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Astronomia. — *Capture of an incoming body by a circumsolar disc.* Nota di VITTORIO BANFI, LUCIANA FENOGLIO e MARIO GIROLAMO FRACASTORO, presentata (*) dal Socio M.G. FRACASTORO.

RIASSUNTO. — A proposito di una vecchia teoria di J.J. See, secondo la quale il sistema solare si sarebbe formato per cattura dei pianeti da parte del sole, H. Poincaré (1911) scriveva che questo processo richiederebbe la presenza di un inviluppo attorno al sole, corotante con esso.

Poiché le attuali teorie astrofisiche prevedono l'esistenza di un tale inviluppo nelle prime fasi evolutive di una stella di tipo solare e si dànno anche valori, sia pure discordanti da un autore all'altro, circa la struttura e la durata di vita di questo inviluppo, è sembrato interessante di riconsiderare tale ipotesi di cattura. Naturalmente, si è preso in esame l'impatto di piccoli corpi di dimensioni asteroidali provenienti da orbite paraboliche. Essi agirebbero come « embrioni » all'interno del disco, da esso rastrellando materia.

L'effetto del frenamento è quello di una diminuizione graduale dell'eccentricità, del semiasse maggiore e del periodo, nonché dell'inclinazione i rispetto al piano equatoriale del disco. Si sono studiati quantitativamente gli effetti di questa decelerazione, in funzione della massa e della densità dell'oggetto impattante, nonché dei vari parametri adottati dai più recenti autori circa la costituzione e la struttura del disco. Ne risulta in particolare che oggetti in arrivo su orbite retrograde ($i > 90°$) vengono catturati, oppure riportati su orbite dirette per successive diminuizioni dell'inclinazione.

La presente ipotesi potrebbe spiegare la non perfetta complanarità delle attuali orbite dei pianeti e rimuovere alcune difficoltà che sorgono quando si tenti di aggregare in un solo pianeta, mediante processi interni, i molti piccoli « embrioni » che si formano sulla stessa orbita secondo i modelli attuali.

1. FOREWORD

In his classic treatise " Hypothèses cosmogoniques " (1911), H. Poincaré refers about a theory concerned with the origin of the solar system, published by T.J.J. See in his book " Researches on the evolution of the stellar systems. II The capture theory of cosmical evolution ".

At variance with current theories, See ascribes the formation of the solar system to a process of capture of the planets by an envelope isotropically surrounding the sun. The same process is proposed by the Author for the formation of the satellite systems of the planets.

Carrying along See's hypothesis, Poincaré shows how, every time the planet crosses this envelope, the orbit, originally parabolic or elliptic with large ellip-

(*) Nella seduta del 10 dicembre 1983.

ticity, reduces progressively the values of a (semimajor axis) and e (eccentricity) down to values comparable to the present ones. About the mass m of the incoming object, Poincaré only says that it is negligible in comparison to the solar mass $M_{\odot}$. Nothing is said about the structure and chemical constitution of the envelope. Finally, he gives the following concluding remarks:

« La théorie de M. See rend bien compte des excentricités des orbites des planètes et des satellites. Mais pourquoi les mouvements de presque tous ces astres sont-ils directs et pourquoi leurs orbites ont-elles de faibles inclinaisons mutuelles? ... Pour essayer d'expliquer la faiblesse des inclinaisons, on peut supposer que l'atmosphère résistante du Soleil a une forme lenticulaire très aplatie. ... On pourrait aussi supposer que le milieu résistant est lui-même en rotation: il tendrait alors, non pas à annuler la vitesse de la planète qui s'y meut, mais à imprimer à cette planète une vitesse d'un certain sense ... le plan de l'orbite pourrait varier et tendre à diminuer son inclinaison sur le plan équatorial de l'atmosphère solaire ».

The present Authors did not find any sequel to See's theory. Yet, Poincaré's remarks, in particular those concerning a discoidal structure of the envelope and its co-rotation with the central body, are generally accepted by the present astrophysical theories about the formation and the early phases of the stellar life.

It can be added that the monogenetic theories about the formation of the planetary system do not explain why the orbits are almost but not completely complanary.

This may justify a renewed interest in See's hypothesis.

In the present paper the parameters of the disc (diameter 2R, thickness, density ρ in terms of distance r from the centre, and z from the equatorial plane) will be specified, according to the models recently proposed by several Authors. Of course, the mass m of the incoming object is supposed to be that of an object of asteroidal size, which successively sweeps the material from the disc and gradually assumes the structure of a planet, as actually meant.

