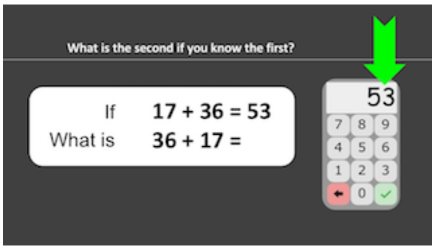
Error analysis

Serial Number	Task	Correct answer	Student's answer

5	Letizia ha 58 euro. Elia ha 17 euro in più che Letizia. Quanti euro ha Elia?	58+17	58:17
7	Daniela e Bianca hanno realizzato 136 braccialetti dell'amicizia. Daniela ha realizzato 87 braccialetti. Quanti braccialetti dell'amicizia ha realizzato Bianca?	136-87	136:87
9	Chiara ha 53 euro. Bruno ha 19 euro. Di quanti euro Bruno ha bisogno per averne come Chiara?	53-19	53*19
10	Denis ha 65 euro. Maria ha 27 euro. Quanti euro ha Denis in più di Maria?	65-27	27+65
11	Da un rubinetto escono 16 litri d'acqua al minuto. Quanti litri d'acqua colano da un rubinetto in 37 minuti?	16*37	16:37

Calculation principles (AC)

A pair of related multi-digit operations, one with the correct answer, the other without the answer given, are presented to the student. Students are asked to type the solution to the second problem without calculating it, but with reference to the first problem. This task intends to examine deductive reasoning skills on basic calculation principles such as the commutativity of the addition for instance (see in the example of the picture). The **ACCURACY** is calculated as a measure of performance.The task stops after three consecutive incorrect responses.



ACCURACY

The ACCURACY is 1/15 which corresponds to the percentile rank of **11th**, ranking the performance of the student at the **POOR** level.

This means that **11%** of the same age students perform in this task equal or below.

11

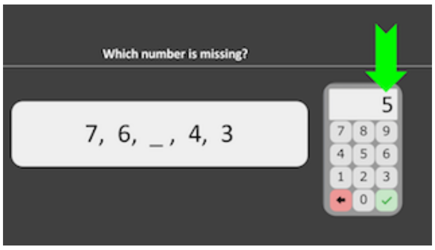
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Error analysis

Serial Number	Task	Correct answer	Student's answer
2	35 + 47 = 82 35 + 48 =	83	82
3	81 - 27 = 54 54 + 27 =	81	54
4	68 + 25 = 93 93 - 68 =	25	93

Numerical patterns (AC)

A series of numbers, with a missing one, is presented horizontally in the center of the computer screen and the student has to find the missing one and type it on the computer screen calculator. This task assesses the inductive reasoning skills on numerical patterns. The **ACCURACY** is calculated as a measure of performance. The task stops after three consecutive incorrect responses.



ACCURACY

The ACCURACY is 10/18 which corresponds to the percentile rank of **76th**, ranking the performance of the student at the **HIGH** level.

This means that **76%** of the same age students perform in this task equal or below.

76

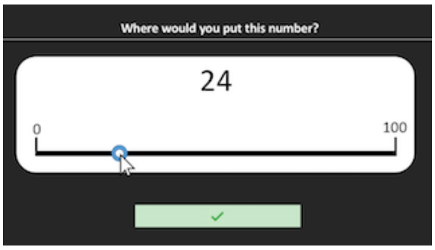
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Error analysis

Serial Number	Task	Correct answer	Student's answer
7	13, 11, 9, _ , 5	7	Geen antwoord
11	15, 13, 12, 10, 9, 7, _	6	5
13	2, 4, 5, 10, 11, 22, _	23	33
14	1, 4, 3, 6, 5, 8, _ , 10	7	9
15	1, 3, 6, 10, 15, _	21	18

Number lines 0-100 (AC)

Number lines with the 0 at the left end and the 100 at the right end are presented in the center of the computer screen. An Arabic target number from 0 to 100 appears above the center of the number line. Students are asked to mark the correct location of that target number on the line by marking it with the mouse. This task intends to examine the mapping skills of a number to a number line. The **ACCURACY** is calculated as a measure of performance.



ACCURACY

The percentile rank for the ACCURACY at the 22 trials of this task is **99th**, ranking the performance of the student at the **EXCELLENT** level.

This means that **99%** of the same age students perform in this task equal or below.

99

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Error analysis

Serial Number	Task	Correct answer	Student's answer

1	8	8	7.67
2	21	21	22.71
3	33	33	30.23
4	79	79	70.98
5	43	43	38.20
6	90	90	89.92
7	25	25	24.66
8	57	57	58.95
9	6	6	3.76
10	81	81	77.74
11	17	17	16.99
12	64	64	66.02
13	29	29	26.47
14	84	84	81.65
15	23	23	22.11
16	61	61	61.80
17	48	48	46.32
18	96	96	88.12
19	12	12	6.02
21	4	4	2.56
22	72	72	70.83

Number lines 0-1000 (AC)

The exact same task is then presented using a line marked this time with 0 at the left end and 1000 at the right end. The **ACCURACY**is calculated as a measure of performance.

ACCURACY

The percentile rank for the ACCURACY at the 22 trials of this task is **99th**, ranking the performance of the student at the **EXCELLENT** level.

This means that **99%** of the same age students perform in this task equal or below.

99

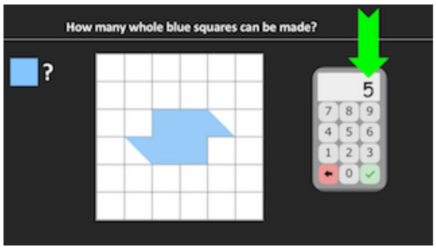
WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

Error analysis

Serial Number	Task	Correct answer	Student's answer
1	18	18	64.66
2	163	163	224.06
3	722	722	688.72
5	818	818	727.82
7	2	2	13.53
9	366	366	324.81
12	78	78	166.92
14	938	938	876.69
16	606	606	646.62

17	34	34	115.79
18	725	725	735.34
20	5	5	19.55
21	754	754	759.40
22	56	56	115.79

Squares (AC)



Geometrical shapes built on a white grid square background by joining a combination of blue squares, half-blue squares triangles and quarter blue squares triangles are presented. Students are asked to type the total number of whole squares that can be made from each geometrical shapeby clicking on the computer screen calculator. This task examines the ability to analyse 2D geometric figures (or subparts of them) in particular visualizing rigid motions such as rotations. The **ACCURACY** is calculated as a measure of performance.

ACCURACY

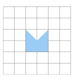
The ACCURACY is 3/10 which corresponds to the percentile rank of **15th**, ranking the performance of the student at the **POOR** level.

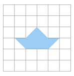
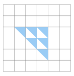
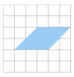
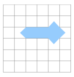
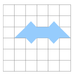
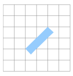
This means that **15%** of the same age students perform in this task equal or below.

15

WEAK	LIMITED	AVERAGE	HIGH	EXCELLENT

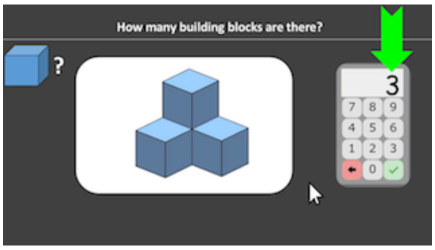
Error analysis

Serial Number	Task	Correct answer	Student's answer
4		3	4

5		4	8
6		3	6
7		6	10
8		4	12
9		5	14
10		2	4

Building blocks (AC)

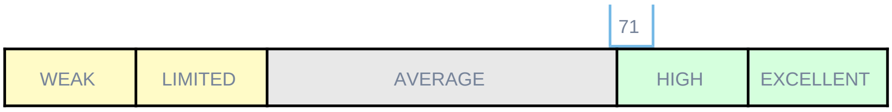
Abstract structures made of cube buildings blocks are presented. Students are asked to type the number of cubes that each 3D structure contains in total (both visible and hidden) by clicking on the computer screen calculator. This task examines the ability to analyse the structure of 3D geometric figures. The **ACCURACY**is calculated as a measure of performance.



