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Sea waves and quasi-determinism of rare events in random processes

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Oceanografia. — *Sea waves and quasi-determinism of rare events in random processes.* Nota di PAOLO BOCCOTTI (*), presentata (**) dal Corrisp. E. MARCHI.

RIASSUNTO. — Alcune registrazioni di onde effettuate nella stazione di misura entro la diga foranea di Genova, producono nuove conferme di una tendenza delle onde più alte verso forme di quasi-determinismo, similmente a quanto è stato recentemente trovato dall'autore per i processi stazionari gaussiani.

NOTATION

In what follows whe shall denote by $\psi(T)$ the autocovariance of a random function. The abscissa of the absolute minimum of ψ, if it exists, will be denoted by T^*. While $\mathscr{T}^*$ will be the interval between the first zero-upcrossing before the origin and the first zero-upcrossing after the origin of the function

$$\psi(T) - \psi(T - T^*) .$$

Parameters T^* and $\mathscr{T}^*$ play an important role in wave statistics of gaussian processes. The ratio between the minimum and the maximum of the autocovariance will be denoted by the symbol a:

$$a = \frac{\psi(T^*)}{\psi(0)} . \tag{1}$$

The parameter a is negative, ranging from - 1 for infinitely narrow energy spectra, to zero for very broad spectra. Finally m_0 will be the lowest moment and T_d the dominant period of the energy spectrum. Clearly for the definitions of autocovariance and energy spectrum it follows that

$$m_o = \psi(0) . \tag{2}$$

As the zero-upcrossing definition of an individual wave will be followed, the wave crest will be the highest relative maximum and the wave trough the

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lowest relative minimum within two consecutive zero-upcrossings. By wave height we mean the difference of crest and trough ordinates, by wave period the interval between two consecutive zero-upcrossings.

A few recently found properties of gaussian processes

Recently it has been shown by the Author [1], [2] that very high waves in stationary gaussian random processes obey a set of quasi-deterministic rules. Two of those rules are:

I) *Given a positive number* β *and taking at random a wave whose crest ordinate is equal to* $\beta\sqrt{m_o}$, *this will have the profile*

$$\zeta(\mathrm{T}) = \frac{\psi(\mathrm{T})}{\psi(0)}\,\beta\sqrt{m_o} \tag{3}$$

with probability approaching one, provided that β *tends to infinity.*

II) *Given a positive number* α *and taking at random a wave whose height is greater than* $\alpha\sqrt{m_o}$, *this will have a period*

$$\mathrm{T}_x = \mathscr{T}^* \tag{4}$$

with probability approaching one, provided that α *tends to infinity.*

The gist of the above properties is: when a condition with an infinitely low probability density obtains, it nearly always shows constant characteristics Hence the term ' quasi-determinism '.

From a set of quasi-deterministic properties a further rule emerges:

III) *Given a positive* α, *the probability that a wave taken at random has an height greater than* $\alpha\sqrt{m_o}$ *is*

$$\mathrm{P}(\alpha) = \exp\left[-\frac{\alpha^2}{4(1-a)}\right] \tag{5}$$

provided that α *tends to infinity.*

The above properties strictly require only: $\psi(\mathrm{T})$ should be twice derivable and have an absolute minimum.

When the energy spectrum is infinitely narrow, the autocovariance $\psi(\mathrm{T})$ approaches a cosine so that

$$a = \frac{\psi(\mathrm{T}^*)}{\psi(0)} = -1 \tag{6}$$

and eq. (5) reduces to

$$P(\alpha) = \exp\left(-\frac{\alpha^2}{8}\right) \tag{7}$$

which is a well known equation due to the basic work of Rice [3] (see Cartwright & Longuet-Higgins [4]).

Properties II and III denote apparent connexions with open sea waves (see paper [1]). The next section shows that these similarities with sea waves apply even under more complex boundary conditions.

