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On the product property of the Carathéodory pseudodistance

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Analisi matematica. — *On the product property of the Carathéodory pseudodistance.*
Nota (*) di José M. Isidro e Jean-Pierre Vigué, presentata dal Socio E. Vesentini.

Abstract. — We prove that, for certain domains $\mathbb{D}$, continuous product of domains D_ω, the Carathéodory pseudodistance satisfies the following product property

$$C_{\mathbb{D}}(f, g) = \sup_\omega C_{D_\omega}(f(\omega), g(\omega)).$$

Key words: Carathéodory pseudodistance; Product domains; Product property.

Riassunto. — *Proprietà del prodotto della pseudodistanza di Carathéodory.* Si prova che per alcuni domini $\mathbb{D}$, che sono prodotti continui di domini D_ω, la pseudodistanza di Carathéodory soddisfa la seguente proprietà:

$$C_{\mathbb{D}}(f, g) = \sup_\omega C_{D_\omega}(f(\omega), g(\omega)).$$

1. Introduction

Let Ω and E respectively be a completely regular topological space and a complex Banach space with open unit ball $B(0)$. Let $\mathbb{E} := \mathcal{C}_b(\Omega, E)$ be the Banach space of all continuous bounded E-valued functions $f: \Omega \to E$, endowed with the pointwise operations and the norm of the supremun. Whenever E is a complex Banach space and $D \subset E$ is a domain, we let C_D denote the Carathéodory distance in D.

Recall [5, Definition 1.5] a domain $\mathbb{D} \subset \mathcal{C}(\Omega, E)$ is the continuous Ω-product of the family $(D_\omega)_{\omega\in\Omega}$ of bounded domains in E if the following two conditions hold : $\mathbb{D}$ is the interior of

$$\{f \in \mathcal{C}(\Omega, E): f(\omega) \in D_\omega,\ (\omega \in \Omega)\}, \qquad D_\omega = \{f(\omega): f \in \mathbb{D}\}, \qquad (\omega \in \Omega).$$

In that case $\mathbb{D}$ consists of continuous sections of $\mathbb{D}_\star := \{(\omega, x) \in \Omega \times E: \omega \in \Omega, x \in D_\omega\}$ with respect to the fibration $p: \mathbb{D}_\star \to \Omega$ given by $(\omega, x) \mapsto \omega$. Let Ω, E and $\|\cdot\|_\omega$, $(\omega \in \Omega)$, respectively be a compact topological space, a complex Banach space and a family of norms in E with open unit balls D_ω, and let $\mathbb{D} := \{f \in \mathcal{C}(\Omega, E): f(\omega) \in D_\omega,\ (\omega \in \Omega)\}$ be a bounded domain in $\mathcal{C}(\Omega, E)$. Then $\mathbb{D}$ is the continuous Ω-product of the family $(D_\omega)_{\omega\in\Omega}$ if and only if there are constants $0 < m \le M < \infty$ such that $m\|\cdot\| \le \|\cdot\|_\omega \le M\|\cdot\|$ for all $\omega \in \Omega$ and the function $N(\omega, x) := \|x\|_\omega$ is upper semicontinuous on $\Omega \times E$.

Definition 1.1. Let $(D_\omega)_{\omega\in\Omega}$ be a family of domains $D_\omega \subset E$ whose Ω-product $\mathbb{D}$ is domain in $\mathcal{C}_b(\Omega, E)$.

(1) We say that the *continuous product property* (the CPP for short) holds for $\mathbb{D}$ if

(*) Pervenuta in forma definitiva all'Accademia il 6 ottobre 1999.

the Carathéodory distance $C_{\mathbb{D}}$ satisfies

$$C_{\mathbb{D}}(f, g) = \sup_{\omega\in\Omega} C_{D_\omega}[f(\omega), g(\omega)], \qquad f, g \in \mathbb{D}. \tag{1}$$

(2) We say that the CPP holds for the space $\mathcal{C}_b(\Omega, E)$ if whenever $(D_\omega)_{\omega\in\Omega}$ is a family whose Ω-product $\mathbb{D}$ is a domain in $\mathcal{C}_b(\Omega, E)$, the CPP holds for $\mathbb{D}$.

