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Holomorphic automorphisms of the tube domain over the Vinberg cone

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RENDICONTI

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SEZIONE I

(Matematica, meccanica, astronomia, geodesia e geofisica)

Geometria. — *Holomorphic automorphisms of the tube domain over the Vinberg cone.* Nota di LAURA GEATTI, presentata (*) dal Corrisp. E. VESENTINI.

RIASSUNTO. — In questo articolo si determina il gruppo di tutti gli automorfismi olomorfi del dominio tubolare sul cono di Vinberg.

Tale dominio ha dimensione complessa 5 ed è il dominio tubolare omogeneo non simmetrico di dimensione più bassa.

Si costruisce esplicitamente un gruppo transitivo di automorfismi olomorfi del dominio; successivamente, dimostrando che tale gruppo contiene l'intero sottogruppo di isotropia di un qualunque punto, si ottiene che esso coincide col gruppo di tutti gli automorfismi olomorfi del dominio.

INTRODUCTION

For bounded symmetric domains, the groups of holomorphic automorphisms have been completely determined by C.L. Siegel, U. Klingen, F. Hirzebruch [S].

In the case of bounded homogeneous non-symmetric domains the structure of such groups is essentially unknown.

In a forthcoming paper [G], we will determine the groups of all holomorphic automorphisms of a class of non-symmetric tubular domains over some affinely homogeneous non-selfadjoint cones. Such cones are characterized in

(*) Nella seduta del 10 maggio 1986.

terms of T-algebras and are the natural generalization of the « Vinberg cone »

$$V = \{Y = (y_1, y_2, y_3, y_4, y_5) \in \mathbf{R}^5 / y_1 y_3 - y_4^2 > 0, y_2 y_3 - y_5^2 > 0, y_3 > 0\}$$

and its dual cone.

The « Vinberg cone » was introduced by E.B. Vinberg in 1960 as the first example of a convex homogeneous non-selfadjoint cone and has the lowest dimension among such cones.

In this paper we will determine the group of all holomorphic automorphisms of the tube domain D (V), over the cone V, since the discussion of this particular case already contains all the ideas and the methods developed in [G].

A transitive group $G \subset \mathrm{Aut}(D(V))$ is first constructed, and then, by proving a « Schwarz Lemma », the entire isotropy subgroup $\mathrm{Aut}(D(V))_p$, at any point $p \in D(V)$, is determined.

The fact that $\mathrm{Aut}(D(V))_p \subset G_p$, ($G_p$ is the isotropy subgroup of p in G), implies that $\mathrm{Aut}(D(V)) = G$.

The proof of the Schwarz Lemma, which is the crucial point of this method, is obtained by computing the maximum and the minimum values of the holomorphic sectional curvature for the Bergman metric of D (V), at the point p.

These values, together with the invariance of the Bergman metric and a result of W. Kaup [KA], yield a complete description of the differential at p of any element of $\mathrm{Aut}(D(V))_p$.

§ 1. A transitive group on D (V).

We identify the real (complex) 5-dimensional euclidean space with the pairs of 2×2 symmetric matrices with a common corner element

$$(X_1, X_2) = \left(\begin{bmatrix} x_1 & x_4 \\ x_4 & x_3 \end{bmatrix}, \begin{bmatrix} x_2 & x_5 \\ x_5 & x_3 \end{bmatrix} \right) \quad , \quad x_i \in \mathbf{R}(\mathbf{C}),\ i = 1, \ldots, 5 .$$

The « Vinberg cone » and the corresponding tube domain are then respectively defined as

$$V = \{(V_1, V_2) \in \mathbf{R}^5 / V_1, V_2 \in H^+(2, \mathbf{R})\} \text{ and}$$

$$D(V) = \{(Z_1, Z_2) \in \mathbf{C}^5 / Y_i \in H^+(2, \mathbf{R}),\ Y_i = \mathrm{Im}(Z_i),\ i = 1, 2\},$$

where $H^+(2, \mathbf{R})$ denotes the cone of 2×2 real symmetric positive definite matrices.

Since V is a homogeneous non-selfadjoint cone, D (V) is a homogeneous non-symmetric tube domain [GPV].

In this section, we explicitely construct a transitive group G of holomorphic automorphisms of D (V). The group G will later be shown to coincide with the full group Aut (D (V)).