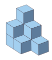
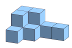
ACCURACY

The ACCURACY is 6/8 which corresponds to the percentile rank of **71th**, ranking the performance of the student at the **HIGH** level.

This means that **71%** of the same age students perform in this task equal or below.



Error analysis

Serial Number	Task	Correct answer	Student's answer
7		9	8
8		10	9

List of Figures

1 Students with DSA by type of disorder - comparison of school years 2013/14 - 2015/16 - 2017/18 (primary, middle and high school are considered). 9
2 My friend and me. 13
3 My friend and our math teacher. 14
1.1 Domains of the four-pronged model (Karagiannakis et al., 2014)- MLD stands for all those cognitive difficulties that students may experience in mathematics. 23
1.2 Numbers typing. 27
1.3 Find the cat. 27
1.4 Single digit numbers comparison - Which is larger? 28
1.5 Two-digit number comparison - Which is larger? 29
1.6 Three-digit number comparison - Which is larger? 29
1.7 Four-digit number comparison - Which is larger? 29
1.8 Decimal numbers comparison - Which is larger? 29
1.9 Dots comparison - Which picture has more dots? 30
1.10 Subitizing - How many dots are there? 30
1.11 Subitizing. 31
1.12 Subitizing - How many dots are there? 31
1.13 Enumeration - How many dots are there? 32
1.14 Numbers dictation - Which number have you heard? 33
1.15 Next number - Which number comes just after? 33

1.16 Previous number - Which number comes just before? 34
1.17 Addition facts retrieval - What is the result? 34
1.18 Multiplication facts retrieval - What is the result? 35
1.19 Mental Calculations - What is the result? 36
1.20 Mental Calculations - What is the result? 36
1.21 Mental Calculations - What is the result? 36
1.22 Word problems - How would you solve this? 37
1.23 Calculation Principles - What is the second if you know the first? . . . 38
1.24 Numerical patterns - Which number is missing? 38
1.25 Number lines 0-100 - Where would you put this number? 39
1.26 Number lines 0-1000 - Where would you put this number? 39
1.27 2D Shapes (Squares) - How many whole blue squares can you do? . . 40
1.28 3D Shapes - How many Building Blocks are there? 40
1.29 Students at work. 41
1.30 Achievement levels.The grey zone corresponds to the AVERAGE performance. The green zone corresponds to HIGH or EXCELLENT performance. The yellow zone corresponds to LIMITED or POOR performance. 44
1.31 Grade 4; LD: dyscalculia; teacher evaluation in maths: moderate; teacher evaluation in literacy: moderate. Gender M 47
1.32 Grade 4; LD: dyscalculia; teacher evaluation in maths: moderate; teacher evaluation in literacy: moderate. Gender: M 47
1.33 Grade 2; LD: no; teacher evaluation in maths: good; teacher evaluation in literacy: good. Gender: F. 48
1.34 Grade 2; LD: no; teacher evaluation in maths: good; teacher evaluation in literacy: good. Gender: F 49

1.35 Gender: male; age:8; grade: 3; date of birth 2009-12-01; LD: no; date of test: 2018-05-21; testing language: Italian; teacher evaluation in maths: good; teacher evaluation in literacy: good. 50
1.36 Grade 1; teacher evaluation in maths: very good; teacher evaluation in literacy: good. 55
1.37 Grade 1; teacher evaluation in maths: poor; teacher evaluation in literacy: moderate. 56
1.38 Traditional school, First-Grade class - MathPro tasks: percentage of failures. 56
1.39 Traditional school, Second-Grade class - MathPro tasks: percentage of failures. 57
1.40 Traditional school, Third-Grade class - MathPro tasks: percentage of failures. 58
1.41 Technological school, Second-Grade class - MathPro tasks: percentage of failures. 59
1.42 Technological school, Fourth-Grade class - MathPro tasks: percentage of failures. 60

2.1 The little girl. 80
2.2 The Bio-tracker. 80
2.3 Stress Index/Cognitive Load of the little girl. 81
2.4 Average GSR values. 81
2.5 GSR Peaks. 82
2.6 'Simple task' activation pattern. 82
2.7 'Difficult task' activation pattern. 82
2.8 Average values GSR. 83
2.9 GSR peaks. 84
2.10 Activation pattern with the teacher. 84
2.11 Activation pattern with the mother. 85

3.1 PRISMA flow diagram of systematic search and study selection. . . . 99

3.2 Forest plot. Each square represents the effect size of the study together with 95% confidence interval. The size of the symbol is proportional to the study's weight. 102

3.3 Funnel plot. Each black dot represents one study included in the meta-analysis. 105

4.1 PRISMA flow diagram of systematic search and study selection. . . . 116

List of Tables

1 Pupils with DSA by type of disorder - school year 2017/2018; the total number of pupils with DSA may not coincide with the sum of pupils by type of disorder since some pupils may have more types of disorders. * for kindergarten and for the first and second year of primary school: these are students at risk of learning difficulties, identified following specific tests at the competent healthcare facilities. 9

1.1 Classification model for MLD, proposing 4 subtypes, possible specific systems, and typical mathematical difficulties - Karagiannakis et al. 2014. 25

1.2 Grouping of the tasks of the MathPro Test. 26

1.3 Sample used for the standardization of MathPro in Italy. 26

1.4 The principle components Analysis (varimax) of the tasks of the MathPro test - Karagiannakis et al. 2017 42

1.5 Pearson's correlation coefficients between the tasks of the MathPro test - Karagiannakis et al. 2017 43

3.1 Descriptive statistics of the studies included in the review 101

3.2 Characteristics of the intervention for each study included in the review 103

3.3 Sensitivity analysis using the *Leave-One-Out Method* 106

3.4 Sensitivity analysis varing r_{tasks} and $r_{pre-post}$ 106

4.1 Characteristics of the selected studies. 117

4.2 The studies included in the review. 118

References

Aarnos, E., & Perkkila, P. (2012, 12). Early signs of mathematics anxiety? *Procedia - Social and Behavioral Sciences*, *46*, 1495-1499. doi: 10.1016/j.sbspro.2012.05.328

Aiken Jr, L. R., & Dreger, R. M. (1961). The effect of attitudes on performance in mathematics. *Journal of Educational psychology*, *52*(1), 19.

Alfano, C. A., Beidel, D. C., & Turner, S. M. (2002). Cognition in childhood anxiety: conceptual, methodological, and developmental issues. *Clinical Psychology Review*, *22*(8), 1209–1238.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in china and the us. *Journal of mathematics teacher education*, *7*(2), 145–172.

Anderson, J. R., Lebiere, C., Lovett, M., & Reder, L. (1998, dec). ACT-r: A higher-level account of processing capacity. *Behavioral and Brain Sciences*, *21*(6), 831–832. Retrieved from `https://doi.org/10.1017%2Fs0140525x98221765` doi: 10.1017/s0140525x98221765

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *Mathematics Educator*, *17*(1), 7–14.

anne LeFevre, J., Shanahan, T., & DeStefano, D. (2004, sep). The tie effect in simple arithmetic: An access-based account. *Memory & Cognition*, *32*(6), 1019–1031. Retrieved from `https://doi.org/10.3758%2Fbf03196878` doi: 10.3758/bf03196878

Ansari, D. (2016). Number symbols in the brain. In *Development of mathematical cognition* (pp. 27–50). Elsevier. Retrieved from `https://doi.org/10.1016%2Fb978-0-12-801871-2.00002-2` doi: 10.1016/b978-0-12-801871-2.00002-2

Arcara, G., Burgio, F., Benavides-Varela, S., Toffano, R., Gindri, P., Tonini, E., ... Semenza, C. (2017). Numerical activities of daily living–financial (NADL-F): A tool for the assessment of financial capacities. *Neuropsychological rehabilitation*, 1–23. doi: 10.1080/09602011.2017.1359188

Arzarello, F., & Robutti, O. (2002). *Matematica*. La Scuola.

Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton University Press. Retrieved from `https://books.google.it/books?id=SOkItEXV3poC`

Ashcraft, M. H., & Christy, K. S. (1995, nov). The frequency of arithmetic facts in elementary texts: Addition and multiplication in grades 1-6. *Journal for Research in Mathematics Education*, *26*(5), 396. Retrieved from `https://doi.org/10.2307%2F749430` doi: 10.2307/749430

Askew, M., & Askew, M. (1997). *Effective teachers of numeracy*. King's College London London.

Aubrey, C., Godfrey, R., & Dahl, S. (2006, 05). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal*, *18*, 27-46. doi: 10.1007/BF03217428

Aunio, P. (2019). Chapter 8 - early numeracy skills learning and learning difficulties-evidence-based assessment and interventions. In D. C. Geary, D. B. Berch, & K. Mann Koepke (Eds.), *Cognitive foundations for improving mathematical learning* (1st ed., Vol. 5, pp. 195–214). Academic Press. doi: 10.1016/B978-0-12-815952-1.00008-6

Aunio, P., & Mononen, R. (2018). The effects of educational computer game on low-performing children's early numeracy skills-an intervention study in a preschool setting. *European Journal of Special Needs Education*, *33*(5), 677–691. doi: 10.1080/08856257.2017.1412640

Baker, S., & Smith, S. (1999). Starting off on the right foot: The influence of four principles of professional development in improving literacy instruction in two kindergarten programs. *Learning Disabilities Research & Practice*, *14*(4), 239–253.

Bakker, M., van den Heuvel-Panhuizen, M., & Robitzsch, A. (2015, 01). Effects of playing mathematics computer games on primary school students' multiplicative reasoning ability. *Contemporary Educational Psychology*, *40*, 55. doi: 10.1016/j.cedpsych.2014.09.001

Bakker, M., Van den Heuvel-Panhuizen, M., Van Borkulo, S., & Robitzsch, A. (2013). Effecten van online mini-games op multiplicatieve vaardigheden van leerlingen in groep 4. *Pedagogische Studiën*, *90*(3), 21–36.

Baldacci, M. (2002). *Una scuola a misura d'alunno: qualità dell'istruzione e successo formativo*. UTET libreria. Retrieved from `https://books.google.it/books?id=OA7IAAAACAAJ`

Ball, D. L. (1991). What's all this talk about" discourse"? *The Arithmetic Teacher*, *39*(3), 44.

Ball, D. L., & Feiman-Nemser, S. (1988). Using textbooks and teachers' guides: A dilemma for beginning teachers and teacher educators. *Curriculum Inquiry*, *18*(4), 401–423.

Ball, D. L., Thames, M. H., Phelps, G., et al. (2008). Content knowledge for teaching: What makes it special. *Journal of teacher education*, *59*(5), 389–407.

Baroody, A. J., Eiland, M. D., Purpura, D. J., & Reid, E. E. (2012). Fostering at-risk kindergarten children's number sense. *Cognition and Instruction*, *30*(4), 435–470. doi: 10.1080/07370008.2012.720152

Baroody, A. J., Eiland, M. D., Purpura, D. J., & Reid, E. E. (2013). Can computer-assisted discovery learning foster first graders' fluency with the most basic addition combinations? *American Educational Research Journal*, *50*(3), 533–573. doi: 10.3102/0002831212473349

Bartelet, D., Ansari, D., Vaessen, A., & Blomert, L. (2014, 01). Cognitive subtypes of mathematics learning difficulties in primary education. *Research in developmental disabilities*, *35*. doi: 10.1016/j.ridd.2013.12.010

Becchi, E. (1963). *Appunti per un'educazione alla creatività*. Nicola.

Bechara, A., Damasio, H., Tranel, D., & Damasio, A. R. (2005). The iowa gambling task and the somatic marker hypothesis: some questions and answers. *Trends in cognitive sciences*, *9*(4), 159–162.

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, *107*(5), 1860–1863.

Bekdemir, M. (2010, 12). The pre-service teachers' mathematics anxiety related to depth of negative experiences in mathematics classroom while they were students. *Educational Studies in Mathematics*, *75*, 311-328. doi: 10.1007/s10649-010-9260-7

Benavides-Varela, S., Butterworth, B., Burgio, F., Arcara, G., Lucangeli, D., & Semenza, C. (2016). Numerical activities and information learned at home link to the exact numeracy skills in 5-6 years-old children. *Frontiers in Psychology*, *7*, 94. Retrieved from `https://www.frontiersin.org/article/10.3389/fpsyg.2016.00094` doi: 10.3389/fpsyg.2016.00094

Benavides-Varela, S., Zandonella Callegher, C., Fagiolini, B., Leo, I., Altoè, G., & Lucangeli, D. (2019-submitted). Effectiveness digital tools designed for children with developmental dyscalculia and at risk for developing numerical deficits: a meta-analysis.

Berkowitz, T., Schaeffer, M., Maloney, E., Petersen, L., Gregor, C., Levine, S., & Beilock, S. (2015, 10). Math at home adds up to achievement in school. *Science*, *350*, 196. doi: 10.1126/science.aac7427

Betz, N. E. (1978). Prevalence, distribution, and correlates of math anxiety in college students. *Journal of counseling psychology*, *25*(5), 441.

Borenstein, M., Hedges, L. V., Higgins, J. P., & Rothstein, H. R. (2009). *Introduction to meta-analysis*. John Wiley & Sons.

Boucsein, W. (2012). *Electrodermal activity*. Springer Science & Business Media.

Boyle, E. A., Connolly, T. M., Hainey, T., & Boyle, J. M. (2012). Engagement in digital entertainment games: A systematic review. *Computers in human behavior*, *28*(3), 771–780.

Breese, J. S., & Ball, J. E. (1999, November 16). *Modeling a user's emotion and personality in a computer user interface.* Google Patents. (US Patent 5,987,415)

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en didactique des mathématiques (Revue)*, *7*(2), 33–115.

Brownfield, S., & Vik, G. (1983). Teaching basic skills with computer games. *Training and Development Journal*, *37*(12), 52–56.

Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., ... Woolcott, G. (2016, dec). Understanding gaps in research networks: using "spatial reasoning" as a window into the importance of networked educational research. *Ed-*

ucational Studies in Mathematics, *95*(2), 143–161. Retrieved from `https://doi.org/10.1007%2Fs10649-016-9743-2` doi: 10.1007/s10649-016-9743-2

Brunda, A., & Bhavithra, J. (2010). Adaptive computer assisted instruction (CAI) for students with dyscalculia (learning disability in mathematics). In *Proceedings of the 1st amrita ACM-w celebration on women in computing in india* (pp. 12:1–12:7). New York, NY, USA: ACM. doi: 10.1145/1858378.1858390

Bryant, D. P., Bryant, B. R., Roberts, G., Vaughn, S., Pfannenstiel, K. H., Porterfield, J., & Gersten, R. (2011). Early numeracy intervention program for first-grade students with mathematics difficulties. *Exceptional Children*, *78*(1), 7–23. doi: 10.1177/001440291107800101

Burns, M. K., Kanive, R., & DeGrande, M. (2012). Effect of a computer-delivered math fact intervention as a supplemental intervention for math in third and fourth grades. *Remedial and Special Education*, *33*(3), 184–191. doi: 10.1177/0741932510381652

Butterworth, B. (2019). *Dyscalculia from science to education*. Routledge Taylor & Francis Group. Retrieved from `https://lccn.loc.gov/2018033493`

Butterworth, B., & Laurillard, D. (2010). Low numeracy and dyscalculia: identification and intervention. *ZDM Mathematics Education*, *42*(6), 527–539. doi: 10.1007/s11858-010-0267-4

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science*, *332*(6033), 1049–1053. doi: 10.1126/science.1201536

Byrne, B., Fielding-Barnsley, R., & Ashley, L. (2000). Effects of preschool phoneme identity training after six years: Outcome level distinguished from rate of response. *Journal of educational psychology*, *92*(4), 659.