In general no information is available about how D_ω depends on $\omega \in \Omega$. If all domains D_ω coincide (say with $D \subset E$) then $s \mapsto C_D[f(\omega), g(\omega)]$ is continuous, hence the supremun in (1) is attainable whenever Ω is compact. In the general case, the evaluation $e_\omega: \mathcal{C}_b(\Omega, E) \to E$ is a holomorphic map, hence it is a contraction for the Carathéodory distances and so

$$\sup_{\omega\in\Omega} C_{D_\omega}[f(\omega), g(\omega)] \le C_{\mathbb{D}}(f, g), \tag{2}$$

holds, hence the CPP for the domain $\mathbb{D}$ is equivalent to

$$C_{\mathbb{D}}(f, g) \le \sup_{\omega\in\Omega} C_{D_\omega}[f(\omega), g(\omega)], \qquad f, g \in \mathbb{D}. \tag{3}$$

In [3], Jarnicki and Pflug have proved that (3) holds whenever Ω is finite and E is finite dimensional. The general case seems to be very difficult, and we prove this property in the following cases:

(*a*) Ω is a finite set and E is a Banach space.

(*b*) $\mathbb{D}$ is contained in a space of sequences converging to zero at infinity.

(*c*) Ω is an infinite set with the discrete topology and we consider an infinite product of copies of the same domain $D \subset \mathbb{C}^n$, with an additional hypothesis on D.

2. Finite products in complex Banach spaces

We get the following result

Proposition 2.1. *Let A and B be domains in the Banach spaces E and F respectively. Then*

$$C_{A\times B}[(a, b), (a', b')] = \max\{C_A(a, a'), C_B(b, b')\}$$

holds for all pairs $a, a' \in A$ and $b, b' \in B$.

Proof. For a domain D in a complex Banach space X and a pair of points $x, x' \in D$, we let $\mathcal{F}(X, x, x')$ denote the family of all vector subspaces $Z \subset X$ such that $\dim Z < \infty$ and $x, x' \in Z$. By [1, Th. 2.1] we have

$$C_D(x, x') = \inf_{Z\in\mathcal{F}(X,x,x')} C_{D\cap Z}(x, x'). \tag{4}$$

Let $\epsilon > 0$ be given. By (4) there are subspaces $X \in \mathcal{F}(E, a, a')$ and $Y \in \mathcal{F}(F, b, b')$ such that

$$C_A(a, a') + \epsilon > C_{A\cap X}(a, a'), \qquad C_B(b, b') + \epsilon > C_{B\cap Y}(b, b').$$

Obviously we have $\mathcal{F}(E\times F,(a,b),(a',b'))\supset\mathcal{F}(E,a,a')\times\mathcal{F}(F,b,b')$. Therefore by [3, Th. 1.1]

$$\begin{aligned}C_{A\times B}[(a,b),(a',b')]&\le\inf_{X,Y}C_{(A\cap X)\times(B\cap Y)}[(a,b),(a',b')]=\\&=\inf_{X,Y}\max\{C_{A\cap X}(a,a'),C_{B\cap Y}(b,b')\}\le\\&\le\max\{C_A(a,a')+\epsilon,C_B(b,b')+\epsilon\}=\max\{C_A(a,a'),C_B(b,b')\}+\epsilon.\end{aligned}$$

Since this is valid for all $\epsilon>0$, the result follows from (3). □

3. Space of continuous sections converging to zero at infinity

Let Ω be a a locally compact space and let $\mathcal{C}_0(\Omega,E)$ be the Banach space of continuous maps $f:\Omega\to E$ converging to 0 at infinity. First, we prove the following proposition

Proposition 3.1. *Let Ω, E and D respectively be a locally compact space, a complex Banach space and a domain $D\subset E$ such that $0\in D$. Let $\mathcal{K}$ be a compact set in $\mathcal{C}_0(\Omega,E)$. Then*

$$\lim_{\omega\to\infty}C_D[f(\omega),g(\omega)]=0$$

holds uniformly for $f,g\in\mathcal{K}$. In particular, for $f,g\in\mathcal{C}_0(\Omega,E)$ with $f(\Omega),g(\Omega)\subset D$, the function $d:\omega\mapsto C_D[f(\omega),g(\omega)]$ satisfies $d\in\mathcal{C}_0(\Omega,\mathbb{R})$.

