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Remarks on materials with memory

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Meccanica dei continui. — *Remarks on materials with memory.* Nota di VICTOR MIZEL, presentata (*) dal Socio C. Truesdell.

ABSTRACT. — The mistaken notion that consistency with fading memory should require uniqueness is refuted by citation of the sources. Indeed, insistence on uniqueness would exclude many examples from non-linear elasticity and would exclude materials capable of exhibiting hysteresis.

KEY WORDS: Materials with memory; Uniqueness; Viscoelastic; Hysteresis.

RIASSUNTO. — *Osservazioni sui materiali con memoria.* Mediante opportune citazioni delle fonti, si rifiuta la nozione ambigua che per ogni materiale con memoria accettabile si debba garantire l'unicità della soluzione. Difatti, esigendo sempre l'unicità, molti casi di elasticità nonlineare e di materiali con isteresi verrebbero scartati.

Some issues have recently been presented by Professor G. Fichera concerning the notion of fading memory in mechanics, as initiated and used by a number of researchers building on the early work of Coleman and Noll. It seems useful to provide some additional comments in an effort to explore these matters further.

It will be assumed that the reader is familiar with the fact that the function spaces in question, which have been proposed as a useful model in the study of materials of «viscoelastic» type, are weighted Lebesgue spaces on the positive real axis. For simplicity the present discussion will be confined to the L^2 context, where

$$\|u\|^2 = \int_0^\infty u^2(s)\,\rho(s)\,ds\,.$$

Here the Lebesgue integrable, nonnegative weight function ρ is required to satisfy conditions ensuring that the rightward or leftward shift of an element in the space is also an element of the space and that the norm of the rightward shift of an element converges to zero as the amount of shift tends to infinity (cf. *e.g.* [1, 4]).

One objection that has been raised is that choice of the density ρ is not subject to experimental determination, especially in its asymptotic behaviour at ∞. This is certainly correct – although the experimental determination of such constitutive quantities as stress relaxation functions does provide some restrictions on the asymptotic behaviour – and has been well understood all along.

On the other hand, the objection based on the existence of linear stress strain relations of Boltzmann type in which (for some choices of ρ) the $L^2(\rho)$ strain associated to a given stress may fail to be unique seems to reflect a confused notion that consistency with fading memory should imply uniqueness. While [3] suggests something to that effect, certainly no such idea can be found in any of the other references listed at the

(*) Nella seduta del 14 giugno 1991.

end of this *Note* (cf. *e.g.* [4]). Insistence on uniqueness would exclude well-known examples from nonlinear elasticity and would totally exclude the treatment of materials exhibiting hysteresis.

Nevertheless it is worth noting that for the example given in [3] it is easily verified that a properly prescribed *dynamical* problem in which the strain history at all times prior to an initial time is specified necessarily provides a *unique* (in fact *stable*) continuation of the strain history to future times (cf. *e.g.* [5, 6 §4B]).

Of course there is no inherent virtue in insisting on a theory of fading memory with a memory of scope extending into the *infinite* past — provided that there is an equally useful theory which confines itself to a memory of finite scope. However I am unaware of any theory of the latter sort which provides an acceptable alternative for the problems of interest in this branch of continuum mechanics. The one deep investigation of this point which I know is that due to Noll [7] in 1972. In that paper it was shown that «semi-elastic» materials necessarily possess a representation in terms of a memory space with infinite scope, although the original definitions were posed purely in terms of a natural state space, using considerations in which the assumptions were physically sound. Thus the use of infinite histories can, if desired, be regarded as the adoption of a mathematical structure arising as a natural artifact after starting from a set of well motivated physical postulates distinct from those provided in 1961 by Coleman and Noll and in 1966 by Coleman and Mizel.

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