Cacioppo, J. T., Tassinary, L. G., & Berntson, G. (2007). *Handbook of psychophysiology*. Cambridge University Press.

Cezarotto, M. A., & Battaiola, A. L. (2016). Game design recommendations focusing on children with developmental dyscalculia. In *International conference on learning and collaboration technologies* (pp. 463–473). doi: 10.1007/978-3-319-39483-1_42

Chandler, C. C., & Kamii, C. (2009). Giving change when payment is made with a dime: The difficulty of tens and ones. *Journal for Research in Mathematics Education*, 97–118.

Chi, M. T., & Klahr, D. (1975). Span and rate of apprehension in children and adults. *Journal of Experimental Child Psychology*, *19*(3), 434–439.

Chodura, S., Kuhn, J.-T., & Holling, H. (2015). Interventions for children with mathematical difficulties. *Zeitschrift für Psychologie*, *223*(2), 129-144. doi: 10.1027/2151-2604/a000211

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ: Erlbaum.

Colangelo, N., & Davis, G. (1997). *Handbook of gifted education*. Allyn and Bacon. Retrieved from `https://books.google.it/books?id=A4LuAAAAMAAJ`

Conti-Ramsden, G., Mok, P. L., Pickles, A., & Durkin, K. (2013). Adolescents with a history of specific language impairment (sli): Strengths and difficulties in social, emotional and behavioral functioning. *Research in developmental disabilities*, *34*(11), 4161–4169.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for research in mathematics education*, 324–336.

Coppola, C., Di Martino, P., Pacelli, T., Sabena, C., et al. (2012). Primary teachers' affect: A crucial variable in the teaching of mathematics. *Nordic Studies in Mathematics Education*, *17*(3-4), 101–118.

Cozza, B., & Oreshkina, M. (2013). Cross-cultural study of cognitive and metacognitive processes during math problem solving. *School Science and Mathematics*, *113*(6), 275–284.

D'Amore, B. (2003). *Le basi filosofiche, epistemologiche e concettuali della didattica della matematica*. Pitagora. Retrieved from https://books.google.it/books?id=lp9XAAAACAAJ

D'Amore, B., & Fandiño Pinilla, M. (2001). *Didattica della matematica*. Pitagora.

D'Amore, B., Fandiño Pinilla, M. I., Marazzani, I., Santi, G., & Sbaragli, S. (2009). Il ruolo dell'epistemololgia dell'insegnante nelle pratiche d'insegnamento. *Insegnamento della matematica e delle scienze integrate*, *32*(2), 171–192.

David, F. (1998). *Games, gods, and gambling: A history of probability and statistical ideas*. Dover Publications. Retrieved from https://books.google.it/books?id=8ddP8zNx9nQC

Dempsey, J. V., Rasmussen, K., & Lucassen, B. (1994). Instructional gaming: Implications for instructional technology. In *Annual meeting of the association for educational communications and technology, 16-20 february 1994.* Nashville, TN. Retrieved from https://eric.ed.gov/?id=ED368345

Denham, S., & Dinn, J. (1998). *Emotional development in young children*. Guilford Publications. Retrieved from https://books.google.it/books?id=eNZxMsUCoewC

Desoete, A., Roeyers, H., & De Clercq, A. (2004). Children with mathematics learning disabilities in belgium. *Journal of learning disabilities*, *37*(1), 50–61. doi: 10.1177/00222194040370010601

DeStefano, D., & LeFevre, J.-A. (2004, may). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, *16*(3), 353–386. Retrieved from https://doi.org/10.1080%2F09541440244000328 doi: 10.1080/09541440244000328

Di Martino, P., & Zan, R. (2013). Where does fear of maths come from? beyond the purely emotional. In *Congress of the european society of research in mathematics education* (pp. 1309–1318).

Di Martino, P., & Zan, R. (2015). The construct of attitude in mathematics education. In *From beliefs to dynamic affect systems in mathematics education* (pp. 51–72). Springer.

Dowker, A. (2005). Early identification and intervention for students with mathematics difficulties. *Journal of learning disabilities*, *38*(4), 324–332.

Dowker, A. (2017). Interventions for primary school children with difficulties in mathematics. In J. Sarama, D. H. Clements, C. Germeroth, & C. Day-Hess (Eds.), *Advances in child development and behavior* (Vol. 53, pp. 255–287). Elsevier. doi: 10.1016/bs.acdb.2017.04.004

Dreger, R. M., & Aiken Jr, L. R. (1957). The identification of number anxiety in a college population. *Journal of Educational psychology*, *48*(6), 344.

Drigas, A. S., Pappas, M. A., & Lytras, M. (2016). Emerging technologies for ICT based education for dyscalculia: Implications for computer engineering education. *International Journal of Engineering Education*, *32*(4), 1604–1610. Retrieved from https://www.researchgate.net/profile/Marios_Pappas/publication/309922334_Emerging_technologies_for_ICT_based_education_for_dyscalculia_Implications_for_computer_engineering_education/links/582ed5ea08aef19cb81521a2.pdf

Dubé, A. K., & Keenan, A. (2016). Are games a viable home numeracy practice? In *Early childhood mathematics skill development in the home environment* (pp. 165–184). Springer.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... others (2007). School readiness and later achievement. *Developmental psychology*, *43*(6), 1428–1446. doi: 10.1037/0012-1649.43.6.1428

Durkin, K., Boyle, J., Hunter, S., & Conti-Ramsden, G. (2015). Video games for children and adolescents with special educational needs. *Zeitschrift für Psychologie*, *221*(2), 79–89. doi: 10.1027/2151-2604/a000138

Durkin, K., Whitehouse, A., Jaquet, E., Ziatas, K., & Walker, A. (2010, 04). Cell phone use by adolescents with asperger syndrome. *Research in Autism Spectrum Disorders*, *4*, 314-318. doi: 10.1016/j.rasd.2009.09.017

Duval, S., & Tweedie, R. (2000). Trim and fill: a simple funnel-plot-based method of testing and adjusting for publication bias in meta-analysis. *Biometrics*, *56*(2), 455–463. doi: 10.1111/j.0006-341X.2000.00455.x

Eccles, J. S., & Roeser, R. W. (2011). Schools as developmental contexts during adolescence. *Journal of research on adolescence*, *21*(1), 225–241.

Eichenbaum, A., Bavelier, D., & Green, C. S. (2014). Video games: play that can do serious good. *American Journal of Play*, *7*(1), 50-72. Retrieved from `https://archive-ouverte.unige.ch/unige:84313` (ID: unige:84313)

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of education for teaching*, *15*(1), 13–33.

Eysenck, M. W., & Calvo, M. G. (1992). Anxiety and performance: The processing efficiency theory. *Cognition & Emotion*, *6*(6), 409–434.

Fandiño Pinilla, M. (2003). *Riflessioni sulla formazione degli insegnanti di matematica: una rassegna internazionale.* Bologna: Pitagora.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In *Affect and mathematical problem solving* (pp. 205–219). Springer.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact.