Proof. Let $f,g\in\mathcal{C}_0(\Omega,E)$ satisfy $f(\Omega),g(\Omega)\subset D$. The evaluations and the Carathéodory distance are continuous functions, hence so is $d:\omega\mapsto C_D[f(\omega),g(\omega)]$. Thus we only have to prove that

$$\lim_{\omega\to\infty}C_D[f(\omega),g(\omega)]=0$$

holds uniformly for $f,g\in\mathcal{K}$. Let $\epsilon>0$ be given. For a suitable $\rho>0$, the ball $B_\rho(0):=\{x\in E:\|x\|<\rho\}$ clearly satisfies $B_\rho(0)\subset\subset D$, hence by [2, Th. IV.2.] there is a constant M such that

$$C_D(z,w)\le M\|z-w\|,\qquad z,w\in B_\rho(0).$$

Let $\epsilon':=\min\{\frac{1}{2},\frac{\epsilon}{2M}\}$. Since $\mathcal{K}$ is a compact subset $\mathcal{C}_0(\Omega,E)$, there is a compact set $S\subset\Omega$ such that $\|h(\omega)\|\le\epsilon'$ for all $\omega\in\Omega\backslash S$ and all $h\in\mathcal{K}$. Therefore

$$C_D[f(\omega),g(\omega)]\le M\|f(\omega)-g(\omega)\|\le M\big(\|f(\omega)\|+\|g(\omega)\|\big)\le 2M\epsilon'=\epsilon,\qquad s\in\Omega\backslash S$$

which completes the proof. □

For every compact subset $K\subset\Omega$ we let $\mathcal{S}(K,E)$ denote the (possibly non closed) normed subspace of $\mathcal{C}_0(\Omega,E)$ consisting of the functions f such that $\mathrm{supp}(f)\subset K^\circ$.

Proposition 3.2. *Let Ω, E and D respectively be a locally compact σ-compact topological space, a complex Banach space and a star-like domain $D\subset E$. Let $\mathbb{D}_0\subset\mathcal{C}_0(\Omega,E)$ denote the $c_0(\Omega)$-power of D. If the CPP holds in $\mathcal{S}(K,E)$ for every compact set $K\subset\Omega$, then the CPP holds for $\mathbb{D}_0$.*

PROOF. For every compact subset $K \subset \Omega$ we define $\mathbb{D}(K, E)$ by

$$\mathbb{D}(K, E) := \mathbb{D}_0 \cap \mathcal{S}(K, E).$$

Clearly $\mathbb{D}(K, E)$ is a domain in $\mathcal{S}(K, E)$ since it an open star-like (hence connected) subset of $\mathcal{S}(K, E)$. Also if K and L are compact subsets of Ω such that $K \subset L^\circ$, then we have the inclusions

$$\mathbb{D}(K, E) \hookrightarrow \mathbb{D}(L, E) \hookrightarrow \mathbb{D}_0.$$

LEMMA 3.3. *There are a sequence $(S_n)_{n\in\mathbb{N}}$ of compact subsets of Ω such that $S_n \subset S_{n+1}^\circ$ for all $n \in \mathbb{N}$ and $\Omega = \cup_{n\in\mathbb{N}} S_n$ and a sequence of functions $(\varphi_n)_{n\in\mathbb{N}}$ in $\mathcal{C}_0(\Omega, E)$ such that $\varphi_n|_{S_n} \equiv 1$ and $\varphi_n|_{\Omega\setminus S_{n+1}^\circ} \equiv 0$ such that the following statement holds: For every $h \in \mathcal{C}_0(\Omega, E)$ we have $h = \lim_{n\to\infty} h\varphi_n$ in the space $\mathcal{C}_0(\Omega, E)$.*

PROOF. Combining the σ-compactness of Ω and Urysohn's lemma we can easily construct sequences $(S_n)_{n\in\mathbb{N}}$ and $(\varphi_n)_{n\in\mathbb{N}}$ meeting the properties required in the first sentence of the lemma.