We start by considering the transformations of $\mathbf{C}^5$

$$T = \{t_{B_1 B_2} : (Z_1, Z_2) \rightarrow (Z_1 + B_1, Z_2 + B_2), B_1, B_2 \in H(2, \mathbf{R})\},$$

$$GL = \{gl_{A_1 A_2} : (Z_1, Z_2) \rightarrow (A_1 Z_1 {}^t A_1, A_2 Z_2 {}^t A_2), A_1, A_2 \in GL(2, \mathbf{R}),$$

$$A_1 \rightarrow \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, A_2 = \begin{bmatrix} d & e \\ 0 & c \end{bmatrix}\},$$

$$R = \{r_C : (Z_1, Z_2) \rightarrow (Z_1 (CZ_1 + I_2)^{-1}, Z_2 (CZ_2 + I_2)^{-1}), C = \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix}, c \in \mathbf{R}\},$$

$$\phi : (Z_1, Z_2) \rightarrow (Z_2, Z_1) \tag{1.1}$$

(${}^tA$ denotes the transpose of the matrix A and I_n the identity matrix of order n).

It is easy to verify that all the above transformations define holomorphic automorphisms of D (V).

We denote by G the group generated by T, GL, R and ϕ, and we denote by S the subgroup generated by T, GL and R.

Then:

a) G acts transitively on D (V).

b) G is isomorphic to the semi-direct product of S and (ϕ), where (ϕ) is the subgroup of order 2 generated by ϕ.

c) S can be identified with the subgroup of $Sp(2, \mathbf{R}) \times Sp(2, \mathbf{R})$, whose elements are the pairs of symplectic matrices of the form

$$(S_1, S_2) = \left(\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}, \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\right)$$

satisfying the conditions

$$A_i = \begin{bmatrix} a_{i1} & a_{i2} \\ 0 & a_{i4} \end{bmatrix} B_i = \begin{bmatrix} b_{i1} & b_{i2} \\ b_{i3} & b_{i4} \end{bmatrix} C_i = \begin{bmatrix} 0 & 0 \\ 0 & c_{i4} \end{bmatrix} D_i = \begin{bmatrix} d_{i1} & 0 \\ d_{i3} & d_{i4} \end{bmatrix},$$

$i = 1, 2$, and $\quad a_{14} = a_{24}, b_{14} = b_{24}, c_{14} = c_{24}, d_{14} = d_{24}$.

The group homomorphism given by

$$p : (S_1, S_2) \rightarrow [(A_1 Z_1 + B_1)(C_1 Z_1 + D_1)^{-1}, (A_2 Z_2 + B_2)(C_2 Z_2 + D_2)^{-1}]$$

is surjective and kernel $(p) = \{\pm (I_4, I_4)\}$.

We now fix the point $(iI, iI) \in D(V)$ and denote by $G_{(iI, iI)}$ the isotropy subgroup of (iI, iI) in G. From *b*) and *c*) we obtain:

PROPOSITION 1.1. $G_{(iI,iI)}$ *is the subgroup generated by* ϕ *and by the transformations of the form*

$$(1.2)\qquad g_\theta : (Z_1, Z_2) \to \left(\begin{bmatrix} z_1 + \dfrac{\sin\theta\, z_4^2}{(\cos\theta - z_3 \sin\theta)} & \pm \dfrac{z_4}{(\cos\theta - z_3\sin\theta)} \\ \pm \dfrac{z_4}{(\cos\theta - z_3\sin\theta)} & \dfrac{(\sin\theta + z_3\cos\theta)}{(\cos\theta - z_3\sin\theta)} \end{bmatrix}, \begin{bmatrix} z_2 + \dfrac{\sin\theta\, z_5^2}{(\cos\theta - z_3 \sin\theta)} & \pm \dfrac{z_5}{(\cos\theta - z_3\sin\theta)} \\ \pm \dfrac{z_5}{(\cos\theta - z_3\sin\theta)} & \dfrac{(\sin\theta + z_3\cos\theta)}{(\cos\theta - z_3\sin\theta)} \end{bmatrix} \right),$$

or $\theta \in [0, 2\pi]$. *In particular,* $G_{(iI,iI)}$ *is isomorphic to a semi-direct product of* Z_2 *by the compact Lie group* $S^1 \times 0(1) \times 0(1)$. *(Here* Z_2 *denotes a group of order two and* $0(1)$ *the orthogonal group of order one).*

COROLLARY 1.2. *Let* $g \in G_{(iI,iI)}$. *The jacobian matrix of* g, *at* (iI, iI), *is given either by the matrix*

$$(1.3)\qquad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & e^{i\theta} & 0 & 0 \\ 0 & 0 & 0 & \pm e^{i2\theta} & 0 \\ 0 & 0 & 0 & 0 & \pm e^{i2\theta} \end{bmatrix}$$

or by the product of (1.3) *and*

$$(1.4)\qquad d\phi = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

§ 2. A SCHWARZ LEMMA FOR D (V).

