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Linear response of the gate system for protection of the Venice Lagoon. Note II: Excitation of transverse suhharmonic modes

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Meccanica dei fluidi. — *Linear response of the gate system for protection of the Venice Lagoon. Note II: Excitation of transverse subharmonic modes.* Nota di PAOLO BLONDEAUX, GIOVANNI SEMINARA e GIOVANNA VITTORI, presentata (*) dal Socio E. Marchi.

ABSTRACT. — We show that the transverse subharmonic modes characterizing the free oscillations of the gate system proposed to defend the Venice Lagoon from the phenomenon of high water (see *Note* I [1]) can be excited when the gate system is forced by plane monochromatic waves orthogonal to the gates with the typical characteristics of large amplitude waves in the Adriatic sea close to the lagoon inlets. A linear stability analysis of the coupled motion of the system sea-gates-lagoon reveals that for typical values of the parameters of the problem modes 4, 5 and 6 may be unstable. The need for a nonlinear analysis of mode competion is pointed out.

KEY WORDS: Gates; Waves; Forced oscillations.

RIASSUNTO. — *Risposta lineare della schiera di ventole per la protezione della Laguna Veneta. Nota II: Modi trasversali forzati subarmonici.* Si dimostra che i modi trasversali subarmonici caratterizzanti le oscillazioni libere del sistema di ventole proposto per la difesa della Laguna di Venezia dalle acque alte (vedi *Nota* I [1]) possono essere eccitati quando il sistema di ventole è sollecitato da onde piane monocromatiche che si propagano ortogonalmente alle ventole con le caratteristiche tipiche delle onde di grande ampiezza del mare Adriatico nel paraggio in esame. Un'analisi di stabilità lineare del moto del sistema mare-laguna-ventole rivela che per valori tipici dei parametri del problema possono risultare instabili i modi 4, 5 e 6. Tali risultati pongono l'esigenza di una analisi non lineare della competizione fra i diversi modi linearmente instabili.

1. INTRODUCTION

In *Note* I [1] we have shown that the free oscillations of the gate system designed to protect Venice from the phenomenon of high water is characterized by a set of transverse modes the frequency of which depends on the aspect ratio of the channel, on the mass density of the gates and on the stiffness constant which models the recoil effect induced on the gates by the action of Archimede's force.

In the present *Note* we investigate the possibility that such transverse modes of oscillation may be excited by the forcing effect associated with the action of an incoming wave.

Notations employed in the present *Note* are identical to those of *Note* I [1] which we will refer to as I.

As in many other non linear oscillatory phenomena, resonance may occur if the frequency of the forcing is close to twice the natural frequency of the free mode (subharmonic response) or to any integral fraction $1/m$ of it (synchronous response for $m = 1$, ultra-harmonic response for $m > 1$).

(*) Nella seduta del 18 giugno 1993.

We will concentrate on the subharmonic case as it is well known that in instability mechanisms of Mathieu type subharmonic perturbations are the most unstable ones. More precisely such perturbations have growth rates of order a, if a is the small parameter which defines the amplitude of the forcing effect.

We then assume the frequency ω^* of the incoming wave to attain a value such that σ_n is close to 1/2. We recall that σ_n is the angular frequency of oscillations of the free mode n, normalized such that the value $\sigma_n = 1$ corresponds to a dimensional angular frequency equal to that of the incident wave. Hence let us set

(1) $$\sigma_n = (1 + \mu a)/2 ,$$

with μ free parameter of order 1.

Furthermore we introduce a «slow» time variable τ defined as

(2) $$\tau = at ,$$

which describes the growth of perturbations and we allow the amplitude of the free perturbation $\hat{\zeta}_0$ to be a function of τ to be determined by solving the differential problem for perturbations at $O(\varepsilon a)$.

2. Excitation of transverse subharmonic modes

The differential problem for the perturbation $V_1 \equiv (e_{L1}, f_{L1}, g_{L1}, e_{S1}, f_{S1}, g_{S1}, \zeta_1)$ is obtained by substituting from [1, (23)] into [1, (21)] and equating terms of order εa. From (1), (2) it follows that

(3) $$\frac{\partial V}{\partial t} = \frac{\partial V_0}{\partial t} + \left[\left(\frac{i}{2}\mu + \frac{\partial}{\partial \tau}\right) V_0 + \frac{\partial V_1}{\partial t}\right] a ,$$

we find

Lagoon

(4) $$e_{L1,t} + h_L f_{L1,X} + h_L g_{L1,y} = -(\eta_{L0} f_{L0})_{,X} - (e_{L0} u_{L0})_{,X} + \\ -(\eta_{L0} g_{L0})_{,y} - e_{L0,\tau} - (i/2)\mu e_{L0} ,$$