Fen-ping, W. (2011). Comparative study of high school mathematics textbook between china and britain—plane vector in high school mathematics textbooks edited by britain aqa examination committee and in the beijing normal university press textbook as examples [j]. *Journal of Mathematics Education*, *6*.

Fletcher, J. (2007). *Learning disabilities: From identification to intervention.* Guilford Publications. Retrieved from `https://books.google.com/books?id=JhpKDwAAQBAJ`

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work.* Heinemann Portsmouth, NH.

Frabboni, F., Guerra, L., & Scurati, C. (1999). *Pedagogia: realtà e prospettive dell'educazione*. Pearson Italia Spa.

Fritz, A., Haase, V. G., & Räsänen, P. (2019). *International handbook of mathematical learning difficulties: From the laboratory to the classroom*. Springer International Publishing. Switzerland. doi: 10.1007/978-3-319-97148-3

Fuchs, L. S., Fuchs, D., Hamlet, C. L., Powell, S. R., Capizzi, A. M., & Seethaler, P. M. (2006). The effects of computer-assisted instruction on number combination skill in at-risk first graders. *Journal of Learning Disabilities*, *39*(5), 467–475. doi: 10.1177/00222194060390050701

Garvis, S., Fluckiger, B., & Twigg, D. (2012). Exploring the beliefs of commencing early childhood education graduate students: Providing insights to improve teacher education programs. *Australian Journal of Teacher Education*, *37*(1), 93–105.

Gavazzi, I., & Crugnola, C. (2011). *Lo sviluppo della competenza emotiva dall'infanzia all'adolescenza. percorsi tipici e atipici e strumenti di valutazione*. Unicopli. Retrieved from https://books.google.it/books?id=1_jNygAACAAJ

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* American Psychological Association.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study. *Developmental Psychology*, *47*(6), 1539–1552. doi: 10.1037/a0025510

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of Learning Disabilities*, *45*(4), 291–307. doi: 10.1177/0022219410392046

Gee, J. P. (2003). What video games have to teach us about learning and literacy. *Computers in Entertainment (CIE)*, *1*(1), 20–20. doi: 10.1145/950566.950595

Georgiou, S. N. (1999). Parental attributions as predictors of involvement and influences on child achievement. *British Journal of Educational Psychology*, *69*(3), 409-429. Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1348/000709999157806 doi: 10.1348/000709999157806

Gilmore, C., Göbel, S. M., & Inglis, M. (2018). *An introduction to mathematical cognition*. Routledge.

Ginsburg, H. P., Jamalian, A., & Creighan, S. (2013). Cognitive guidelines for the design and evaluation of early mathematics software: The example of MathemAntics.

In L. English & J. Mulligan (Eds.), *Reconceptualizing early mathematics learning* (pp. 83–120). Springer, Dordrecht. doi: 10.1007/978-94-007-6440-8_6

Gottfried, M. A., & Gee, K. A. (2017). Identifying the determinants of chronic absenteeism: A bioecological systems approach. *Teachers College Record*, *119*(7), n7.

Gouet, C., Silva, C. A. G., Guedes, B., & Peña, M. (2018). Cognitive and neural effects of a brief nonsymbolic approximate arithmetic training in healthy first grade children. *Frontiers in integrative neuroscience*, *12*, 28. doi: 10.3389/fnint.2018.00028

Graeber, A., & Tirosh, D. (2008). Pedagogical content knowledge: Useful concept or elusive notion. In *The handbook of mathematics teacher education: Volume 1* (pp. 115–132). Brill Sense.

Griffiths, M. D. (2002). The educational benefits of videogames. *Education and health*, *20*(3), 47–51.

Halberda, J., & Feigenson, L. (2008, 10). Developmental change in the acuity of the ”number sense”: The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental psychology*, *44*, 1457-65. doi: 10.1037/a0012682

Hampden-Thompson, G., & Galindo, C. (2017). School–family relationships, school satisfaction and the academic achievement of young people. *Educational Review*, *69*(2), 248–265.

Handal, B. (2003). Teachers’ mathematical beliefs: A review. *The Mathematics Educator*, *13*(2).

Hasselbring, T. S., & Glaser, C. H. W. (2000). Use of computer technology to help students with special needs. *The Future of Children*, *10*(2), 102. Retrieved from https://doi.org/10.2307%2F1602691 doi: 10.2307/1602691

Hassler Hallstedt, M., Klingberg, T., & Ghaderi, A. (2018). Short and long-term effects of a mathematics tablet intervention for low performing second graders. *Journal of Educational Psychology*, *110*(8), 1127–1148. doi: 10.1037/edu0000264

Hedges, L. V., & Olkin, I. (2014). *Statistical methods for meta-analysis*. Academic press.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for research in mathematics education*, 33–46.

Hendel, D. D., & Davis, S. O. (1978). Effectiveness of an intervention strategy for reducing mathematics anxiety. *Journal of Counseling Psychology*, *25*(5), 429.

Hoffman, B. (2010, 06). "i think i can, but i'm afraid to try": The role of self-efficacy beliefs and mathematics anxiety in mathematics problem-solving efficiency. *Learning and Individual Differences - LEARN INDIVID DIFFER*, *20*, 276-283. doi: 10.1016/j.lindif.2010.02.001

Hwa, S. P. (2018). Pedagogical change in mathematics learning: Harnessing the power of digital game-based learning. *Educational Technology & Society*, *21*, 259-276.

Immordino-Yang, M., & Damasio, A. (2007, 03). We feel, therefore we learn: The relevance of affective and social neuroscience to education. *Mind, Brain, and Education*, *1*, 3 - 10. doi: 10.1111/j.1751-228X.2007.00004.x

Jackson, J. B., Paratore, J. R., Chard, D. J., & Garnick, S. (1999). An early intervention supporting the literacy learning of children experiencing substantial difficulty. *Learning Disabilities Research & Practice*, *14*(4), 254–267.

Jitendra, A. K., Lein, A. E., Im, S.-h., Alghamdi, A. A., Hefte, S. B., & Mouanoutoua, J. (2018). Mathematical interventions for secondary students with learning disabilities and mathematics difficulties: A meta-analysis. *Exceptional Children*, *84*(2), 177–196. doi: 10.1177/0014402917737467

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child development*, *74*(3), 834–850.

Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014, 01). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience*, *8*. doi: 10.3389/fnhum.2014.00057

Karagiannakis, G., Baccaglini-Frank, A., & Roussos, P. (2017, 02). Detecting strengths and weaknesses in learning mathematics through a model classifying mathematical skills. *Australian Journal of Learning Difficulties*, *21*, 115-141. doi: 10.1080/19404158.2017.1289963

Käser, T., Baschera, G.-M., Kohn, J., Kucian, K., Richtmann, V., Grond, U., … von Aster, M. (2013a). Design and evaluation of the computer-based training program calcularis for enhancing numerical cognition. *Frontiers in psychology*, *4*, 489. doi: 10.3389/fpsyg.2013.00489

Käser, T., Baschera, G.-M., Kohn, J., Kucian, K., Richtmann, V., Grond, U., … von Aster, M. (2013b). Design and evaluation of the computer-based training program calcularis for enhancing numerical cognition. *Frontiers in psychology*, *4*, 489.

Kaufmann, L., Kucian, K., & von Aster, M. (2014). *Development of the numerical brain* (R. C. Kadosh & A. Dowker, Eds.). Oxford University Press. Retrieved from `https://doi.org/10.1093%2Foxfordhb%2F9780199642342.013.008` doi: 10.1093/oxfordhb/9780199642342.013.008

Kelly, A. E. (2008). Reflections on the national mathematics advisory panel final report. *Educational Researcher*, *37*(9), 561–564.

Kesici, S., & Erdogan, A. (2010, 01). Mathematics anxiety according to middle school students' achievement motivation and social comparison. *Education*, *131*, 54-63.