Let $h \in \mathcal{C}_0(\Omega, E)$ and $\epsilon > 0$ be given. Then there is a compact set $K \subset \Omega$ such that $\sup_{s\in\Omega\setminus K} \|h(\omega)\| \leq \epsilon$, and for $n \in \mathbb{N}$ large enough we have $K \subset S_n$. Therefore

$$\|h - h\varphi_n\| = \sup_{\omega\in\Omega\setminus S_n} \|h(\omega) - h(\omega)\varphi_n(\omega)\| \leq \sup_{\omega\in\Omega\setminus K} \|h(1-\varphi_n)\| \leq 2\epsilon$$

which shows that $\lim_{n\to\infty} h\varphi_n = h$ in the space $\mathcal{C}_0(\Omega, E)$. □

Now we prove the proposition. Take sequences $(S_n)_{n\in\mathbb{N}}$ and $(\varphi_n)_{n\in\mathbb{N}}$ in accordance with (3.3). Note that the products $f\varphi_n$, $g\varphi_n$ belong to $\mathbb{D}_0$ due to the star-likeness. Since the Carathéodory distance in $\mathbb{D}_0$ is continuous, we have

$$C_{\mathbb{D}_0}(f, g) = \lim_{n\to\infty} C_{\mathbb{D}_0}(f\varphi_n, g\varphi_n), \qquad f, g \in \mathbb{D}_0. \tag{5}$$

To simplify the notation, write $\mathbb{D}_n$ instead of $\mathbb{D}(S_n, E)$. Consider the maps $\mathbb{D}_0 \xrightarrow{\varphi_n} \mathbb{D}_{n+1} \xrightarrow{i} \mathbb{D}_0$, where the arrows are the operator of multiplication by φ_n and the canonical inclusion respectively. Note that $\mathrm{supp}(\varphi_n h) \subset S_{n+1}$ so that $\varphi_n h \in \mathbb{D}_{n+1}$ for all $h \in \mathbb{D}_0$. By the contractive property of $\mathbb{D}_{n+1} \xrightarrow{i} \mathbb{D}_0$

$$C_{\mathbb{D}_0}(f\varphi_n, g\varphi_n) \leq C_{\mathbb{D}_{n+1}}(f\varphi_n, g\varphi_n), \qquad n \in \mathbb{N}. \tag{6}$$

By taking upper limits and using (5) we get $C_{\mathbb{D}_0}(f, g) \leq \limsup_{n\in\mathbb{N}} C_{\mathbb{D}_{n+1}}(f\varphi_n, g\varphi_n)$. We shall prove that $\limsup_{n\in\mathbb{N}} C_{\mathbb{D}_{n+1}}(f\varphi_n, g\varphi_n) \leq \sup_{\omega\in\Omega} C_D[f(\omega), g(\omega)]$ from which the result follows. By assumption the CCP holds for every $\mathcal{S}(K, E)$. Hence for every fixed $n \in \mathbb{N}$ we have

$$\begin{aligned} C_{\mathbb{D}_{n+1}}(f\varphi_n, g\varphi_n) &= \max_{\omega\in\Omega} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)] = \max_{\omega\in S_{n+1}} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)] = \\ &= \max\Big[\sup_{\omega\in S_n} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)], \sup_{\omega\in S_{n+1}\setminus S_n} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)]\Big]. \end{aligned} \tag{7}$$