SOURCE AND NATURE OF DATA

A caisson of Genoa breakwater at 25 m depth is equipped for measuring wave pressures. Facilities were first described by Marchi *et al.* [5]. They consist of a laboratory room 3.2 × 3.0 × 22 m (see fig. 1) with 10 windows

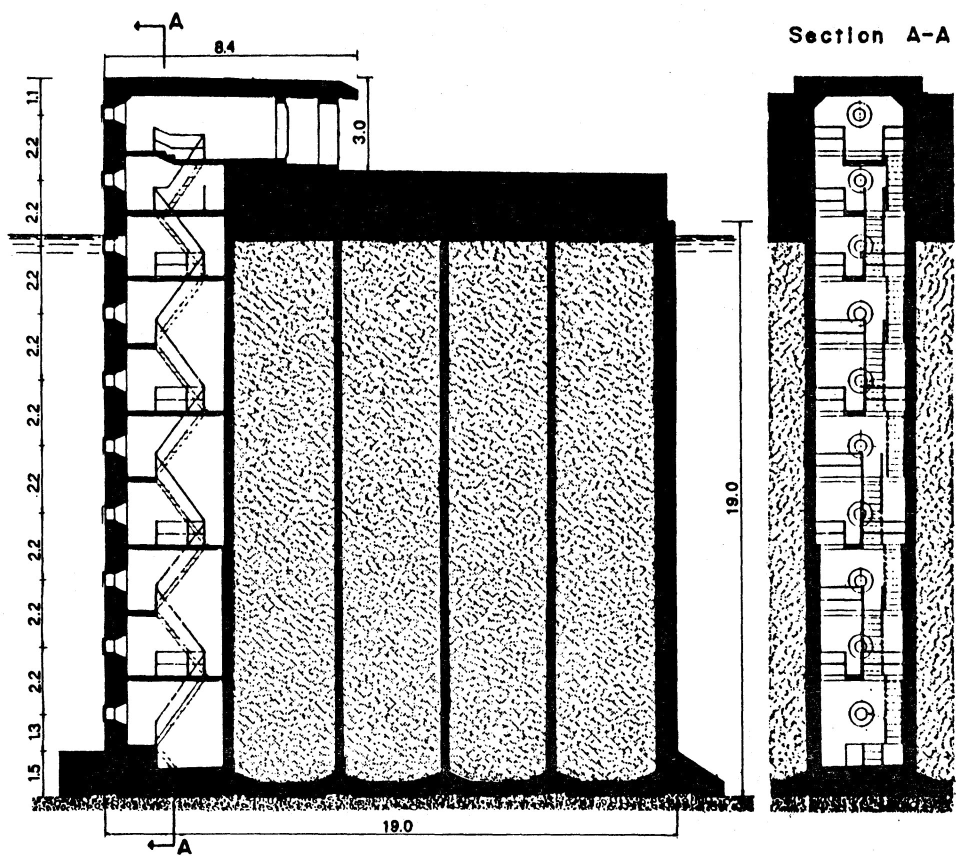

Fig. 1. – Station for measuring wave pressure against Genoa breakwater. The caisson is based on a rubble mound. Depth of the caisson base is nearly 17 m, depth of natural bottom is 25 m.

facing seawards. The windows are closed by bronze flanges. A tube crosses each flange and connects the sea and the laboratory room, permitting measurement of the pressure at various depths in nearly ideal laboratory conditions.

The data are now digitized in the laboratory room at a rate of 2/s.

Below we shall consider two different kinds of sea states recorded by the station. One consisted of storm waves generated by stable winds nearly parallel to the dam front. The second consisted of swells normal to the breakwater. Two records were taken during the storm sea (records A and B, 10 9 1983, h. 12 and h. 15) and one record during swells (record C, 12.9.1983, h. 9).

Waves were calculated through a double extrapolation formula

$$\zeta_{iii} = \frac{2\, p_{iii}}{4\, p_{ii} - 3\, p_{iii} - p_i}\, \Delta z \,. \tag{8}$$

Where *i*, *ii* and *iii* denote the three windows, for increasing ordinates, just below the free surface, p_i, p_{ii} and p_{iii} are the pressures on those windows, ζ_{iii} is the free surface elevation above the window *iii* and Δz is the interaxis between two windows, that is $2.2\,m$ (see fig. 1).

Eq. (8) should yield satisfactory degrees of precision for waves of the size concerned, significant wave heights and periods such that

$$1.5\,m < \mathrm{H}_{1/3} < 2.0\,\mathrm{m} \;,\; 5.0 < \mathrm{T}\,(\mathrm{H}_{1/3}) < 7.5\,s\,. \tag{9}$$

For regular waves to the third order, both progressive (Skjelbreia [6]) and stationary (Carry and Chabert D'Hieres [7]) having the above parameters, eq. (8) involves a possible error less than 2% on wave height. Negligible errors on wave periods.