Kilpatrick, J., & Quinn, H. (2009). Science and mathematics education. education policy white paper. *National Academy of Education (NJ1)*.

Kliman, M. (2006, 01). Math out of school: Families' math game playing at home. , *16*.

Kline, M. (2012). *Mathematics and the physical world*. Dover Publications. Retrieved from `https://books.google.it/books?id=DeSzakR5LKoC`

Kovas, Y., Harlaar, N., Petrill, S., & Plomin, R. (2005, sep). 'generalist genes' and mathematics in 7-year-old twins. *Intelligence*, *33*(5), 473–489. Retrieved from `https://doi.org/10.1016%2Fj.intell.2005.05.002` doi: 10.1016/j.intell.2005.05.002

Kroeger, L. A., Brown, R. D., & O'Brien, B. A. (2012). Connecting neuroscience, cognitive, and educational theories and research to practice: A review of mathematics intervention programs. *Early Education & Development*, *23*(1), 37–58. doi: 10.1080/10409289.2012.617289

Kroesbergen, E. H., & Van Luit, J. E. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education*, *24*(2), 97–114. doi: 10.1177/07419325030240020501

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., … von Aster, M. (2011a). Mental number line training in children with developmental dyscalculia. *NeuroImage*, *57*(3), 782–795. doi: 10.1016/j.neuroimage.2011.01.070

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., … von Aster, M. (2011b). Mental number line training in children with developmental dyscalculia. *NeuroImage*, *57*(3), 782–795.

Kucian, K., Kaufmann, L., & von Aster, M. (2014). *Brain correlates of numerical disabilities* (R. C. Kadosh & A. Dowker, Eds.). Oxford University Press. Retrieved from `https://doi.org/10.1093%2Foxfordhb%2F9780199642342.013.009` doi: 10.1093/oxfordhb/9780199642342.013.009

Kucian, K., & von Aster, M. (2014, dec). Developmental dyscalculia. *European Journal of Pediatrics*, *174*(1), 1–13. Retrieved from `https://doi.org/10.1007%2Fs00431-014-2455-7` doi: 10.1007/s00431-014-2455-7

Kulik, J. A. (1994). Meta-analytic studies of findings on computer-based instruction. In E. L. Baker & H. F. O'Neil (Eds.), *Technology assessment in education and training* (pp. 9–34). Hillsdale, NJ: Lawrence Erlbaum.

Lang, P. J. (1995). The emotion probe: studies of motivation and attention. *American psychologist*, *50*(5), 372.

Lefevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010, 03). Do home numeracy and literacy practices of greek and canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education*, *18*, 55-70. doi: 10.1080/09669761003693926

Leinhardt, G., & Glaser, R. (1993). Advances in instructional psychology.

Lewis, K., & Fisher, M. (2016, 07). Taking stock of 40 years of research on mathematical learning disability: Methodological issues and future directions. *Journal for Research in Mathematics Education*, *47*, 338. doi: 10.5951/jresematheduc.47.4.0338

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review*, *22*(3), 215–243. doi: 10.1007/s10648-010-9125-8

López-Morteo, G., & López, G. (2007). Computer support for learning mathematics: A learning environment based on recreational learning objects. *Computers & Education*, *48*(4), 618–641. doi: 10.1016/j.compedu.2005.04.014

Maloney, E., Ansari, D., & Fugelsang, J. (2011, 01). The effect of mathematics anxiety on the processing of numerical magnitude. *Quarterly journal of experimental psychology (2006)*, *64*, 10-6. doi: 10.1080/17470218.2010.533278

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational effects of parents' math anxiety on children's math achievement and anxiety. *Psychological Science*, *26*(9), 1480–1488.

Mancini, G. (2011). *L'intelligenza emotiva in età evolutiva* (Doctoral dissertation, Alma Mater Studiorum Bologna). doi: 10.6092/unibo/amsdottorato/3412

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of teacher education*, *41*(3), 3–11.

Mark W, S., Frederick G, F., Peter J, M., Raja, P., Francesco Di, N., & Lawrence J Prinzel, I. (2001). The efficacy of psychophysiological measures for implementing adaptive technology.

Mason, J. (2001). Teaching for flexibility in mathematics: Being aware of the structures of attention and intention. *Quaestiones Mathematicae*, *24*(Suppl 1), 1–15.

Mason, J. (2006). The intelligence-led enterprise: Taking the first 'baby steps'. *Journal of Medical Marketing*, *6*(2), 140–145.

Mason, L., Boldrin, A., & Zurlo, G. (2006). Epistemological understanding in different judgment domains: Relationships with gender, grade level, and curriculum. *International Journal of Educational Research*, *45*(1-2), 43–56.

Mastropieri, M. A., Scruggs, T. E., & Shiah, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research &*

Practice, *6*(2), 89–98. Retrieved from `https://psycnet.apa.org/record/1994-19528-001`

Mazzocco, M., & Räsänen, P. (2013, 06). Contributions of longitudinal studies to evolving definitions and knowledge of developmental dyscalculia. *Trends in Neuroscience and Education*, *2*, 65-73. doi: 10.1016/j.tine.2013.05.001

Mazzocco, M. M., & Myers, G. F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of dyslexia*, *53*(1), 218–253.

Mercenaro, S. (2006). *La mente emotiva*. Carocci. Retrieved from `https://books.google.it/books?id=ZjbIAAAACAAJ`

Miller, S. P., Butler, F. M., & Kit-hung, L. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of literature. *Focus on Exceptional Children*, *31*(1), 1–24. Retrieved from `https://search.proquest.com/openview/ccbba8a7e4230ed732a58064e6fea3bf/1?cbl=48488&pq-origsite=gscholar`

Mitchell, A., & Savill-Smith, C. (2004). *The use of computer and video games for learning: a review of the literature*. Learning and Skills Development Agency, London. (OCLC: 62244824)

Miyake, A., & Shah, P. (Eds.). (1999). *Models of working memory*. Cambridge University Press. Retrieved from `https://doi.org/10.1017%2Fcbo9781139174909` doi: 10.1017/cbo9781139174909

Mohd Syah, N. E., Hamzaid, N. A., Murphy, B. P., & Lim, E. (2016). Development of computer play pedagogy intervention for children with low conceptual understanding in basic mathematics operation using the dyscalculia feature approach. *Interactive Learning Environments*, *24*(7), 1477–1496. doi: 10.1080/10494820.2015.1023205

Moher, D., Liberati, A., Tetzlaff, J., & Altman, D. G. (2009). Preferred reporting items for systematic reviews and meta-analyses: The PRISMA statement. *Annals of Internal Medicine*, *151*(4), 264–269. doi: 10.7326/0003-4819-151-4-200908180-00135

Monks, F., & Ferguson, T. (1983, 02). Gifted adolescents: An analysis of their psychosocial development. *Journal of youth and adolescence*, *12*, 1-18. doi: 10.1007/BF02092110

Morris, D., Tyner, B., & Perney, J. (2000). Early steps: Replicating the effects of a first-grade reading intervention program. *Journal of Educational Psychology*, *92*(4), 681.

Morris, S. B. (2008). Estimating effect sizes from pretest-posttest-control group designs. *Organizational Research Methods*, *11*(2), 364–386. doi: 10.1177/1094428106291059

Morton, A., Rae, J., Pentleton, S., Stuart, R., Murdoch, R., MacAdam, R., & Boyle, J. (1988). Educational computing in schools: The report of a working party. *Glasgow, UK: Strathclyde Regional Council Department of Education*.

