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Sismologia. — *The change in the moment of inertia due to faulting in a self-gravitating layered Earth model with viscoelastic mantle.* Nota (*) di MICHELE DRAGONI, DAVID A. YUEN e ENZO BOSCHI, presentata dal Corrisp. E. BOSCHI.

RIASSUNTO. — Si considera un modello di Terra stratificato e autogravitante, costituito da una litosfera elastica e da un mantello viscoelastico di Maxwell, allo scopo di calcolare l'evoluzione nel tempo del momento di inerzia in seguito al verificarsi di un terremoto. Si trova che il flusso viscoelastico nel mantello produce una amplificazione della variazione del momento di inerzia rispetto ai modelli elastici, ma questa non è in grado di eccitare il moto di precessione libera della Terra per valori della viscosità del mantello uguali a quelli dedotti dal rilassamento post-glaciale.

1. INTRODUCTION

The change in the moment of inertia of the Earth due to earthquakes has been studied by several authors in the case of an elastic planet (Mansinha and Smylie, 1967; Smylie and Mansinha, 1971; Dahlen, 1971, 1973; O'Connell and Dziewonski, 1976). These efforts were mainly directed to ascertain whether this mechanism is capable of exciting the Earth's free precession, or Chandler wobble (Munk and MacDonald, 1960). However, the results have been unsatisfactory so far. In the present work we shall study the role played in this problem by a viscoelastic structure of the Earth's mantle in the presence of an elastic lithosphere. As a result, we obtain the temporal evolution of the inertia tensor as a consequence of earthquake faulting.

2. THE CHANGE IN THE MOMENT OF INERTIA

Calculation of the change ΔI_{ij} in the moment of inertia of the Earth would require, in principle, the knowledge of the entire displacement field due to faulting. A simple way of obtaining ΔI_{ij} has been found by Rice and Chinnery (1972). The moment of inertia of a sphere about an axis having the orientation of a unit vector $\hat{n}$ is

$$I^{(\hat{n})} = \int_M X^2 \, dm \tag{1}$$

(*) Pervenuta all'Accademia il 4 ottobre 1983.

where X is the perpendicular distance from the axis to the mass element d*m*, and M is the total mass. When the sphere is subjected to the displacement field $\mathbf{U}(\boldsymbol{r})$ due to the presence of a dislocation, the change in the moment of inertia is, to first order,

$$\Delta I^{(\hat{n})} = 2 \int_M X U_X \, dm \tag{2}$$

where U_X is the displacement component in the radially outward direction from the axis. One may note that eq. (2) is nothing but twice the work done by a centrifugal body force $\omega^2 X$ per unit mass on the displacement $\mathbf{U}$, when $\boldsymbol{\omega} = \hat{\mathbf{n}}$. Since the centrifugal force is derivable from a potential field which is formed by spherical harmonics of degrees zero and two, the result obtained means that only the terms of degrees zero and two in the displacement field produced by faulting contribute to a change in the moment of inertia, as should be expected.

We now apply Betti's theorem (Love, 1927) to an elastic sphere. This theorem yields a relationship between two different configurations of the same elastic body. Consider the following configurations:

(1) A sphere has a shear dislocation $\Delta \boldsymbol{u}$ across an internal surface Σ. In this situation we have no body force, a displacement field U_i and a stress field Σ_{ij} associated with the dislocation.

(2) A sphere is rotating with angular velocity ω around an axis directed as $\hat{\mathbf{n}}$: in this case there is a centrifugal body force F_i per unit mass, a displacement field u_i and a stress field σ_{ij}.