For $\omega \in S_n$ we have $\varphi_n(\omega) = 1$, therefore

$$\sup_{\omega\in S_n} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)] = \sup_{\omega\in S_n} C_D[f(\omega), g(\omega)] \leq \sup_{\omega\in\Omega} C_D[f(\omega), g(\omega)], \quad n \in \mathbb{N}.$$

On the other hand, the set $\mathcal{K} := \{h\} \cup \{h\varphi_n : n \in \mathbb{N}\}$ is compact for every $h \in \mathcal{C}_0(\Omega, E)$, hence (3.1) applies. Let $\epsilon > 0$ be given. There is a compact subset $K \subset \Omega$ such that

$$C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)] < \epsilon, \qquad \omega \in \Omega \backslash K, \ n \in \mathbb{N}.$$

For n large enough (say $n \geq n_0$)) we have $K \subset S_n \subset S_{n+1} \subset \Omega$, therefore $S_{n+1} \backslash S_n \subset \Omega \backslash K$ and so

$$\sup_{\omega \in S_{n+1} \backslash S_n} C_D[(f\varphi_n)(\omega), (g\varphi_n)(\omega)] \leq \epsilon, \qquad n \geq n_0.$$

Replacing this in (7) we get $C_{D_{n+1}}(f\varphi_n, g\varphi_n) \leq \max\{\sup_{\omega\in\Omega} C_D[f(\omega), g(\omega)], \epsilon.\}$ for $n \geq n_0$. Since ϵ was arbitrary, $\limsup C_{D_{n+1}}(f\varphi_n, g\varphi_n) \leq \sup_{\omega\in\Omega} C_D[f(\omega), g(\omega)]$ which completes the proof. □

Example 3.4. Take $\Omega := \mathbb{N}$ with the discrete topology, and let D be a balanced domain in E. Then $\mathbb{D}_0$, the $c_0(\mathbb{N})$-power of D, is a balanced domain in $c_0(\mathbb{N}, E)$ and it is easy to see that the assumptions in (3.2) are satisfied. Hence the CPP holds for $C_{\mathbb{D}}$.

4. Infinite product of a finite dimensional domain

Let I and E respectively be a set of indices and a normed space. As usually, we let $\ell^\infty(I, E)$ be the vector space of all bounded sequences $(x_\iota)_{\iota\in I}$ with the supremun norm $\|(x_\iota)_{\iota\in I}\| := \sup_{\iota\in I} \|x_\iota\|$. In this case, we can prove the following theorem

Theorem 4.1. *Let E be a* finite dimensional *vector space with a norm. Let D be a bounded domain in E such that, for every $r \geq 0$ and for every $a \in D$, the ball $B_C(a, r)$ for the Carathéodory distance is relatively compact in D. Let $\mathbb{D} := \Pi_{\iota\in I} D_\iota$ where D_ι is a copy of D. More precisely,*

$$\mathbb{D} := \{(x_\iota)_{\iota\in I} : x_\iota \in D \ \textit{and} \ \exists\, \eta > 0 \ \textit{such that} \ \forall\, \iota \in I \quad d(x_\iota, \partial D) > 0\}.$$

Then

$$C_{\mathbb{D}}\big((x_\iota)_{\iota\in I}, (y_\iota)_{\iota\in I}\big) = \sup_{\iota\in I} C_D(x_\iota, y_\iota).$$

Proof. The inequality $\geq$ is trivial. Let $\epsilon > 0$. We have to prove that

$$C_{\mathbb{D}}\big((x_\iota)_{\iota\in I}, (y_\iota)_{\iota\in I}\big) \leq \sup_{\iota\in I} C_D(x_\iota, y_\iota) + \epsilon.$$

First we get the following lemma

Lema 4.2. *Let* $\mathbf{a} := (a, \ldots, a, \ldots)$ *and* $\mathbf{b} := (b, \ldots, b, \ldots)$ *be constant sequences equal to* a (resp. *to* b) *in* $\mathbb{D}$. *Then* $C_{\mathbb{D}}(\mathbf{a}, \mathbf{b}) = C_D(a, b)$