Naranjo, C., Cheli, E., & Callegari, A. (2011). *Amore, coscienza e psicoterapia. verso una nuova educazione dell'essere umano*. Xenia. Retrieved from `https://books.google.it/books?id=Ch6kuAAACAAJ`

Nelson, G., & Powell, S. R. (2018). A systematic review of longitudinal studies of mathematics difficulty. *Journal of Learning Disabilities*, *51*(6), 523–539. doi: 10.1177/0022219417714773

Nelson, P. M., Burns, M. K., Kanive, R., & Ysseldyke, J. E. (2013). Comparison of a math fact rehearsal and a mnemonic strategy approach for improving math fact fluency. *Journal of School Psychology*, *51*(6), 659–667. doi: 10.1016/j.jsp.2013.08.003

Netz, R. (2003). *The shaping of deduction in greek mathematics: A study in cognitive history*. Cambridge University Press. Retrieved from `https://books.google.it/books?id=VwggGX0ORLkC`

Nunes, T., Schliemann, A., & Carraher, D. (1993, 01). Street mathematics and school mathematics.

OECD. (2016). *Pisa 2015 results (volume i): Excellence and equity in education*. PISA: OECD Publishing, Paris. doi: 10.1787/9789264266490-en

Outhwaite, L. A., Faulder, M., Gulliford, A., & Pitchford, N. J. (2019). Raising early achievement in math with interactive apps: A randomized control trial. *Journal of Educational Psychology*, *111*(2), 284–298. doi: 10.1037/edu0000286

Pellerey, M. (2006). *Dirigere il proprio apprendimento: autodeterminazione e autoregolazione nei processi di apprendimento*. La scuola.

Peltenburg, M., Van Den Heuvel-Panhuizen, M., & Doig, B. (2009). Mathematical power of special-needs pupils: An ict-based dynamic assessment format to reveal weak pupils' learning potential 1. *British Journal of Educational Technology*, *40*(2), 273–284.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. *Second handbook of research on mathematics teaching and learning*, *1*, 257–315.

Phillipson, S., & Phillipson, S. (2007, 06). Academic expectations, belief of ability, and involvement by parents as predictors of child achievement: A cross-cultural comparison. *Educational Psychology*, *27*, 329-348. doi: 10.1080/01443410601104130

Piaget, J., & Campbell, R. (2001). *Studies in reflecting abstraction*. Psychology Press. Retrieved from `https://books.google.it/books?id=HrZvQgAACAAJ`

Piazza, M., Facoetti, A., Trussardi, A., Berteletti, I., Conte, S., Lucangeli, D., ... Zorzi, M. (2010, 04). Developmental trajectoy of number acuity revels a severe impairment in developmental dyscalculia. *Cognition*, *116*, 33-41. doi: 10.1016/j.cognition.2010.03.012

Pisa assessing scientific, reading and mathematical literacy a framework for pisa 2006: A framework for pisa 2006. (2006). OECD Publishing. Retrieved from https://books.google.it/books?id=37HVAgAAQBAJ

Prendinger, H., Mori, J., & Ishizuka, M. (2005). Using human physiology to evaluate subtle expressivity of a virtual quizmaster in a mathematical game. *Int. J. Hum.-Comput. Stud.*, *62*, 231-245.

Prensky, M. (2003, 10). Digital game-based learning. *Computers in Entertainment (CIE) - Theoretical and Practical Computer Applications in Entertainment*, *1*. doi: 10.1145/950566.950596

R Core Team. (2019). R: A language and environment for statistical computing [Computer software manual]. Vienna, Austria. Retrieved from https://www.R-project.org/

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development*, *14*(2), 187–202.

Räsänen, P., Laurillard, D., Käser, T., & von Aster, M. (2019). Perspectives to technology-enhanced learning and teaching in mathematical learning difficulties. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the classroom* (pp. 733–754). Springer International Publishing. doi: 10.1007/978-3-319-97148-3_42

Räsänen, P., Salminen, J., Wilson, A. J., Aunio, P., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. *Cognitive Development*, *24*(4), 450–472. doi: 10.1016/j.cogdev.2009.09.003

Raths, J. (2001). Teachers' beliefs and teaching beliefs.

Re, A. M., Benavides-Varela, S., Pedron, M., & Lucangeli, D. (2019-submitted). Web app - i bambini contano.

Reis, S. M., & McCoach, D. B. (2000, jul). The underachievement of gifted students: What do we know and where do we go? *Gifted Child Quarterly*, *44*(3), 152–170. Retrieved from `https://doi.org/10.1177%2F001698620004400302` doi: 10.1177/001698620004400302

Ricci, K. E. (1994). The use of computer-based videogames in knowledge acquisition and retention. *Journal of Interactive Instruction Development*, *7*(1), 17–22.

Ritchie, D., & Dodge, B. (1992). Integrating technology usage across the curriculum through educational adventure games. In *the annual conference on technology and teacher education, 12-15 march 1992.* Houston, TX. Retrieved 2019-05-31, from `https://eric.ed.gov/?id=ED349955`

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. doi: 10.1177/0956797612466268

Rivera-Batiz, F. L. (1992). Quantitative literacy and the likelihood of employment among young adults in the united states. *The Journal of Human Resources*, *27*(2), 313–328. doi: 10.2307/145737

Rosas, R., Nussbaum, M., Cumsille, P., Marianov, V., Correa, M., Flores, P., ... Salinas, M. (2003). Beyond Nintendo: design and assessment of educational video games for first and second grade students. *Computers & Education*, *40*(1), 71–94. doi: 10.1016/S0360-1315(02)00099-4

Rothstein, H. R., Sutton, A. J., & Borenstein, M. (2005). *Publication bias in meta-analysis: prevention, assessment and adjustments*. John Wiley & Sons, Ltd. doi: 10.1002/0470870168

Rubinsten, O., & Tannock, R. (2010, 01). Mathematics anxiety in children with developmental dyscalculia. *Behavioral and Brain Functions*, 1-13.

Ryan, A. M., & Pintrich, P. R. (1997). " should i ask for help?" the role of motivation and attitudes in adolescents' help seeking in math class. *Journal of educational psychology*, *89*(2), 329.

Salminen, J., Koponen, T., Räsänen, P., & Aro, M. (2015). Preventive support for kindergarteners most at-risk for mathematics difficulties: Computer-assisted intervention. *Mathematical Thinking and Learning*, *17*(4), 273–295. doi: 10.1080/10986065.2015.1083837

Schneider, W., Roth, E., & Ennemoser, M. (2000). Training phonological skills and letter knowledge in children at risk for dyslexia: A comparison of three kindergarten intervention programs. *Journal of Educational Psychology*, *92*(2), 284.

Schütte, M. (2014). Language-related learning of mathematics: a comparison of kindergarten and primary school as places of learning. *ZDM*, *46*(6), 923–938.

Scriven, M. (1967). The methodology of evaluation. in. tyler, rw, gagne, rm, scriven, m.(ed.): Perspectives of curriculum evaluation. *'^'eds.'): Book The Methodology of Evaluation. In. Tyler, Rw, Gagne, Rm, Scriven, M.(Ed.): Perspectives of Curriculum Evaluation, Rand McNally, Chicago*.

Seo, Y.-J., & Bryant, D. P. (2009). Analysis of studies of the effects of computer-assisted instruction on the mathematics performance of students with learning disabilities. *Computers & Education*, *53*(3), 913–928. doi: 10.1016/j.compedu.2009.05.002

Seo, Y.-J., & Woo, H. (2010). The identification, implementation, and evaluation of critical user interface design features of computer-assisted instruction programs in mathematics for students with learning disabilities. *Computers & Education*, *55*(1), 363–377. doi: 10.1016/j.compedu.2010.02.002

Shalev, R. S., Auerbach, J., Manor, O., & Gross-Tsur, V. (2000). Developmental dyscalculia: prevalence and prognosis. *European Child & Adolescent Psychiatry*, *9*(2), S58–S64. doi: 10.1007/s007870070009

Shirvani, H. (2010). The effects of using computer technology with lower-performing students: Technology and student mathematics achievement. *International Journal of Learning*, *17*(1).