Applying Betti's theorem to these two configurations we get the following relationship

$$\int_M F_i U_i \, dm = \int_\Sigma \upsilon_s \sigma_{jk} s_k \Delta u \, dS \tag{3}$$

where $\hat{\upsilon} = \hat{\upsilon}^- = - \hat{\upsilon}^+$, $\hat{\upsilon}^+$ and $\hat{\upsilon}^-$ being the unit outward normals to the sides Σ^+ and Σ^- respectively of the fault surface Σ and $\hat{\mathbf{s}}$ is a unit vector in the slip direction at every point of Σ:

$$\Delta \boldsymbol{u} = \Delta u \hat{\mathbf{s}} \; . \tag{4}$$

We see that eq. (3) represents just the work done by the centrifugal body force $\mathbf{F}$ on the displacement $\mathbf{U}$ due to faulting, which is required from eq. (2) to obtain the change in the moment of inertia. We have then

$$\Delta I^{(\hat{n})} = 2 \int_\Sigma \tau^{(\hat{n})} \Delta u \, dS \tag{5}$$

where

$$\tau^{(\hat{n})} = \nu_j \, \sigma_{jk} \, s_k \tag{6}$$

is the shear stress induced on the (unslipped) fault surface by the centrifugal force **F** (case 2 above). It can be shown (Rice and Chinnery, 1972) that the change in a general component I_{ij} of the inertia tensor is given by

$$\Delta I_{ij} = 2 \int_{\Sigma} \tau^{(ij)} \, \Delta u \, dS \tag{7}$$

where $\tau^{(ij)}$ is defined in such a way that

$$\tau^{(\hat{\mathbf{n}})} \doteq n_i \, n_j \, \tau^{(ij)} \, . \tag{8}$$

Since $\tau^{(ij)}(\boldsymbol{r})$ will be a smooth function of position over size scales much larger than typical fault dimensions, one can approximate eq. (7) by

$$\Delta I_{ij} = 2 \, \tau^{(ij)} \, \Delta u \, \Sigma \tag{9}$$

where $\tau^{(ij)}$ is evaluated at a representative point of Σ.

3. The model

We consider a two-layer, spherical elastic Earth model. The inner layer, the mantle, has radius r_1, density ρ_1 and rigidity μ_1, while the outer layer, the lithosphere, has an outer radius r_2, density ρ_2 and rigidity μ_2 (fig. 1). The di-

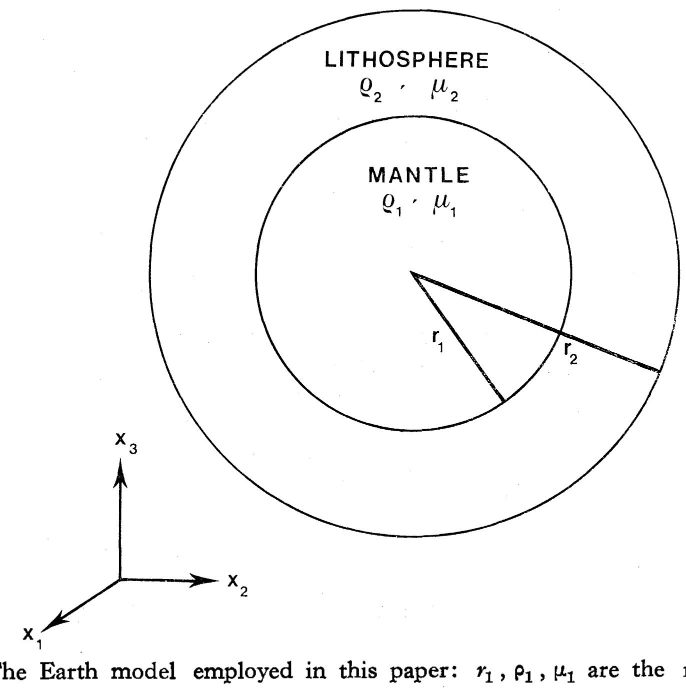

Fig. 1. – The Earth model employed in this paper: r_1, ρ_1, μ_1 are the mantle radius density and rigidity, respectively; r_2, ρ_2, μ_2 are the lithosphere outer radius, density and rigidity, respectively.