(5) $$f_{L1,t} + e_{L1,X} = -(u_{L0} f_{L0})_{,X} - f_{L0,\tau} - (i/2)\mu f_{L0} ,$$

(6) $$g_{L1,t} + e_{L1,y} = -(u_{L0} g_{L0})_{,X} - g_{L0,\tau} - (i/2)\mu g_{L0} ,$$

(7) $$f_{L1}|_{X=0} = \zeta_{1,t} - f_{L0,X}|_{X=0}\, x_{G1} + \zeta_{0,\tau} + (i/2)\mu\zeta_0 - u_{L0,X}|_{X=0}\, \zeta_0 .$$

The solution of the above system can be expressed in the form

(8) $$(e_{L1}, f_{L1}, \zeta_1) = \exp((i/2)t)\cos(n\pi y/\beta)(\hat{e}_{L1}, \hat{f}_{L1}, \hat{\zeta}_1) + c.c. + \\ + \text{terms. proport. to } \exp(\pm(3/2)it) ,$$

where $(\hat{e}_{L1}, \hat{f}_{L1}, \hat{\zeta}_1)$ can be decomposed into homogeneous and non homogeneous components:

(9) $$(\hat{e}_{L1}, \hat{f}_{L1}, \hat{\zeta}_1) = (E_{LH}(\tau), F_{LH}(\tau), 0)\exp(i\alpha_L X) + (E_{L1}(X, \tau), F_{L1}(X, \tau), \zeta_{11}(\tau)) .$$

Some tedious algebra shows that $E_{L1}(X, \tau)$ is a particular solution of the equation

(10) $$-h_L E_{L1,XX} + (h_L n^2\pi^2/\beta^2 - 1/4)E_{L1} = b_{1L}\bar{\bar{\zeta}}_0 \exp i(\lambda - \bar{\alpha}_L)X + \\ + [b_{2L} d\hat{\zeta}_0/d\tau + b_{3L}\hat{\zeta}_0]\exp(i\alpha_L X),$$

where

(11) $$b_{1L}=\{(2\bar{\alpha}_L^2-\bar{\alpha}_L\lambda-\lambda^2)/(4\bar{\alpha}_L)+n^2\pi^2/(2\alpha_L\beta^2)-h_L\lambda[n^2\pi^2/\beta^2+(\lambda-\bar{\alpha}_L)^2]\}(i/2)A_L,$$

(12) $$b_{2L} = -[1/(4\alpha_L) + h_L\alpha_L + (h_L/\alpha_L)(n\pi/\beta)^2]/2,$$

(13) $$b_3 = (i/2)\mu b_{2L}.$$

Hence

(14) $$E_{L1} = E_{L11} b_{1L}\bar{\bar{\zeta}}_0 \exp[i(\lambda - \bar{\alpha}_L)X] + E_{L12}(b_{2L} d\hat{\zeta}_0/d\tau + b_{3L}\hat{\zeta}_0)X\exp(i\alpha_L X) + c.c.$$

where

(15) $$E_{L11} = \{h_L[n^2\pi^2/\beta^2 + (\lambda - \bar{\alpha}_L)^2] - 1/4\}^{-1},$$

(16) $$E_{L12} = i/(2\alpha_L h_L).$$

Substituting from (14) into (5) we solve for $F_{L1}(X, \tau)$ in the form

(17) $$F_{L1} = F_{L11}\bar{\bar{\zeta}}_0 \exp[i(\lambda - \bar{\alpha}_L)X] + (F_{L12} d\hat{\zeta}_0/d\tau + F_{L13}\hat{\zeta}_0)X\exp(i\alpha_L X) + \\ + (F_{L14} d\hat{\zeta}_0/d\tau + F_{L15}\hat{\zeta}_0)\exp(i\alpha_L X),$$

and

(18) $$F_{L11} = (\lambda - \bar{\alpha}_L)(-2E_{L11}b_{1L} - i\lambda A_L),$$

(19) $$F_{L12} = -2\alpha_L E_{L12} b_{2L},$$

(20) $$F_{L13} = -2\alpha_L E_{L12} b_{3L},$$

(21) $$F_{L14} = 2iE_{L12}b_{2L} - 1,$$

(22) $$F_{L15} = i(2E_{L12}b_{3L} - \mu/2).$$

Finally the boundary condition (7) allows us to determine the homogeneous component $F_{LH}(\tau)$. We find

