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SEZIONE I

(Matematica, meccanica, astronomia, geodesia e geofisica)

Logica matematica. — *General operators binding variables in the interpreted modal calculus* $\mathscr{MC}^\nu$. Nota di ALDO BRESSAN e ALBERTO ZANARDO (*), presentata (**), dal Corrisp. A. BRESSAN.

RIASSUNTO. — Si considera il calcolo modale interpretato MC^ν, che è basato su un sistema di tipi con infiniti livelli, contiene descrizioni, ed è dotato di una semantica di tipo generale - v. [2], o [3], o [4], o [5]. In modo semplice e naturale si introducono in MC^ν operatori vincolanti variabili, di tipo generale. Per teorie basate sul calcolo logico risultante $\mathscr{MC}^\nu$ vale un teorema di completezza, che si dimostra in modo immediato sulla base dell'estensione del teorema parziale di completezza stabilito in [11], fatta in [12].

1. INTRODUCTION

In extensional logic variable binding term operators, vbtos, or formula-term operators—i.e. expressions of the form $(\Omega\, y_1, \cdots, y_n)$ that, if applied to a wff (well formed formula) generate a term—are studied from the semantical point of view in e.g. [7], and from the syntactical one in e.g. [8] and [9],

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where also completeness theorems are proved. In [9], where a survey on the subject is also presented, it is said—on p. 158—that future papers are planned to treat the vbtos in a modal logic, by means of a modified kind of Kripke models.

Here we consider general variable binding operators that apply to wffs or terms and generate wffs or terms; and we introduce them in a very simple way within the frame of the general modal calculus MC^{ν} whose underlying language ML^{ν} is based on a type system with infinitely many levels. There is no need of changing the semantics for ML^{ν}. We use as operator signs functors and attributes already existing in ML^{ν}. We simply add a new rule to form operator expressions and two natural axioms. Thus the language $\mathcal{ML}^{\nu}$ and the calculus $\mathcal{MC}^{\nu}$ are obtained.

The completeness theorem for the 1^{st} order fragment of MC^{ν}, deprived of the iota operator ι for descriptions, is proved in [11]. This result is extended to the full calculus MC^{ν} in [12]. Here this extension is easily carried over to $\mathcal{MC}^{\nu}$. Incidentally the fact that this extension can deal with descriptions is essential. Indeed, from the present paper it appears that general operators can be reduced to descriptions; furthermore that the use of types simplifies the introduction of the above operators (1).

2. The modal language ML^{ν} and the modal calculus MC^{ν} based on it

The language ML^{ν} ($\nu \in Z^+$, the set of positive integers) is based on a *type system* τ^{ν} which is the smallest set such that $\{1, \cdots, \nu\} \subset \tau^{\nu}$ and $\langle t_1, \cdots, t_n, t_0\rangle \in \tau^{\nu}$ whenever $t_1, \cdots, t_n \in \tau^{\nu}$ and $t_0 \in \overline{\tau}^{\nu} = \tau^{\nu} \cup \{0\}$.

For every $t \in \tau^{\nu}$, the *constants* c_{tn} and the *variables* v_{tn} ($n \in Z^+$) are primitive symbols of ML^{ν} in addition to the usual logical symbols $=, \sim, \wedge, \forall, \Box, \iota$, comma, and left and right parentheses. For $t \in \overline{\tau}^{\nu}$ the set $\mathcal{E}_t$ of the *well formed expressions* (wfes) of type t of ML^{ν} is defined recursively by formation rules (f_1) to (f_8) below, where $t, t_1, \cdots, t_n$ run over τ^{ν} and t_0 [n] runs over $\overline{\tau}^{\nu}$ [Z^+].

($f_{1,2}$) $\quad v_{tn}, c_{tn} \in \mathcal{E}_t$; $\Delta_1, \Delta_2 \in \mathcal{E}_t \Rightarrow \Delta_1 = \Delta_2 \in \mathcal{E}_0$;

(f_3) $\quad \Delta_i \in \mathcal{E}_{t_i}$ $(i = 1, \cdots, n)$ and $\Delta \in \mathcal{E}_{\langle t_1, \ldots, t_n, t_0\rangle} \Rightarrow \Delta(\Delta_1, \cdots, \Delta_n) \in \mathcal{E}_{t_0}$;

(f_{4-7}) $\quad p, q \in \mathcal{E}_0 \Rightarrow (\sim p), (p \wedge q), ((\forall v_{tn}) p)$, and $(\Box p) \in \mathcal{E}_0$;

(f_8) $\quad p \in \mathcal{E}_0 \Rightarrow (\iota\, v_{tn}) p \in \mathcal{E}_t$.