splacement field $\boldsymbol{u}(\boldsymbol{r})$ within the elastic sphere can be written in the form (Love, 1909):

$$\boldsymbol{u}(\boldsymbol{r}) = \sum_{l=-\infty}^{\infty} [\boldsymbol{u}_l(\boldsymbol{r}) - (r^2/2\,l)\,\nabla\psi_l(\boldsymbol{r})] \tag{10}$$

where $\boldsymbol{u}_l$ is a set of three spherical solid harmonics of degree l and $\psi_l(\boldsymbol{r})$ is another representation of the spherical solid harmonic, defined as

$$\psi_l(\boldsymbol{r}) = \nabla \cdot \boldsymbol{u}_{l+1}(\boldsymbol{r})\,. \tag{11}$$

We also introduce a spherical solid harmonic $\varphi_{-l-2}(\boldsymbol{r})$ of degree $-l-2$

$$\varphi_{-l-2}(\boldsymbol{r}) = \nabla \cdot [\boldsymbol{u}_l(\boldsymbol{r})/r^{2l+1}]\,. \tag{12}$$

The body force to which the sphere is subjected is the centrifugal force due to steady rotation around the x_3 axis, as shown in fig. 1. The centrifugal potential

$$\mathrm{W}(\boldsymbol{r}) = \frac{1}{2}\,\omega^2\,(x_1^2 + x_2^2) \tag{13}$$

can be expressed as a sum of a degree-zero spherical solid harmonic W_0 plus a degree-two spherical solid harmonic W_2:

$$\mathrm{W}_0(r) = \frac{1}{3}\,\omega^2\,r^2 \tag{14}$$

$$\mathrm{W}_2(\boldsymbol{r}) = \frac{1}{6}\,\omega^2\,(x_1^2 + x_2^2 - 2\,x_3^2) \tag{15}$$

where $r^2 = x_1^2 + x_2^2 + x_3^2$ and ω is the angular frequency of rotation. W_0 gives a purely radial force field, so that it does not produce any displacement in an incompressible sphere. Thus we have to compute only the displacement due to W_2. It can be seen that, if the applied potential is a spherical harmonic of degree two, only six harmonic functions appear in the expressions for displacement and stress. By means of eq. (10) and the corresponding expression for stress, we can write down the boundary conditions to be satisfied at the interface between the two layers (the surface $r = r_1$) and at the Earth's surface $(r = r_2)$. We impose boundary conditions of continuity of displacement and traction across the lithosphere-mantle interface and stress-free boundary conditions at the surface. These conditions can be reduced to an algebraic system of six equations in six unknowns. According to eq. (10), the displacement field in the lithosphere is found to be

$$\boldsymbol{u}(\boldsymbol{r}) = f(r)\,\nabla\mathrm{W}_2 + g(r)\,\mathrm{W}_2\,\boldsymbol{r} \tag{16}$$

where f and g are functions of r. The shear stress $\sigma_{ij}(\boldsymbol{r})$ is then given by Hooke's law:

$$\sigma_{ij}(\boldsymbol{r}) = \mu_2 (u_{i,j} + u_{j,i}) , \qquad r_1 \leq r \leq r_2 . \tag{17}$$

From eq. (16)

$$\sigma_{ij}(\boldsymbol{r}) = \mu_2 [f_{,j} \mathrm{W}_{2,j} + f \mathrm{W}_{2,ij} + g_{,j} \mathrm{W}_2 x_i + g \mathrm{W}_{2,j} x_i + g \mathrm{W}_2 \delta_{ij} + (i \leftrightarrow j)] \tag{18}$$

where $(i \leftrightarrow j)$ represents the same expression as before with the indices i and j interchanged. Let us now consider a strike-slip fault across which a dislocation $\Delta \boldsymbol{u}$ is imposed. Here we denote a representative point of the fault surface Σ by $\boldsymbol{r}'$. The component of the shear stress projected upon Σ in the slip direction $\hat{\mathbf{s}}$ is given by eq. (6) to be:

$$\tau^{(\hat{\mathbf{n}})}(\boldsymbol{r}') = \mu_2 f(r') \mathrm{W}_{2,ij} (\nu_i s_j + \nu_j s_i) . \tag{19}$$