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review*, *57*(1), 1–23.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4–14.

Sorrentino, C., & Pinnelli, S. (2017, mar). UNIVERSAL DESIGN FOR LEARNING APPROACH AND SCHOOLWIDE ENRICHMENT MODEL: A CRITICAL PERSPECTIVE. In *EDULEARN17 proceedings.* IATED. Retrieved from `https://doi.org/10.21125%2Fedulearn.2017.2219` doi: 10.21125/edulearn.2017.2219

Spence, I., & Feng, J. (2010, jun). Video games and spatial cognition. *Review of General Psychology*, *14*(2), 92–104. Retrieved from `https://doi.org/10.1037%2Fa0019491` doi: 10.1037/a0019491

Stanmore, E., Stubbs, B., Vancampfort, D., de Bruin, E. D., & Firth, J. (2017). The effect of active video games on cognitive functioning in clinical and non-clinical populations: A meta-analysis of randomized controlled trials. *Neuroscience & Biobehavioral Reviews*, *78*, 34–43. doi: 10.1016/j.neubiorev.2017.04.011

Stultz, S. L. (2013). The effectiveness of computer-assisted instruction for teaching mathematics to students with specific learning disability. *The Journal of Special Education Apprenticeship*, *2*(2), Article 7. Retrieved from `https://scholarworks.lib.csusb.edu/josea/vol2/iss2/7`

Suinn, R. M. (1972). *Mathematics anxiety rating scale*. Rocky Mountain Behavioural Sciences Institute.

Szücs, D. (2016). Chapter 11 - subtypes and comorbidity in mathematical learning disabilities: Multidimensional study of verbal and visual memory processes is key to understanding. In M. Cappelletti & W. Fias (Eds.), *The mathematical brain across the lifespan* (Vol. 227, p. 277 - 304). Elsevier. Retrieved from `http://www.sciencedirect.com/science/article/pii/S007961231630053X` doi: https://doi.org/10.1016/bs.pbr.2016.04.027

Taylor, L., Adelman, H. S., & Kaser-Boyd, N. (1983). Perspectives of children regarding their participation in psychoeducational decisions. *Professional Psychology: Research and Practice*, *14*(6), 882–894. Retrieved from `https://doi.org/10.1037%2F0735-7028.14.6.882` doi: 10.1037/0735-7028.14.6.882

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational studies in mathematics*, *15*(2), 105–127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research.

Tirosh, D. (2009). What do we know about mathematics teacher education? what evidence do we have? what comes next? *Journal of mathematics teacher education*, *12*(2), 83–87.

Tornar, C. (2001). *Il processo didattico tra organizzazione e controllo*. Monolite Editrice.

Vadasy, P. F., Jenkins, J. R., & Pool, K. (2000). Effects of tutoring in phonological and early reading skills on students at risk for reading disabilities. *Journal of Learning Disabilities*, *33*(6), 579–590.

Valle-Lisboa, J., Cabana, Á., Eisinger, R., Mailhos, Á., Luzardo, M., Halberda, J., & Maiche, A. (2016). Cognitive abilities that mediate SES's effect on elementary mathematics learning: The uruguayan tablet-based intervention. *PROSPECTS*, *46*(2), 301–315. doi: 10.1007/s11125-017-9392-y

Vance, W. R., & Watson, T. S. (1994). Comparing anxiety management training and systematic rational restructuring for reducing mathematics anxiety in college students. *Journal of College Student Development*.

Van De Rijt, B. A. M., & Van Luit, J. E. H. (1998). Effectiveness of the additional early mathematics program for teaching children early mathematics. *Instructional Science*, *26*(5), 337–358. doi: 10.1023/A:1003180411209

Viechtbauer, W. (2010). Conducting meta-analyses in *R* with the `metafor` package. *Journal of Statistical Software*, *36*(3). doi: 10.18637/jss.v036.i03

Viechtbauer, W., & Cheung, M. W.-L. (2010). Outlier and influence diagnostics for meta-analysis. *Research Synthesis Methods*, *1*(2), 112–125. doi: 10.1002/jrsm.11

Voida, A., & Greenberg, S. (2009, 04). Wii all play: The console game as a computational meeting place. In (p. 1559-1568). doi: 10.1145/1518701.1518940

Vygotsky, L., Rieber, R., Hall, M., Carton, A., & Glick, J. (1997). *The collected works of l. s. vygotsky: The history of the development of higher mental functions*. Plenum. Retrieved from `https://books.google.it/books?id=Wvy8rt0kzf0C`

Watson, S. M. R., & Gable, R. A. (2013). Cognitive development of adolescents at risk or with learning and/or emotional problems implications for teachers..

Weber, R., Ritterfeld, U., & Mathiak, K. (2006). Does playing violent video games induce aggression? empirical evidence of a functional magnetic resonance imaging study. *Media psychology*, *8*(1), 39–60.

Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of educational Psychology*, *80*(2), 210.

Wilson, A. J., Dehaene, S., Dubois, O., & Fayol, M. (2009). Effects of an adaptive game intervention on accessing number sense in low-socioeconomic-status kindergarten children. *Mind, Brain, and Education*, *3*(4), 224–234. doi: 10.1111/j.1751-228X.2009.01075.x

Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of "the number race", an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions*, *2*(1), 20. doi: 10.1186/1744-9081-2-20

Wilson, B., & Cole, P. (1991). A review of cognitive teaching models. *Educational Technology Research and Development*, *39*(4), 47–64.

Wolfle, J. A. (1986). Enriching the mathematics program for middle school gifted students. *Roeper Review*, *9*(2), 81–85.

Woodward, J., & Carnine, D. (1993). Uses of technology for mathematics assessment and instruction: Reflection on a decade of innovations. *Journal of Special Education Technology*, *12*(1), 38–48. doi: 10.1177/016264349301200103

Xolocotzin Eligio, U. (2017). *Understanding emotions in mathematical thinking and learning*. doi: 10.1016/B978-0-12-802218-4.10000-7

Yuan, B., Folmer, E., & Harris, F. C. (2011). Game accessibility: a survey. *Universal Access in the Information Society*, *10*(1), 81–100.

Zan, R. (2006). *Difficoltà in matematica: Osservare, interpretare, intervenire*. Springer Milan. Retrieved from `https://doi.org/10.1007%2F978-88-470-0584-6` doi: 10.1007/978-88-470-0584-6

Ringraziamenti

Vorrei esprimere la mia gratitudine a tutti coloro che mi hanno aiutato in questo percorso.
Alla prof.ssa Michela Prest perché so sempre dove trovarla, senza di lei non avrei neanche iniziato.
Alla prof.ssa Daniela Lucangeli per avermi offerto la possibilità di collaborare con una professionista di altissimo livello e di lavorare su tematiche di grande interesse.
Al prof. Stefano Serra Capizzano per la sua guida, per la sua fiducia e per la grande libertà che mi ha lasciato per affrontare questo cammino.
Alla prof.ssa Silvia Benavides Varela per avermi permesso di vedere le cose da prospettive diverse, per avermi aiutato a scrivere e, a volte, anche a cancellare.
Al prof. Cristiano Termine per la sua professionalità e per avermi coinvolto in un progetto di ricerca multicentrico davvero innovativo.
Al prof. Erik Vallazza per il suo tempo e il suo supporto.
Al dott. Andrea Ciceri, presidente della start-up Sense Catch srl, per il suo progetto ambizioso e straordinario di esplorare la dimensione inconsapevole degli esseri umani.
Ringrazio inoltre gli insegnanti e i dirigenti scolastici che hanno coordinato il lavoro negli istituti in cui abbiamo sperimentato il software MathPro.
Un ringraziamento speciale ai bambini che ho avuto la fortuna di incontrare, per tutto quello che mi hanno insegnato.