(1) General operators are treated within an extensional first order theory in [6]. The completeness theorem for them proved there has an essential role in [1]. The present treatment of operators in MC^{ν} is much simpler.

The elements of $\mathscr{E}_0$ are called *well formed formulas*, and those of $\mathscr{E}_r\,(r \in \{1, \cdots, \nu\})$ are called *individual terms*. Furthermore, the elements of $\mathscr{E}_{\langle t_1, \ldots, t_n, t_0\rangle}$ are said to be *relation* [*function*] *terms* if $t = 0$ [$t_0 \in \tau^\nu$].

The symbols $\vee$, $\supset$, $(\exists v_{tn})$, $\Diamond$, and other metalinguistic abreviations are understood to be introduced in the usual way. In particular $(\exists_1 x)\, p$ will stand for $(\exists x)\,(p \wedge (y)\,(p\,[x/y] \supset x = y))$, where y is the first variable, of the same type of x, not free in p; and λ-expressions are introduced by the definitions

$$(2.1)\qquad \begin{cases} (\lambda x_1, \cdots, x_n)\, p =_d (\iota \mathrm{F})\,(\forall x_1, \cdots, x_n)\,(\mathrm{F}\,(x_1, \cdots, x_n) \equiv p) \\ (\lambda x_1, \cdots, x_n)\, \Delta =_d (\iota f)\,(\forall x_1, \cdots, x_n)\, f\,(x_1, \cdots, x_n) = \Delta \end{cases}$$

where x_1 to x_n are n (distinct) variables in $\mathscr{E}_{t_1}$ to $\mathscr{E}_{t_n}$ respectively, p [Δ] is a wff [a term of type t_0], and F [f] is the first variable of type $\langle t_1, \cdots, t_n, 0\rangle$ [$\langle t_1, \cdots, t_n, t_0\rangle$] not free in p [Δ]. Furthermore, every expression used in the sequel is assumed to be well formed, which makes several explanations unnecessary.

A formula p will be said to be *modally closed* if it is constructed from wffs $\Box p_1, \cdots, \Box p_n$ by means of $\sim$, $\wedge$, (v_{tn}), and $\Box$.

The basic axiom schemes for MC^ν, i.e. those which determine the minimal version of this calculus, are A2.1–17 below. In them p and q denote wffs, Δ denotes a term, and $x, y, z, x_1, \cdots, x_n, \mathrm{F}, \mathrm{G}, f,$ and g denote variables of suitable types.

A2.1–6 The axioms of the predicate calculus;

A2.7, 8 $\Box\,(p \supset q) \cdot \supset \cdot \Box p \supset \Box q \quad ; \quad \Box p \supset p$;

A2.9 $p \supset \Box p$, where p is modally closed;

A2.10–12 $x = x \; ; \; x = y \wedge y = z \cdot \supset \cdot x = z \; ; \; \Box\, x = y \supset \Delta\,[z/x] = \Delta\,[z/y]$;

A2.13 $\mathrm{F} = \mathrm{G} \equiv (\forall x_1, \cdots, x_n)\,(\mathrm{F}\,(x_1, \cdots, x_n) \equiv \mathrm{G}\,(x_1, \cdots, x_n))$;

A2.14 $f = g \equiv (\forall x_1, \cdots, x_n)\, f\,(x_1, \cdots, x_n) = g\,(x_1, \cdots, x_n)$;

A2.15 $(\exists \mathrm{F})\,(\forall x_1, \cdots, x_n)\,(\mathrm{F}\,(x_1, \cdots, x_n) \equiv p)$;

A2.16 $(\exists f)\,(\forall x_1, \cdots, x_n)\, f\,(x_1, \cdots, x_n) = \Delta$;

A2.17 (a) $(\exists_1 v_{tn})\, p \wedge p\,[v_{tn}/y] \cdot \supset \cdot y = (\iota v_{tn})\, p$;

(b) $\sim (\exists_1 v_{tn})\, p \supset (\iota v_{tn})\, p = (\iota v_{tn})\,(v_{tn} \neq v_{tn})$.