In this section we determine the entire isotropy subgroup

$$\mathrm{Aut}\,(D\,(V))_{(iI,iI)}\ ;$$

in particular we show that

$$\mathrm{Aut}\,(\mathrm{D}\,(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})} = \mathrm{G}_{(i\mathrm{I},i\mathrm{I})}\,,$$

obtaining as a consequence that

$$\mathrm{Aut}\,(\mathrm{D}\,(\mathrm{V}) = \mathrm{G}\,.$$

The proof of these facts proceeds along the following line:

a) starting from the Bergman kernel of D (V), we compute the components of the metric tensor and of the Riemann curvature tensor at the point $(i\mathrm{I}\,,i\mathrm{I})$ (cf. [KO]);

b) we determine the maximum and the minimum values of the holomorphic sectional curvature on the tangent space at $(i\mathrm{I}\,,i\mathrm{I})$;

c) we use the invariance properties of the Bergman metric and of the holomorphic sectional curvature to show that the jacobian matrix of a generic element in $\mathrm{Aut}\,(\mathrm{D}\,(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})}$, at $(i\mathrm{I}\,,i\mathrm{I})$, is given by (1.3) or by a product of (1.3) and (1.4).

Cartan's uniqueness theorem completes the argument [N].

The Bergman kernel function of D (V) is

$$\mathrm{K}\,((\mathrm{Z}_1\,,\mathrm{Z}_2)\,,(\overline{\mathrm{Z}}_1\,,\overline{\mathrm{Z}}_2)) = \frac{c\,y_3^2}{(y_1\,y_3 - y_4^2)^3\,(y_2\,y_3 - y_5^2)^3} \quad (c \in \mathbf{R}\,,c > 0)\,. \quad (\text{cf. [V]})$$

Direct computations show that, at the point $(i\mathrm{I}\,,i\mathrm{I}) \in \mathrm{D}\,(\mathrm{V})$, the only non-zero components of the Bergman metric are

$$g_{1\bar{1}} = g_{2\bar{2}} = 3/4 \qquad g_{3\bar{3}} = 1 \qquad g_{4\bar{4}} = g_{5\bar{5}} = 3/2\,, \tag{2.1}$$

while the non-zero elements which determine the Riemann curvature tensor are

$$\begin{aligned}
&\mathrm{R}_{1\bar{1}1\bar{1}} = \mathrm{R}_{1\bar{4}3\bar{4}} = \mathrm{R}_{1\bar{1}4\bar{4}} = -\,3/8\\
&\mathrm{R}_{2\bar{2}2\bar{2}} = \mathrm{R}_{2\bar{5}3\bar{5}} = \mathrm{R}_{2\bar{2}5\bar{5}} = -\,3/8\\
&\mathrm{R}_{3\bar{3}3\bar{3}} = -\,1/2 \qquad \mathrm{R}_{3\bar{3}4\bar{4}} = \mathrm{R}_{3\bar{3}5\bar{5}} = -\,3/8\\
&\mathrm{R}_{4\bar{4}4\bar{4}} = \mathrm{R}_{5\bar{5}5\bar{5}} = -\,15/16 \qquad \mathrm{R}_{4\bar{4}5\bar{5}} = 9/16
\end{aligned}$$

(the other non-zero components are given by the symmetry identities).