(23) $$F_{LH}(\tau) = (i/2)\zeta_{11}(\tau) + F_{LH1}\hat{\zeta}_0 + F_{LH2}\bar{\bar{\zeta}}_0 + F_{LH3} d\hat{\zeta}_0/d\tau,$$

where

(24) $$F_{LH1} = i(\mu - 2E_{L12}b_{3L}),$$

(25) $$F_{LH2} = (\bar{\alpha}_L/2)\mathcal{a} - F_{L11} + i\lambda^2 A_L,$$

(26) $$F_{LH3} = 2 - 2iE_{L12}b_{2L}.$$

From the homogeneous part of (5) we than derive $E_{LH}(\tau)$ which reads

(27) $$E_{LH}(\tau) = -F_{LH}(\tau)/(2\alpha_L).$$

Sea

The differential problem at order εa in the sea region is identical with (4)-(7) except for the pedix S replacing L. Its solution can be similarly cast in the form (8), (9). However in the sea region the basic flow involves a reflected wave besides the incident wave. This implies that the governing equation for E_{S1} reads:

$$(28)\qquad - E_{S1,xx} + (n^2\pi^2/\beta^2 - 1/4)E_{S1} = [b'_{1S}\exp[i(1-\bar{\alpha}_S)X] + b''_{1S}\exp[i(-1-\bar{\alpha}_S)X]]\,\bar{\hat{\zeta}}_0 + [b_{2S}d\hat{\zeta}_0/d\tau + b_{3S}\hat{\zeta}_0]\exp(i\alpha_S X),$$

where

$$(29a)\qquad b'_{1S}(1, 1, 1, \alpha_S) \to b_{1L}(\lambda, h_L, A_L, \alpha_L),$$

$$(29b)\qquad b''_{1S}(-1, 1, \bar{B}_S, \alpha_S) \to b_{1L}(\lambda, h_L, A_L, \alpha_L),$$

$$(29c)\qquad b_{2S}(1, \alpha_S) \to b_{2L}(h_L, \alpha_L),$$

$$(29d)\qquad b_{3S}(1, \alpha_S) \to b_{3L}(h_L, \alpha_L).$$

Hence the entire solution can be written at once in the form

$$(30)\qquad E_{S1} = [b'_{1S}E'_{S11}\exp[i(1-\bar{\alpha}_S)X] + b''_{1S}E''_{S11}\exp[i(-1-\bar{\alpha}_S)X]]\bar{\hat{\zeta}}_0 + E_{S12}X\exp(i\alpha_S X)(b_{2S}d\hat{\zeta}_0/d\tau + b_{3S}\hat{\zeta}_0),$$

where

$$(31a)\qquad E'_{S11}(1, 1, \alpha_S, b'_{1S}) \to E_{L11}(\lambda, h_L, \alpha_L, b_{1L}),$$

$$(31b)\qquad E''_{S11}(-1, 1, \alpha_S, b''_{1S}) \to E_{L11}(\lambda, h_L, \alpha_L, b_{1L}),$$

$$(31c)\qquad E_{S12}(\alpha_S, 1) \to E_{L12}(\alpha_L, h_L),$$

and

$$(32)\qquad F_{S1} = [F'_{S11}\exp[i(1-\bar{\alpha}_S)X] + F''_{S11}\exp[i(-1-\bar{\alpha}_S)X]]\,\bar{\hat{\zeta}}_0 + [F_{S12}d\hat{\zeta}_0/d\tau + F_{S13}\hat{\zeta}_0]X\exp(i\alpha_S X) + [F_{S14}d\hat{\zeta}_0/d\tau + F_{S15}\hat{\zeta}_0]\exp(i\alpha_S X),$$

with

$$(33a)\qquad F'_{S11} = (1-\bar{\alpha}_S)(-2E'_{S11}b'_{1S} - i),$$

$$(33b)\qquad F''_{S12} = (-1-\bar{\alpha}_S)(-2E''_{S11}b''_{1S} + i\bar{B}_S),$$

$$(33c)\qquad F_{S12} = -2\alpha_S E_{S12}b_{2S},$$

$$(33d)\qquad F_{S13} = -2\alpha_S E_{S12}b_{3S},$$

$$(33e)\qquad F_{S14} = 2iE_{S12}b_{2S} - 1,$$

$$(33f)\qquad F_{S15} = i(2E_{S12}b_{3S} - \mu/2).$$

Finally

$$(34)\qquad F_{SH}(\tau) = (i/2)\zeta_{11}(\tau) + F_{SH1}\hat{\zeta}_0 + F_{SH2}\bar{\hat{\zeta}}_0 + F_{SH3}d\hat{\zeta}_0/d\tau,$$

where

$$(35a)\qquad F_{SH1} = i(\mu - 2E_{S12}b_{3S}),$$

$$(35b)\qquad F_{SH2} = (\bar{\alpha}_S/2)\mathcal{a} - F'_{S11} - F''_{S11} + i(1+\bar{B}_S),$$

$$(35c)\qquad F_{SH3} = 2 - 2iE_{S12}b_{2S}.$$

Similarly

$$E_{SH}(\tau) = -F_{SH}(\tau)/(2\alpha_S).$$

Gate motion

The equation of gate motion reads

$$(36)\qquad m\zeta_{11,tt} + k\zeta_{11} = [h_L e_{L1} - e_{S1} + (h_L e_{L0,X} - e_{S0,X})x_{G1} + (h_L \eta_{L0,X} - \eta_{S0,X})\zeta_0 + \eta_{L0}e_{L0} - \eta_{S0}e_{S0}]_{X=0} - 2m\zeta_{0,\tau t} + (\mu/2)m\zeta_0 .$$