Our deduction rules are the *Modus Ponens* and the *generalization* and *necessitation* rules restricted to axioms: if $\mathscr{A}$ is an axiom and x_1 to x_n are variables, then $\Box\,(\forall x_1, \cdots, x_n)\,\mathscr{A}$ is a direct consequence of $\mathscr{A}$.

3. Standard and general ML^{ν}-interpretations. Completeness of MC_{ν}

For every choice of the $\nu + 1$ sets $D_1, \cdots, D_{\nu}$, and Γ, we say that the set $\mathscr{S} = \{\mathscr{QT}_t : t \in \overline{\tau}^{\nu}\}$ is a QI-*structure* in case the following conditions (3.1–3) hold (2)

$$(3.1\,,\,2) \qquad \mathscr{QT}_0 \subseteq \mathscr{P}(\Gamma) \quad ; \quad \mathscr{QT}_r \subseteq (\Gamma \to D_r) \;;$$

$$(3.3) \qquad \mathscr{QT}_{\langle t_1,\ldots,t_n,t_0\rangle} \subseteq ((\Pi_n^i \mathscr{QT}_{t_i}) \to \mathscr{QT}_{t_0})$$

where $\alpha \in \mathscr{P}(\beta) \Longleftrightarrow \alpha \subseteq \beta$ and $\Pi_n^i \alpha_i = \alpha_1 \times \cdots \times \alpha_n$. If a^{ν} is a function of domain τ^{ν}, such that $(a_t^{\nu} =_d) \, a^{\nu}(t) \in \mathscr{QT}_t$ (for all $t \in \tau^{\nu}$), then we say that the ordered pair $\langle \mathscr{S}, a^{\nu} \rangle$ is a QI-*system*. We shall call a_t^{ν} the *non-existing object* of type t, looking forward to identify it with the designatum of every description (in $\mathscr{E}_t$) which does not fulfil its exact uniqueness condition—cf. rule (d_8) below. The elements of $\mathscr{QT}_t$ are called quasi-intensions (QI's) of type t.

An ML^{ν}-*interpretation* is an ordered triple $\mathfrak{I} = \langle \mathscr{S}, a^{\nu}, \mathscr{T} \rangle$ in which $\mathscr{T}$ is a valutation of the constants of ML^{ν} in $\mathscr{S}$, that is, a function assigning each c_{tn} a QI $\mathscr{T}(c_{tn})$ in $\mathscr{QT}_t$. If in (3.1–3) the relation $\subseteq$ holds as an equality, then $\mathfrak{I}$ is said to be *standard*.

Definition 3.1. *Let* $\mathfrak{I}$ *be an* ML^{ν}*-interpretation and let* $\xi, \zeta \in \mathscr{QT}_t$ $(t \in \overline{\tau}^{\nu})$. *Then* ξ *and* ζ *are said to be equivalent in the case* $\gamma \, (\in \Gamma)$*—briefly* $\xi =_{\gamma}^{t} \zeta$*—if one of the following conditions holds.*

(i) $[(ii)]$ $\quad t = 0 \; [t \in \{1, \cdots, \nu\}]$ *and* $\xi \cap \{\gamma\} = \zeta \cap \{\gamma\} \; [\xi(\gamma) = \zeta(\gamma)]$;

(iii) $\quad t = \langle t_1, \cdots, t_n, t_0 \rangle$ and, *for all* $\alpha \in \Pi_i^n \mathscr{QT}_{t_i}$, $\xi(\alpha) =_{\gamma}^{t_0} \zeta(\alpha)$.

Let now $\mathfrak{I}$ be any ML^{ν}-interpretation. The set of all valuations of the variables of ML^{ν} in $\mathfrak{I}$ will be denoted by $\mathrm{Val}_{\mathfrak{I}}$. The following rules (d_1) to (d_8) define the *designatum* $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Delta)$ in $\mathfrak{I}$ of the wfe Δ, in correspondence with the valuation $\mathscr{V} \in \mathrm{Val}_{\mathfrak{I}}$. In these rules we assume $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Delta') = \tilde{\Delta}'$ for all subexpressions Δ' of Δ, and $x \in \mathscr{E}_t \, (t \in \tau^{\nu})$.