If $\mathrm{B} = \{\xi = (\xi^1\,,\xi^2\,,\xi^3\,,\xi^4\,,\xi^5) \in \mathbf{C}^5 / 3/4\,(|\,\xi^1\,|^2 + |\,\xi^2\,|^2) + |\,\xi^3\,|^2 + 3/2\,(|\,\xi^4\,|^2 + |\,\xi^5\,|^2) = 1\}$,

denotes the unit sphere for the Bergman metric in the tangent space to

D (V), at $(i\mathrm{I}, i\mathrm{I})$, the holomorphic sectional curvature, along the vector $\xi \in \mathrm{B}$, is given by

$$\begin{aligned} \mathrm{C}(\xi) = &-3/8 \,|\xi^1|^4 - 3/2 \,\mathrm{Re}\, \xi^1 \xi^3 (\bar{\xi}^4)^2 - 3/2 \,|\xi^1|^2 |\xi^4|^2 - 3/8 \,|\xi^2|^4 \\ &- 3/2 \,\mathrm{Re}\, \xi^2 \xi^3 (\bar{\xi}^5)^2 - 3/2 \,|\xi^2|^2 |\xi^5|^2 - 1/2 \,|\xi^3|^4 - 3/2 \,|\xi^3|^2 |\xi^4|^2 - \\ &- 3/2 \,|\xi^3|^2 |\xi^5|^2 - 15/16 \,|\xi^4|^4 - 15/16 \,|\xi^5|^4 + 9/8 \,\mathrm{Re}\, (\xi^4)^2 (\bar{\xi}^5)^2 . \end{aligned}$$

LEMMA 2.1. *The maximum value of* C *is* $\mathrm{M} = -1/12$, *and all the vectors at which the maximum is attained are*

$$\{(0, 0, 0, \xi^4, \xi^5) : |\xi^4|^2 = |\xi^5|^2 = 1/3\} .$$

The minimum is $m = -2/3$ *and all the vectors where the minimum is assumed are*

$$\{(\xi^1, 0, 0, 0, 0) : |\xi^1|^2 = 4/3 \text{ and } (0, \xi^2, 0, 0, 0) : |\xi^2|^2 = 4/3\} .$$

The proof of this Lemma, although elementary, is rather complicated and is omitted here. It can be found in [G], as a particular case of a more general statement.

PROPOSITION 2.2. *The stability subgroup* $\mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I}, i\mathrm{I})}$ *coincides with the group generated by the transformations* g_θ *and* ϕ, *defined in* (1.2) *and* (1.1). *In particular, it is isomorphic to a semidirect product of* Z_2 *by the compact Lie group* $\mathrm{S}^1 \times 0(1) \times 0(1)$.

Proof: Let g be a generic element in $\mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I}, i\mathrm{I})}$ and let $dg_{(i\mathrm{I}, i\mathrm{I})} = \{a_{ij} = \partial g_i / \partial z_j (i\mathrm{I}, i\mathrm{I})\}_{i,j=1,\dots,5}$ be the jacobian matrix of g at $(i\mathrm{I}, i\mathrm{I})$.
We shall show that $dg_{(i\mathrm{I}, i\mathrm{I})}$ is given either by the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & e^{2i\theta} & 0 & 0 \\ 0 & 0 & 0 & \pm e^{i\theta} & 0 \\ 0 & 0 & 0 & 0 & \pm e^{i\theta} \end{bmatrix} \tag{2.2}$$

or by the product of (2.2) and (1.3).

The proof of the proposition is divided into several steps, in which the various coefficients a_{ij} are determined.

a) By the invariance of the holomorphic sectional curvature, Lemma 2.1 implies that either

$$a_{i1} = a_{i2} = 0 \qquad i > 2$$
$$a_{i4} = a_{i5} = 0\,, \qquad i = 1\,,2\,,3$$
$$a_{45} = a_{54} = 0$$
$$a_{44} = e^{i\theta}\,,\ a_{55} = e^{i\tau}\,,\ \theta\,,\tau \in [0\,,2\pi]$$

or

$$a_{i1} = a_{i2} = 0\,, \qquad i > 2$$
$$a_{i4} = a_{i5} = 0\,, \qquad i = 1\,,2\,,3$$
$$a_{44} = a_{55} = 0$$
$$a_{45} = e^{i\theta}\,,\ a_{54} = e^{i\tau}\,,\ \theta\,,\tau \in [0\,,2\pi]\,.$$

Moreover, the matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ must coincide either with

$$\begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 & e^{i\alpha} \\ e^{i\beta} & 0 \end{bmatrix}, \text{ for } \alpha\,,\beta \in [0\,,2\pi].$$

b) Since, the scalar product induced by the Bergman metric on the tangent space at $(i\mathrm{I}\,,i\mathrm{I})$ must be preserved by $dg_{(i\mathrm{I},i\mathrm{I})}$, it follows from (2.1) that

$$a_{13} = {}_{23} = a_{43} = a_{53} = 0$$
$$a_{33} = e^{i\lambda} \quad , \quad \lambda \in [0\,,2\pi]\,.$$

c) By the invariance of the holomorphic sectional curvature, evaluated on the vectors

$$\xi = (0\,,\xi^2\,,0\,,\xi^4\,,0) \text{ and } \chi = (\chi^1\,,0\,,0\,,0\,,\chi^5)$$