Proof. Clear because there exists an inverse mapping $D \to \mathbb{D}$. □

Lemma 4.2. *Let* $\mathbf{c} := (c, \ldots c, \ldots) \in \mathbb{D}$. *Let* $B_C(\mathbf{c}, r)$ *be a ball for the Carathéodory distance in* $\mathbb{D}$. *Let* $\mathbf{a} = (a, \ldots, a, \ldots)$ *and* $\mathbf{b} = (b, \ldots, b, \ldots)$ *be two points in* $B_C(\mathbf{c}, r)$. *Then for all* $\epsilon > 0$ *there is an* $\eta > 0$ *such that, if* $(a_\iota)_{\iota\in I}$ *and* $(b_\iota)_{\iota\in I}$ *satisfy* $\|a_\iota - a\| < \eta$ *and* $\|b_\iota - b\| < \eta$ *for all* $\iota \in I$, *then we have*

$$C_{\mathbb{D}}\big((a_\iota)_{\iota\in I}, (b_\iota)_{\iota\in I}\big) \leq C_D(a, b) + \epsilon.$$

PROOF. By the triangle inequality, we get

$$C_{\mathbb{D}}\big((a_\imath)_{\imath\in I}\,,\ (b_\imath)_{\imath\in I}\big) \le C_{\mathbb{D}}\big((a_\imath)_{\imath\in I}\,, \mathbf{a}\big) + C_{\mathbb{D}}(\mathbf{a}\,, \mathbf{b}) + C_{\mathbb{D}}\big(\mathbf{b}\,,\ (b_\imath)_{\imath\in I}\big)\,.$$

But there is some r_0, $0 < r_0 < r_1$, such that for all $\mathbf{d} = (d, \dots, d, \dots) \in B_C(\mathbf{c}\,, r)$ we have $B(\mathbf{d}\,, r_0) \subset D \subset B(\mathbf{d}\,, r_1)$, and it is easy to prove the existence of $\eta > 0$ such that

$$\|a_\imath - a\| < \eta \quad \forall\, \imath \in I \Longrightarrow C_D(a_\imath\,, a) < \frac{\epsilon}{2}\,.$$

This implies the result. □

Now we can end the proof of the theorem. For every $(a\,, b) \in \overline{B_C(\mathbf{c}\,, r)}^2$ the ball $B(a\,, \eta) \times B(b\,, \eta)$ covers $\overline{B_C(\mathbf{c}\,, r)}^2$ which is compact. We can extract a finite cover

$$\overline{B_C(\mathbf{c}\,, r)}^2 \subset \bigcup_{\substack{j=1,\dots,n\\ k=1,\dots,m}} B(d_j\,, \eta) \times B(e_k\,, \eta)\,.$$

This enables us to define a partition $I = \bigcup_{\substack{j=1,\dots,n\\ k=1,\dots,m}} I_{j,k}$ with the property that, for all $\imath \in I_{j,k}$ we have $|x_\imath - d_j| < \eta$ and $|y_\imath - e_k| < \eta$. Of course,

$$C_{\mathbb{D}}\big((x_\imath)_{\imath\in I}\,,\ (y_\imath)_{\imath\in I}\big) = \sup_{\jmath\,,k} C_{\mathbb{D}_{\jmath\,,k}}\big((x_\imath)_{\imath\in I_{j,k}}\,,\ \ (y_\imath)_{\imath\in I_{\jmath\,,k}}\big)$$

(where $\mathbb{D}_{\jmath\,,k}$ is the product of copies of D over $I_{j,k}$) by the finite product property. But

$$C_{\mathbb{D}_{\jmath\,,k}}\big((x_\imath)_{\imath\in I_{\jmath\,,k}}\,,\ \ (y_\imath)_{\imath\in I_{\jmath\,,k}}\big) \le C_D(d_j\,, e_k) + \epsilon$$

and this proves the result. □

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