(d_1) $\quad \mathrm{des}_{\mathfrak{I}\mathscr{V}}(v_{tn}) = \mathscr{V}(v_{tn})$, $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(c_{tn}) = \mathscr{T}(c_{tn})$, $(t \in \tau^{\nu}, n \in Z^+)$;

(d_2) $\quad \mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Delta_1 = \Delta_2) = \{\gamma \in \Gamma : \tilde{\Delta}_1 =_{\gamma}^{t} \tilde{\Delta}_2\}$, $(\Delta_1, \Delta_2 \in \mathscr{E}_t)$;

(d_3) $\quad \mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Delta_0(\Delta_1, \cdots, \Delta_n)) = \tilde{\Delta}_0(\tilde{\Delta}_1, \cdots, \tilde{\Delta}_n)$;

$(d_{4,5})$ $\quad \mathrm{des}_{\mathfrak{I}\mathscr{V}}(\sim p) = \Gamma - \tilde{p}$; $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(p \wedge q) = \tilde{p} \cap \tilde{q}$;

(d_6) $\quad \mathrm{des}_{\mathfrak{I}\mathscr{V}}((x)\,p) = \{\gamma \in \Gamma :$ for all $\xi \in \mathscr{QT}_t, \gamma \in \mathrm{des}_{\mathfrak{I}\mathscr{V}'}(p)$ if $\mathscr{V}' = \mathscr{V}\binom{x}{\xi}\}$;

(2) In e.g. [3] to [5] and [12], $\langle t_1, \cdots, t_n, t_0 \rangle$ is denoted by $(t_1, \cdots, t_n)$ for $t_0 = 0$ and by $(t_1, \cdots, t_n : t_0)$ for $t_0 \in \tau^{\nu}$. Furthermore, $\mathscr{QT}_{(t_1,\ldots,t_n)}$ is defined to be a certain set isomorphic with the right hand side of (3.3).

(d_7) $\text{des}_{\Im\mathscr{V}}(\Box p) = \Gamma[\varnothing]$ if $\tilde{p} = \Gamma[\tilde{p} \neq \Gamma]$;

(d_8) $\text{des}_{\Im\mathscr{V}}((\iota x)\, p) =$ the only QI ζ such that:

(*a*) $\gamma \in \text{des}_{\Im\mathscr{V}}((\exists_1 x)\, p)$ and $\gamma \in \text{des}_{\Im\mathscr{V}'}(p)$ for $\mathscr{V}' = \mathscr{V}\binom{x}{\xi} \Rightarrow \zeta =^t_\gamma \xi$,

(*b*) $\gamma \in \text{des}_{\Im\mathscr{V}}(\sim (\exists_1 x)\, p) \Rightarrow \zeta =^t_\gamma a^\vee_t$.

The exact uniqueness of the QI ζ that fulfils (*a*) and (*b*) is proved in [2] (N 11); let us remark however that ζ (as well as other designata) may fail to be in $\bigcup_{t \in \tau^\vee} \mathscr{Q}\mathscr{T}_t$. In any case, this unsatisfactory situation does not happen when general ML$^\vee$-interpretations are dealt with—cf. def. 3.3 below—, and we shall consider only these interpretations.

As usual we say that a wff p is *true* in $\Im$ if $\text{des}_{\Im\mathscr{V}}(p) = \Gamma$ for all $\mathscr{V} \in \text{Val}_\Im$.

The language ML$^\vee$ has been constructed with a view to use it endowed with standard ML$^\vee$-interpretations (which are the most natural among the above interpretations). However, with respect to them, the completeness theorem obviously fails to hold, because the analogous fact occurs for extensional theories based on a type system—cf. [10] (3).

However, a wider class of ML$^\vee$-interpretations—the so-called *general interpretations* (cf. [10] for the extensional case)—can be defined, which turns out to be sound for the completeness of MC$^\vee$.