The expression of $\tau^{(kl)}$ (eq. 8) which will be used in Rice and Chinnery's formula becomes

$$\tau^{(kl)}(\boldsymbol{r}') = - \mu_2 f(r') (\nu_k s_l + \nu_l s_k) . \tag{20}$$

For shallow earthquakes, one can make the approximation $r' \simeq r_2$ (the Earth's radius). The rheology of the mantle is modelled as a Maxwell viscoelastic solid with shear modulus μ_1 and viscosity η. This rheology model behaves initially as an elastic solid and becomes a Newtonian fluid after a characteristic time $\tau_{\mathrm{M}} = \eta / \mu_1$ (the Maxwell time). The temporal change in the moment of inertia can be readily obtained from the elastic increment, eq. (9), by using the correspondence principle for linear viscoelastic materials (see e.g. Fung, 1965). In our case. this is equivalent to the transformation

$$\mu_1 \rightarrow \mu_1 (s) = \mu_1 \eta s / (\eta s + \mu_1) \tag{21}$$

where s is the complex Laplace variable. A Heaviside function $\mathrm{H}(t)$ is introduced to describe the jump discontinuity of the displacement across the fault

$$\Delta u (t) = \Delta u \, \mathrm{H}(t) . \tag{22}$$

From substituting eq. (22) into (9) and using expressions from eqs. (20) and (21), the change $\Delta \tilde{\mathrm{I}}_{ij}(s)$ of the inertia tensor in the Laplace transform domain is obtained. $\Delta \tilde{\mathrm{I}}_{ij}(s)$ is characterized by three poles: a pole at $x = 0$ which is associated with the permanent deformation in the final state and two poles s_1 and s_2 which are designated respectively to be the mantle and lithospheric poles. The relaxation time τ_i of each mode is defined to be $- 1/s_i$ $(i = 1 , 2)$.

In fig. 2, for a mantle viscosity of 10^{22} P, we display the relaxation times of the two modes as a function of the shell thickness $R = r_1/r_2$. As $R \rightarrow 1$, the relaxation time of the lithospheric mode approaches infinity, whereas that of the mantle mode approaches a value associated with a homogeneous viscoelastic sphere. For a thick elastic shell (or $R \rightarrow 0$) the relaxation time of the litho-

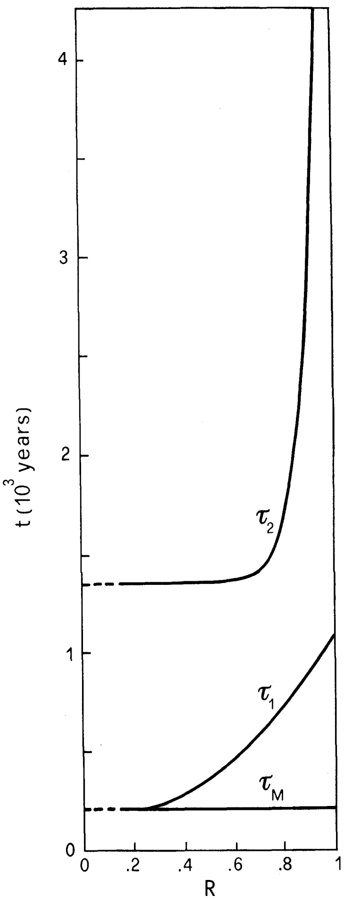

Fig. 2. – Viscoelastic relaxation times as functions of the ratio $R = r_1/r_2$.

spheric mode approaches a constant value, whereas the mantle mode relaxes with a time-scale identical to the Mawxell time τ_M. The Earth, whose lithospheric thickness is about 100 km, lies in the thin-shell regime with $R \simeq 0.98$. By taking the inverse Laplace transform of $\Delta \tilde{I}_{ij}(s)$ we obtain the following expression for the temporal evolution of the change in the moment of inertia

$$\Delta I_{ij}(t) = T_{ij} \, \Delta u \, \Sigma \left(C_1 e^{-t/\tau_1} + C_2 e^{-t/\tau_2} + C_3 \right) \tag{21}$$

where C_1, C_2 and C_3 are coefficients independent of time and T_{ij} is a second order tensor describing the geometry of faulting.