Data analysis

Figs. 2 to 4 show the autocovariances $\psi\,(\mathrm{T})$ for the three sea states. They have been calculated directly from ζ, the elevation of free surface above M.W.L.:

$$\psi\,(\mathrm{T}) = \langle\, \zeta\,(t) \cdot \zeta\,(t + \mathrm{T})\,\rangle \tag{10}$$

where angle brackets denote the mean values. Values of $\psi\,(\mathrm{T})$ evaluated for increments $\Delta\mathrm{T} = 0.5\,s$ have been interpolated according to Fourier.

The ratio a between the absolute minimum and absolute maximum of $\psi\,(\mathrm{T})$ has been marked. In the two storm seas

$$a \sim -0.7 \tag{11}$$

that seems to be a very characteristic value of storm seas, recalling that

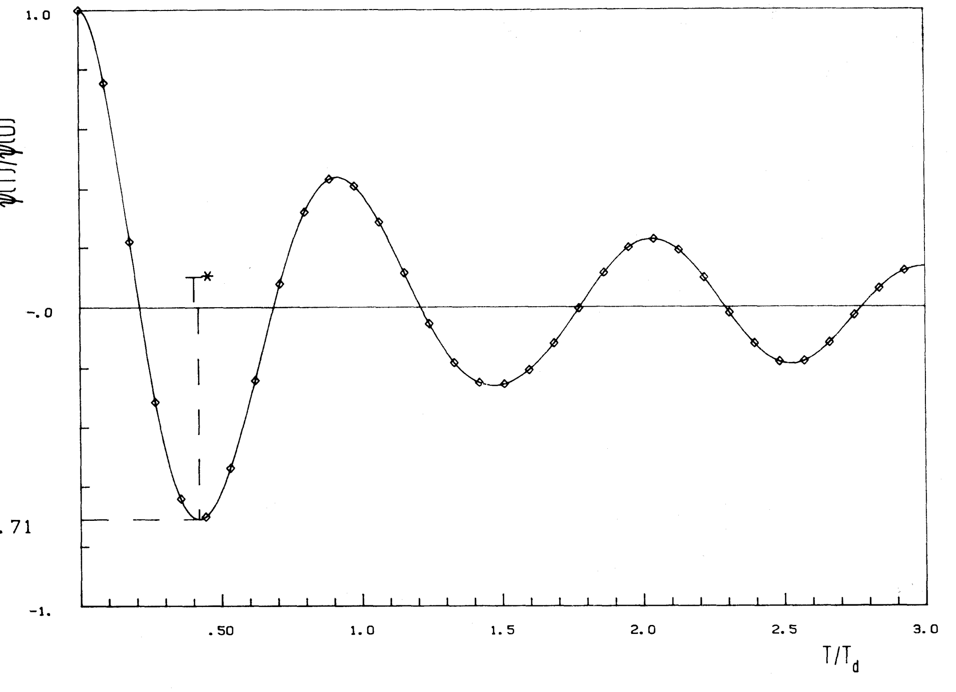

Fig. 2. – Record A: autocovariance of the free surface.