DEFINITION 3.2. *The* QI ξ *of type* $\langle t_1, \cdots, t_n, t_0\rangle$ *is said to be definable in the* ML$^\vee$*-interpretation* $\Im$ *if there exist* (i) *a* $\mathscr{V} \in \text{Val}_\Im$, (ii) *a finite set* $X = \{x_1, \cdots, x_n\}$ *of variables of type* $t_1, \cdots, t_n$, *respectively, and* (iii) *a wfe* $\Delta \in \mathscr{E}_{t_0}$, *such that*

$$\xi = d(\Delta, X, \Im, \mathscr{V}) = \left\{ \langle\langle \xi_1, \cdots, \xi_n\rangle, \text{des}_{\Im\mathscr{V}'}(\Delta)\rangle : \xi_i \in \mathscr{Q}\mathscr{T}_{t_i}\ (i = 1, \cdots, n) \quad \text{and} \quad \mathscr{V}' = \mathscr{V}\begin{pmatrix} x_1, \cdots, x_n \\ \xi_1, \cdots, \xi_n \end{pmatrix} \right\} .$$

DEFINITION 3.3. *The* ML$^\vee$*-interpretation* $\Im$ *is said to be general if, for all* $t \in \overline{\tau}^\vee$, *every* QI *of type t definable in* $\Im$ *belongs to* $\mathscr{Q}\mathscr{T}_t$.

Incidentally, if $\Im$ is a general ML$^\vee$-interpretation and $\mathscr{V} \in \text{Val}_\Im$, then the equality: $\text{des}_{\Im\mathscr{V}}((\lambda x_1, \cdots, x_n)\, \Delta) = d(\Delta, \{x_1, \cdots, x_n\}, \Im, \mathscr{V})$ holds for every wfe Δ and every set $\{x_1, \cdots, x_n\}$ of variables (4). This shows that even

(3) A theory $\mathscr{T}_N$ of natural numbers can be developed in MC$^\vee$ – c. [2] NN 27,45 –, and hence, by the Gödel's incompleteness theorem, there are infinitely many ways of choosing a formula p, true in a given standard model $\mathscr{M}$ and unprovable in MC$^\vee$. If $\mathscr{M}'$ is any other model, then the sets of the (intensional) designata in $\mathscr{M}$ and $\mathscr{M}'$, of the terms (or constants) of $\mathscr{T}_N$ are isomorphic in a suitable sense. Therefore p is true also in $\mathscr{M}'$.

(4) The proof of this statement is similar to that of Theor. 6.1. in [12]; and it is based on the fact that $(\exists_1 F)\,(\forall x_1, \cdots, x_n)\, \Box\, (F(x_1, \cdots, x_n) \equiv p)$ and $(\exists_1 f)\,(\forall x_1, \cdots, x_n)\, \Box\, f(x_1, \cdots, x_n) = \Delta$ are theorems of MC$^\vee$ for every wff p and every term Δ.

if $\mathfrak{I}$ is a non-standard general ML^{ν}-interpretation, λ-expressions have in it designata of a rather usual kind; in particular they cannot be the "non-existing object" as, a priori, could be surmised on the basis of (2.1).

The following theorem is proved (by a Henkin's method) in [12].

THEOREM 3.1. (*Completeness of* MC^{ν}). *Every wff* p *of* ML^{ν} *is provable in* MC^{ν} [*in a theory* $\mathscr{T}$ *based on* MC^{ν} *an having proper axioms*] *iff it is true in every general* ML^{ν}-*interpretation* [*general model of* $\mathscr{T}$].

4. GENERAL OPERATORS IN ML^{ν}

Let us assume that $t_1, \cdots, t_n \in \tau^{\nu}$ and $\vartheta, \varepsilon \in \overline{\tau}^{\nu}$, with either $n > 0$ or $n = 0 = \vartheta$. Intuitively an *operator* of type $w = (t_1, \cdots, t_n ; \vartheta, \varepsilon)$ is an expression Ω such that, if $\Delta \in \mathscr{E}_{\vartheta}$ and y_1 to y_n are n variables of the respective types t_1 to t_n, then $(\Omega y_1, \cdots, y_n)\,\Delta \in \mathscr{E}_{\varepsilon}$. It is natural and useful to identify such operators with the elements of $\mathscr{E}_w$, under the definition

$$(4.1) \qquad w = (t_1, \cdots, t_n ; \vartheta, \varepsilon) = \begin{cases} \langle\langle t_1, \cdots, t_n, \vartheta\rangle, \varepsilon\rangle & (n > 0), \\ \langle\langle 1, \vartheta\rangle, \varepsilon\rangle & (n = 0 = \vartheta). \end{cases}$$