Starting from a parabolic orbit, the progressive reduction of a and e is examined, and it is verified whether the total time (ΣP_i) taken by the process is compatible with the lifetime assigned to the circumsolar disc. The variation of the inclination i after every crossing is also examined.

2. Models of a circumsolar disc

Several theories about the origin of the solar system are based on the collapse of a cloud, with the formation of the sun at its centre, while the cloud itself takes the aspect of a flattened disc. However, they greatly differ according to the assumptions made about the constitution of the disc, namely:

i) mass M of the contracting cloud;

ii) its thermal properties;

iii) mechanism of transfer of the angular momentum;

iv) size of the disc and trend of the function $\rho\ (r, z)$;

v) possible contribution of turbulence and mass loss mechanisms.

Taking into account the present mass of the whole planetary system and supposing that originally the chemical composition of the contracting cloud was of the solar type, most Authors assume $M = 0.01 \div 0.07\, M_\odot$ and R as $30 \div 40$ a.u.

However, Cameron (1978a, 1978b) proposes a model where the disc attains a mass $M = 1.25\, M_\odot$ and extends up to $R = 700$ a.u., assuming that planets were formed at much greater distances, where the specific angular momentum was nevertheless the same as the present one.

In a physical theory for a circumsolar envelope, Lynden-Bell and Pringle (1974) conclude that the viscous dissipation inside the disc leads to an outward flux of angular momentum and kinetic energy, while the material flows inwards at small distances and outwards for large values of r.

Viscosity is generally attributed to turbulence, which, according to Cameron (1976) is originated inside the cloud by meridional currents, mass accretion and gravitational effects produced by gaseous protoplanets.

According to Lin and Papaloizou (1980), turbulence should arise from radiative transport of granules and subsequent vertical instability of the cloud.

The vertical structure of a self-dissipating disc in thermal and hydrostatic equilibrium depends on the adopted model: presence of a dense and thin dust layer on the equatorial plane of the disc (Safronov, 1969); isothermal (Cameron, 1978a; Pringle, 1981) or superadiabatic (Lin and Papaloizou, 1980) structure along z.

The various models assume different values for the density and the thickness of the disc. In his model (Cameron, 1978) assumes that the surface density σ depends on r as

$$\sigma = \frac{M}{2\,\pi\,R\,r}\ \text{sen}^{-1}\left(1 - \frac{r^2}{R^2}\right)^{1/2}$$

while ρ varies with z as

$$\rho = \rho_0 \exp\left(-\frac{z^2}{H^2}\right)$$

where ρ_0 corresponds to $z = 0$ and H is the main scale-height inside the isothermal structure.

Using the Lynden-Bell and Pringle (1974) findings, Lin (1981) considers the omologous stationary solutions of the convective model for a viscous disc. The time-dependent solutions formally approximate those valid for a stationary state in the inner region of the disc, provided that a sufficient time has elapsed since the beginning of the diffusion of the cloud. These stationary solutions appear as a reasonable approximation of the disc structure, when the sun has acquired most of its present mass. Because of their analytical form, they can be easily adopted for studying the structure of the cloud. Furthermore, the functional form of these solutions generally agrees with the numerical resuits.

From the analytical model, it results that on the equatorial plane the density varies with the distance as $\rho_0 \propto r^{-3/4}$, and it is constant within the two regions A and B, in which Lin divides the disc. He also considers two different values for the opacity, depending whether the temperature $T > 160$ K or < 160 K. The average values given for the two regions are

$$\sigma_A = 354.1 \text{ g}\cdot\text{cm}^{-2} \quad ; \quad \sigma_B = 53.1 \text{ g}\cdot\text{cm}^{-2} .$$

The fact that σ decreases with r is also accepted by other Authors. Assuming that the radial displacement of all dust particles which will eventually form the planets has been the smallest, Kusaka, Nakano and Hayashi (1970) find that σ varies as r^{-k}, k being between 1.5 and 2.0.

With considerations of thermic and dynamic equilibrium, Hayashi (1981) gives a disc model where σ and T in terms of r and z are:

r (a.u.)	z_0 (a.u.)	T (K)	ρ_0 (g·cm^{-3})	σ (g·cm^{-2})
1.0	0.04	230	6×10^{-9}	7180
5.2	0.33	100	2×10^{-10}	1975

where z_0 is the half-thickness of the disc.

Weidenschilling (1977) builds a model of a circumsolar cloud adding to the present mass of each planet that of the light elements needed for a chemical composition of solar type. The mass obtained in this way, distributed around the present planetary orbits, is adopted as the mass of the disc. He obtains generally $\sigma \propto r^{-3/2}$, but the lower densities found in connection with Mercury and Mars-Asteroids regions are ascribed to processes subtracting matter from these regions.