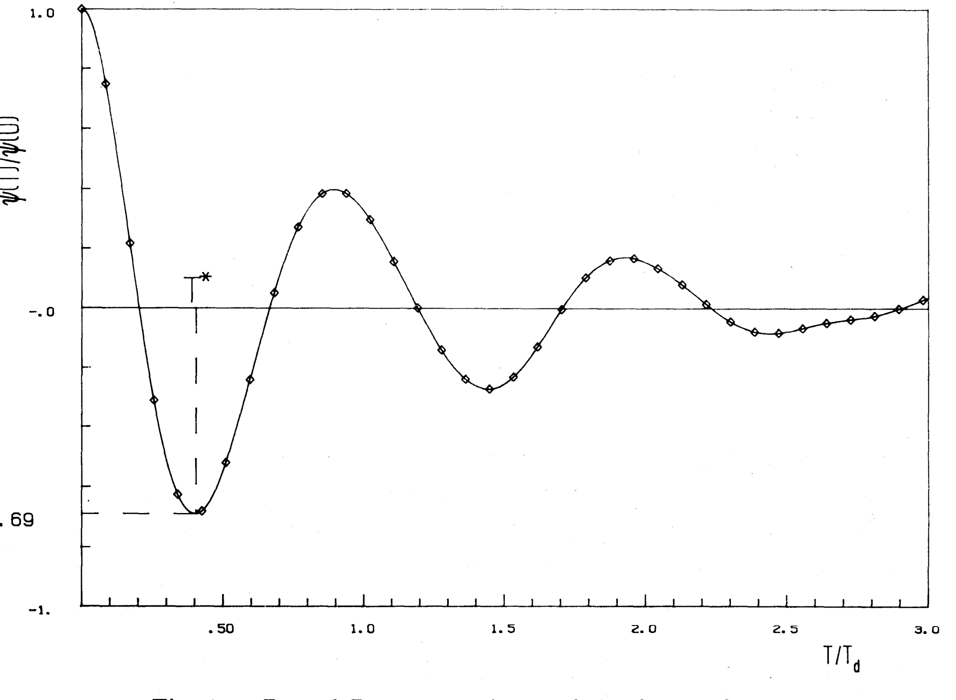

Fig. 3. – Record B: autocovariance of the free surface.

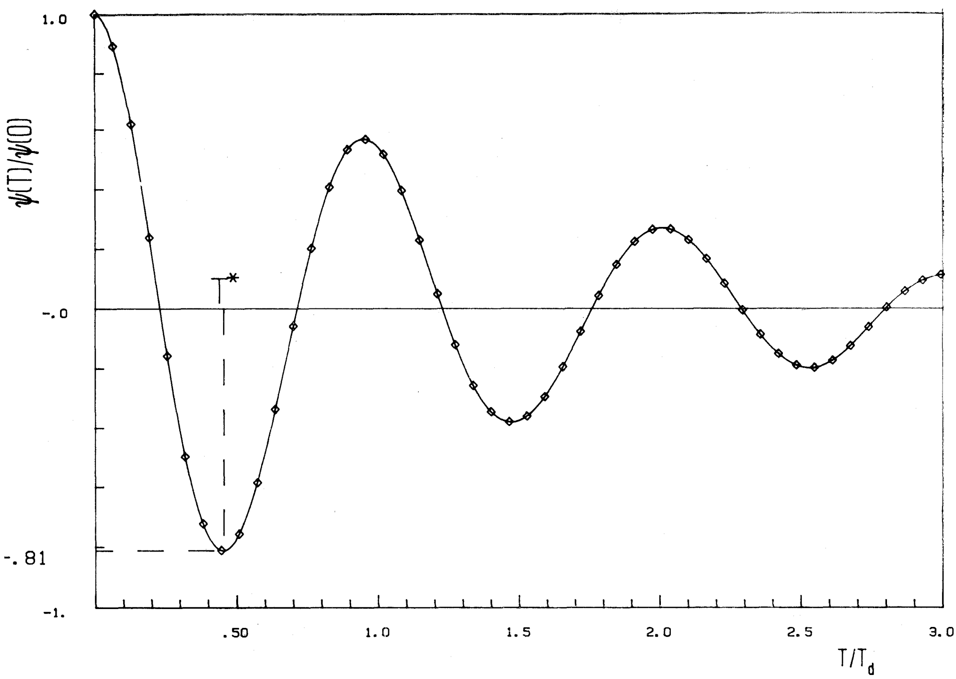

Fig. 4. – Record C: autocovariance of the free surface.

(12.*a*) for Pierson & Moskowitz' spectrum [8]: $a = -0.65$,

(12.*b*) for JONSWAP spectrum [9]: $a = -0.73$.

A lower value ($a = -0.81$) appears in the third record for swells, which usually tend to present narrower spectra

Characteristic periods are

Record	$\mathcal{T}^*$	T_d
A	5.00	5.63
B	5.02	5.87
C	7.36	7.85

Figs 5 and 6 show the probability of excess P (α) for wave height α (that is related to the standard deviation of the free surface). Continuous lines represent expressions (7) and (5) with appropriate values of the parameter a. Waves from storm seas have been grouped in a single set—fig. 5—because parameters a for those seas are very close to each other ($\simeq -0.7$).