4. Conclusions

It has been suggested (Slade *et al.*, 1979) that viscoelastic flow in the mantle induced by earthquakes might provide an additional mechanism sufficient to maintain the excitation of the Chandler wobble. For a homogeneous viscoelastic Earth with Maxwell rheology, the initial excitation arising from the elastic contribution vanishes as the planet tends to a fluid limit (Yuen and Peltier, 1982). This is not the case, however, for a two-layer model with elastic lithosphere. The results are shown in fig. 3 for a range of shell thicknesses. The presence of the lithosphere affects the dynamics quite differently from that of a homogeneous model. For the thick-shell situation ($R \leq 0.5$), the viscoelastic solutions do not differ greatly from their elastic counterpart in that the thick elastic shell prevents any surface deformation from transient creep from taking place.

However, as the thickness of the elastic shell is reduced, the stress exerted by the viscoelastic flow upon the lithosphere gives rise to a further increase in

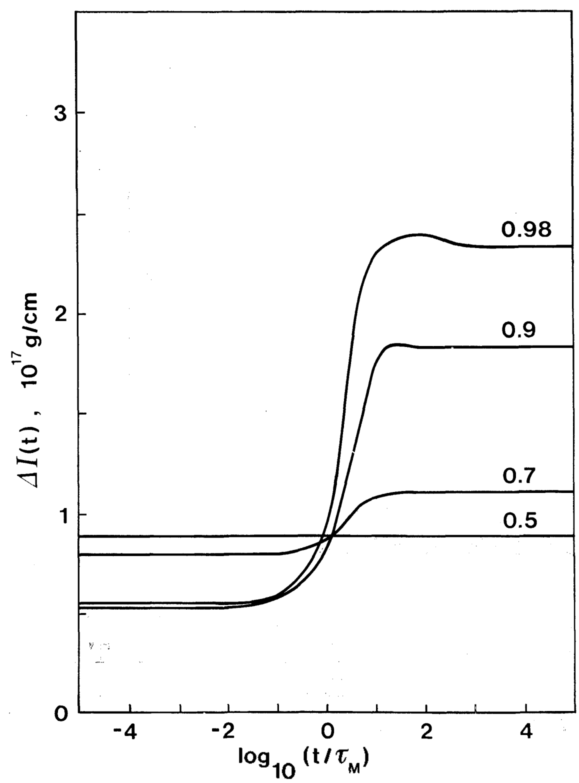

Fig. 3. – Temporal changes in the scalar moment of inertia for two-layer Earth models with a Maxwell-viscoelastic mantle and an elastic lithosphere. Different curves refer to different values of $R = r_1/r_2$. $\mu_1 = 1.45 \times 10^{12}$ dyne/cm^2, $\mu_2 = 2.82 \times 10^{11}$ dyne/cm^2, $\rho_2 = 2.69$ g/cm^3, while ρ_1 is chosen so that the Earth model has always a mass $M = 5.976 \times 10^{27}$ g.

the change of the moment of inertia. For a shell thickness similar to that of the Earth ($R \simeq 0.98$) an enhancement of a factor of about four is obtained. Of course, in the limit of an extremely thin lithosphere ($R \rightarrow 1$), some other physical processes, such as rupturing, would occur.

The time scales associated with the rapid changes in the moment of inertia are of the order of 10 Maxwell times. Since an oscillation can only be excited substantially by functions whose time scales are comparable to the period of the oscillation itself, an upper bound of 5×10^{18} P is required for mantle viscosity in order for the Chandler wobble to be enhanced by viscoelasticity. However, these values for the mean mantle viscosity are about four orders of magnitude lower than that inferred from post-glacial rebound (e.g. Cathles, 1975). Hence, we would not expect the excitation of the Chandler wobble to be enhanced for values of the long-term viscosity derived from post-glacial rebound.

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