$dg_{(i\mathrm{I},i\mathrm{I})}$ has one of the two following expressions

$$\begin{bmatrix} e^{i\alpha} & 0 & 0 & 0 & 0 \\ 0 & e^{i\beta} & 0 & 0 & 0 \\ 0 & 0 & e^{i\lambda} & 0 & 0 \\ 0 & 0 & 0 & e^{i\theta} & 0 \\ 0 & 0 & 0 & 0 & e^{i\tau} \end{bmatrix}, \begin{bmatrix} 0 & e^{i\alpha} & 0 & 0 & 0 \\ e^{i\beta} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{i\lambda} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{i\theta} \\ 0 & 0 & 0 & e^{i\tau} & 0 \end{bmatrix}. \tag{2.3}$$

In the same way, starting from the vectors

$$\xi = (\xi^1\,,0\,,\xi^3\,,\xi^4\,,0)\,, \quad \zeta = (0\,,\zeta^2\,,\zeta^3\,,0\,,\zeta^5)\,, \chi = (0\,,0\,,0\,,\chi^4\,,\chi^5)$$

the relations $\alpha = \beta$, $\lambda = 2\theta - \alpha$, $2\theta \equiv 2\tau \pmod{2\pi}$ are obtained.

Now, all that remains to be shown is that $\alpha = 0$.

Remark 2.3. The elements of $\mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})}$ which are linear, are given by the real matrices of type (2.3) preserving the cone V. An easy check shows that they only occur for $\alpha = 0$, $\theta = 0$, 2π.

Remark 2.4. Remark 2.3 implies the inclusion

$$\mathrm{GL}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})} = \mathrm{Aff}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})} \subset \mathrm{G}_{(i\mathrm{I},i\mathrm{I})}$$

and, since G already contains a transitive subgroup of affine transformations of D (V), it implies also

$$\mathrm{GL}(\mathrm{D}(\mathrm{V})) \subset \mathrm{G} \text{ and } \mathrm{Aff}(\mathrm{D}(\mathrm{V})) \subset \mathrm{G}.$$

(Aff (D (V)) is the subgroup of affine automorphisms of D (V)).

In order to show that $\alpha = 0$, by multiplying matrices in (2.3) by suitable matrices in (1.3) and (1.4), we reduce our discussion to the elements of the form

$$\begin{bmatrix} e^{i\alpha} & 0 & 0 & 0 & 0 \\ 0 & e^{i\alpha} & 0 & 0 & 0 \\ 0 & 0 & e^{-i\alpha} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \alpha \in [0, 2\pi]. \tag{2.3}$$

We first observe that $\alpha \neq (2p+1)/q$, $p, q \in \mathrm{Z}$, $q \neq 0$, since otherwise there would exist an automorphism $h \in \mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})}$ with $dh_{(i\mathrm{I},i\mathrm{I})} = -\mathrm{Id}$, having $(i\mathrm{I}, i\mathrm{I})$ as an isolated fixed point.

Such an automorphism would be a symmetry of D (V) at $(i\mathrm{I}, i\mathrm{I})$, which is absurd.

On the other hand, since the subgroup $\mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})}$ is compact, α can assume only a finite number of values.

In this way $\mathrm{Aut}(\mathrm{D}(\mathrm{V}))_{(i\mathrm{I},i\mathrm{I})}$ is generated by $\mathrm{G}_{(i\mathrm{I},i\mathrm{I})}$ and by a finite number of elements $h_\alpha \notin \mathrm{G}_{(i\mathrm{I},i\mathrm{I})}$ $(\alpha \neq 0)$ while the group Aut (D (V)) is generated by G and by the same elements h_α. In particular, G is union of connected components of Aut (D (V)). For each h_α, $\alpha \neq 0$, $h_\alpha \mathrm{G}$ defines then a connected component of Aut (D (V)) containing no affine transformations. This is in contradiction with a result by W. Kaup [KA], asserting that Aff (D (V)) has a non-empty intersection with each connected component of Aut (D (V)), and concludes the proof.

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