It appears that in both cases, fig. 5 and fig. 6, the highest waves are smaller than predicted by the classic eq. (7), line 1. This agrees with most measurements in the last few years (see among others Borgman [10], Haring *et al.*, [11],

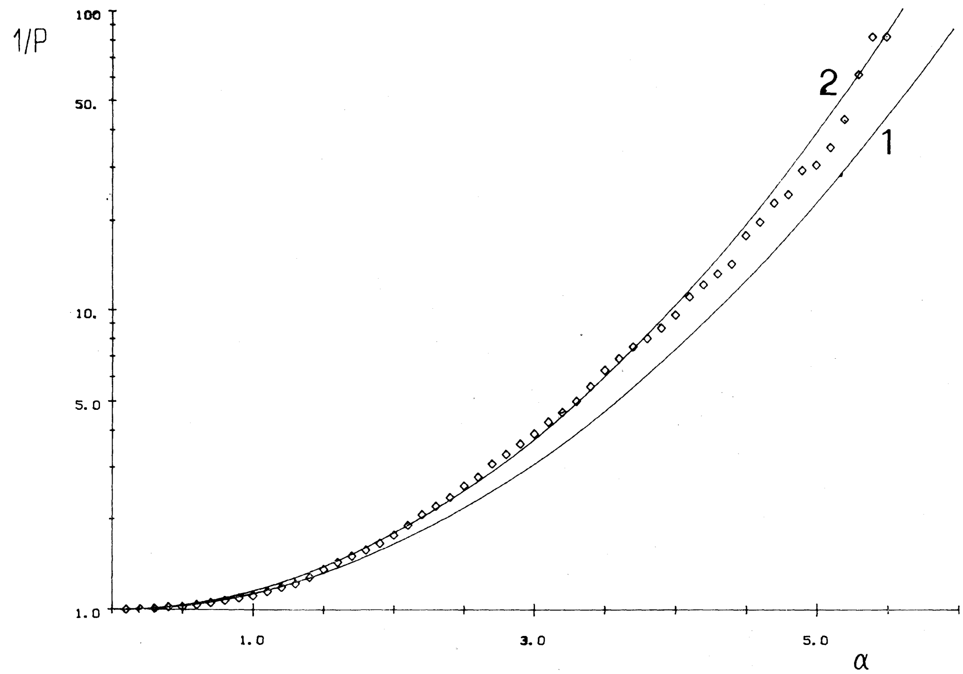

Fig. 5. – Storm seas: probability P (α) that a wave taken at random is greater than $\alpha\ \sqrt{m_0}$. Line 1 represents eq. (7) and line 2 eq. (5) with $a = -0.7$.

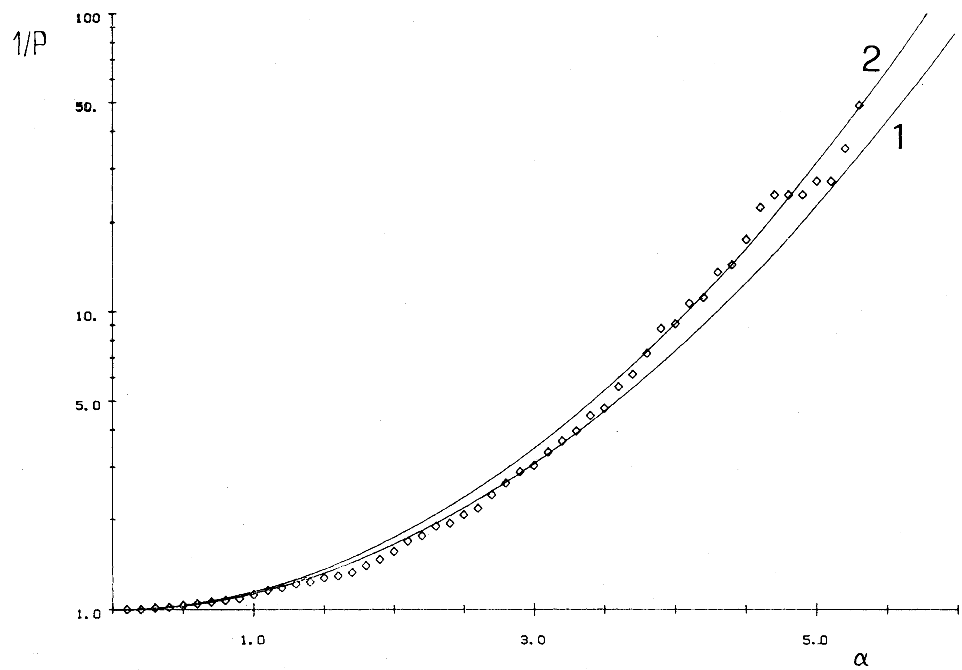

Fig. 6. – Swells: probability P (α) that a wave taken at random is greater than $\alpha\ \sqrt{m_0}$. Line 1 represents eq. (7) and line 2 eq. (5) with $a = -0.81$.