Using the expansion (8), (9) for ζ_1 and the solutions just derived in the lagoon and sea regions we can reduce the equation for ζ_{11} to the following form:

$$(37)\qquad [-m/4 + k + i(h_L/(4\alpha_L) - 1/(4\alpha_S))]\zeta_{11} = c_0 d\hat{\zeta}_0/d\tau + c_1\hat{\zeta}_0 + c_2\bar{\hat{\zeta}}_0$$

where

$$(38)\qquad c_0 = -mi - h_L/(2\alpha_L)F_{LH3} + F_{SH3}/(2\alpha_S),$$

$$(39)\qquad c_1 = \mu m/2 - h_L/(2\alpha_L)F_{LH1} + F_{SH3}/(2\alpha_S),$$

$$(40)\qquad c_2 = -(h_L/(2\alpha_L))F_{LH2} + F_{SH2}/(2\alpha_S) + h_L E_{L11} - E'_{S11} - E''_{S11} + (\mathcal{A}/4)(h_L - 1) + (i/4)(A_L/\bar{\alpha}_L - (1 + \bar{B}_S)/\bar{\alpha}_S) + i(\lambda A_L h_L - 1 + \bar{B}_S).$$

Since eq. [1, (29)] forces the left hand side of (37) to vanish it follows that the following amplitude equation for $\hat{\zeta}_0$ has to be satisfied

$$(41)\qquad c_0 d\hat{\zeta}_0/d\tau + c_1\hat{\zeta}_0 + c_2\bar{\hat{\zeta}}_0 = 0 .$$

Since c_0 is purely imaginary and c_1 is real, equation (41) admits of solutions proportional to $\exp(\gamma\tau)$ with

$$(42)\qquad \gamma = \pm\sqrt{\left|\frac{c_2}{c_0}\right|^2 - \left|\frac{c_1}{c_0}\right|^2}\,.$$

Hence the response of transverse modes on the slow time scale is either exponentially growing (unstable) or periodic (stable). This behaviour is typical of instability with respect to inviscid perturbations (see for instance [2]).

3. Results

From (42), using the expressions for c_1 and c_2 in terms of the solution derived in sect. 2, we can evaluate γ as a function of T and a for each mode n. In fact for any T and n the dispersion relationship [1, (29)] determines σ_n; then eq. (1) gives μ as a function of a.

Following the latter procedure and using the choice of typical values for $\mathcal{M}$, K and b employed in *Note* I [1] we find the stability chart of fig. 1 which is clearly of Mathieu type. It shows that for values of T in the range 5-10 which is typical of the Adriatic sea in the range of interest, modes 4, 5 and 6 may be unstable.

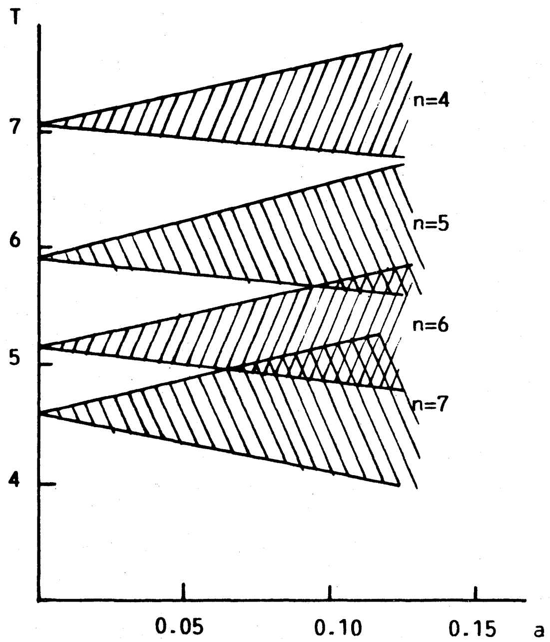

Fig. 1. – Regions (dashed regions) in the plane (a, T) where the n-th subharmonic transverse mode turns out to be unstable ($\mathfrak{M} = 2.0, K = 0.3, b = 160, h_L = 1$).

As the amplitude of the incident wave increases more than one mode may be simultaneously excited. Needless to say the present theory, being linear, does not allow one to predict the preferred mode, assuming that one of them would prevail. A nonlinear theory able to allow for modal competition would then be needed. Such a theory would be of considerable interest as it is well known that in similar nonlinear oscillatory systems various types of interesting responses, periodic, quasi periodic and chaotic may arise as a result of modal competition (see [3-8]).

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