Thus, first, we add (f_1) to (f_8) (N 2) with the formation rule (f_9). *If* (i) $\Omega \in \mathscr{E}_{(t_1,\ldots,t_n;\vartheta,\varepsilon)}$, (ii) y_1 *to* y_n *are* n *variables in* $\mathscr{E}_{t_1}$ *to* $\mathscr{E}_{t_n}$, *respectively, and* (iii) $\Delta \in \mathscr{E}_{\vartheta}$, *then* $(\Omega\, y_1, \cdots, y_n)\,\Delta \in \mathscr{E}_{\varepsilon}$ ($n > 0$, *or* $\vartheta = 0 = n$);

and, secondly, we add A2.1–17 with

$$\begin{matrix} \text{A4.1} \\ \text{A4.2} \end{matrix} \qquad (\Omega\, y_1, \cdots, y_n)\,\Delta \begin{cases} \equiv \\ = \end{cases} \Omega\,((\lambda\, y_1, \cdots, y_n)\,\Delta) \quad \textit{for } n > 0 \textit{ and} \begin{cases} \varepsilon = 0 \\ \varepsilon \in \tau^{\nu}, \end{cases}$$

$$\begin{matrix} \text{A4.3} \\ \text{A4.4} \end{matrix} \qquad (\Omega)\,\Delta \begin{cases} \equiv \\ = \end{cases} \Omega\,((\lambda\, y)\,\Delta) \quad \textit{for } n = 0 = \vartheta \quad \textit{and} \begin{cases} \varepsilon = 0 \\ \varepsilon \in \tau^{\nu}, \end{cases}$$

where (i) *to* (iii) *in* (f_9) *hold and* (iv) y *is the first variable of type* 1 *that fails to occur in* Δ. Let us call $\mathscr{ML}^{\nu}$ [$\mathscr{MC}^{\nu}$] what ML^{ν} [MC^{ν}] becomes by the addition of (f_9) [AA4.1–4].

In order to construct the semantics for $\mathscr{ML}^{\nu}$, we assume that every [every general] ML^{ν}-interpretation $\mathfrak{I}$ is also a [a general] $\mathscr{ML}^{\nu}$-interpretation and that, for all $\mathscr{V} \in \mathrm{Val}_{\mathfrak{I}}$,

$$(\mathrm{d}_9) \quad \begin{cases} \mathrm{des}_{\mathfrak{I}\mathscr{V}}((\Omega\, y_1, \cdots, y_n)\,\Delta) = \mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Omega\,((\lambda y_1, \cdots, y_n)\,\Delta)) & (n > 0), \\ \mathrm{des}_{\mathfrak{I}\mathscr{V}}((\Omega)\,\Delta) = \mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Omega\,(\lambda\, y\Delta)) \quad \textit{where (iv) holds} & (n = 0 = \vartheta). \end{cases}$$

In this way we can immediately conclude that *the analogue for* $\mathscr{ML}^{\nu}$ *and* $\mathscr{MC}^{\nu}$ *of Theorem 3.1, the completeness theorem, holds.*

The case $n = 0 = \vartheta$ is interesting by the following reasons. If F $[f]$ is a variable not free in the wff p, of the type $(; 0, 0)$ $[(; 0, t_{\mathbf{R}})$, where $t_{\mathbf{R}}$ is the type for real numbers], then a suitable $\mathscr{V}$ exists, for which (*a*) [(*b*)] below holds.

(*a*) $\mathrm{des}_{\mathfrak{I}\mathscr{V}}((\mathrm{F})\,p)$ equals any preassigned of the designata $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Box\, p)$ and $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(\Diamond\, p)$.

(*b*) $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(q) = \Gamma$, where q is $(f)\,p = 1 \wedge p \vee (f)\,p = 0 \wedge \sim p$.

In case (*b*) $\mathrm{des}_{\mathfrak{I}\mathscr{V}}((f)\,p)$ is the indicator of p (in $\mathfrak{I}$ and $\mathscr{V}$), i.e., the characteristic function of $\mathrm{des}_{\mathfrak{I}\mathscr{V}}(p)$.

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