Forristall [12] and Longuet-Higgins [13]). Eq. (5) seems to present a discrepancy from eq. (7) of the same order as the data. This at least in the range of the highest waves, as occurs in gaussian processes (property III).

Finally fig. 7 shows the set of all measured waves, in the abscissa: nondimensional wave height α, in the ordinate: nondimensional wave period τ_z ($\tau_z =$ = ratio between zero-upcrossing period T_z and parameter $\mathcal{T}^*$).

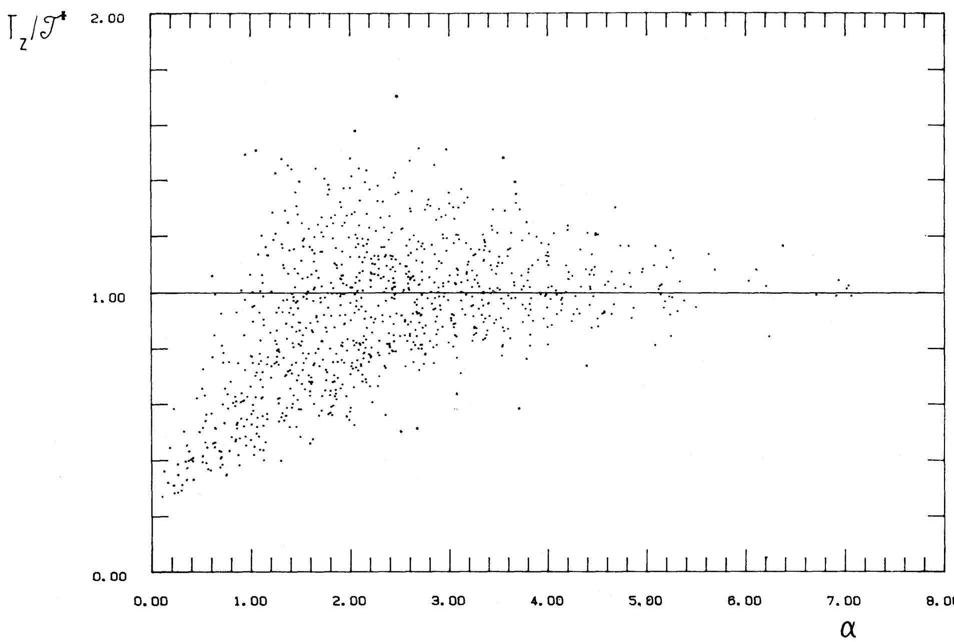

Fig. 7. – Storm sea and swells: nondimensional wave periods versus nondimensional wave heights.

From property II of gaussian processes, a wave with a very large nondimensional height α has a nondimensional period τ_z equal to 1, with probability close to one. A similar property seems to emerge from the data in fig. 7.

Conclusions and comment

The Author has shown that very high waves in stationary gaussian processes obey a set of compact quasi-deterministic rules. The similarity between this finding and the known properties of highest waves in storm seas was immediately apparent. Further evidence has been supplied here through an analysis of sea waves under rather complex boundary conditions.

It should be noted that sea waves are overbounded, in that an exceptionally high wave, say 20 times the standard deviation of the free surface elevation, in most cases has no chance to exist since it would break. Thus we cannot investigate whether quasi-determinism in fact holds for sea waves and we must confine ourselves in taking note of a few apparent trends like that of fig. 7.

Finally it should be emphasized that gaussian processes and sea waves differ markedly in many respects, especially in the range of the highest waves, a point that has been raised many times in the literature. Thus a proved tendency of the highest sea waves towards a few quasi-deterministic rules of stationary gaussian processes suggests that those rules could apply to other random processes.

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