Horedt (1982) assumes that the temperature T of the inner regions of the solar cloud varies with r according to the relation

$$T = \left(\frac{L}{16\,\pi b\, r^2}\right)^{1/4}$$

where L is the luminosity of the sun and b the Stefan-Boltzmann constant. The density distribution inside a disc in hydrostatic equilibrium is given as

$$\rho(r, z) = \rho(r_0, 0)\left(\frac{r_0}{r}\right)^{11/4} \cdot \exp\left(-\frac{GM\,(\mu b/L)^{1/4} z^2}{R\, r^{5/2}}\right)$$

with $z \leq r/3$, $r_0 = 0.25$ *a.u.* being the inner radius and $R = 39.5$ a.u. the outer radius of the disc, and μ the average molecular weight.

On the equatorial plane

$$\rho(r, 0) = \rho(r_0, 0)\left(\frac{r_0}{r}\right)^{11/4}$$

and the surface density results

$$\sigma = \int_{-\infty}^{+\infty} \rho(r, z)\, dz = \sigma_0 \left(\frac{r_0}{r}\right)^{3/2}$$

For the disc model considered, Horedt obtains $M = 0.011\, M_\odot$, $\rho(r_0, 0) = 4.75 \times 10^{-8}\ g \cdot cm^{-3}$, $\sigma_0 = 1.03 \times 10^4\ g \cdot cm^{-2}$.

At the distances from the sun corresponding to the various planets, the average density $\bar{\rho}$ and the limiting thickness z_b of the disc result as follows:

r (a.u.)	$\bar{\rho}$ (g·cm^{-3})	z_b (a.u.)	r (a.u.)	$\bar{\rho}$ (g·cm^{-3})	z_b (a.u.)
0.39	1.04×10^{-8}	0.014	9.54	1.54×10^{-12}	0.78
0.72	1.88×10^{-9}	0.090	19.19	2.28×10^{-13}	1.87
1.00	7.64×10^{-10}	0.047	30.07	6.59×10^{-14}	3.27
1.52	2.42×10^{-10}	0.079	39.50	3.16×10^{-14}	4.58
2.80	4.54×10^{-11}	0.168			
5.20	8.27×10^{-12}	0.364			

z_b is assumed as the limit beyond which the density is lower than that found in the interstellar clouds. Horedt' surface densities are given in Table I, last column.

TABLE I

Values of σ (g·cm^{-2})

Planet	Cameron	Lin	Hayashi	Weidensch.	Horedt
Mercury . . .	4.69×10^6	} 354.1 (Mercury–Mars)		880	5290
Venus				4750	2070
Earth	1.79×10^6		7180	3200	1270
Mars				95	678
Asteroids . . .				0.13	272
Jupiter	3.18×10^5	} 53.1 (Jupiter–Neptune)	1975	120 ÷ 2400	107
Saturn				55 ÷ 330	44
Uranus				15 ÷ 40	15
Neptune . . .	2.04×10^4			10 ÷ 25	8
Pluto					5

Finally, the life-times of the processes depend on the values adopted for the parameters. The collapse of the cloud to form the disc lasts 5×10^4 years

(Cameron, 1976), or $10^5 \div 10^6$ years (Cassen and Moosman, 1981; Hayashi, 1981).

For the subsequent phase of planetary formation and disc depletion, times are proposed ranging from 1.3×10^5 years (Cameron, 1978b), to $10^5 \div 10^7$ years (Lin and Bodenheimer, 1982). Horedt (1982) finds 10^9 years when the mass loss from the disc is attributed to EUV solar radiation, or 10^6 years when the mass loss is due to solar wind of T Tauri type.

In conclusion, all Authors generally feel the necessity of explaining the lower mass and the different composition of the terrestrial planets, in spite of the fact that they have been formed inside the denser region of the disc, in contrast with the composition of the large planets, which formed in the thinner region of the disc itself. However, in favour of the larger planets, stand:

i) the lower local temperature and the smaller effects of the direct action of the sun (wind, EUV etc.), the acquisition or maintenance of light elements being consequently favoured;

ii) the small gravitational effects by the sun, and consequently the enlarged volume allowed for capturing material by the protoplanets;

iii) the larger thickness of the disc, which extends the effects of accretion during each crossing of the disc itself.

3. Dynamic considerations

We now describe the dynamical treatment of the problem. The originating orbit of the incoming body is assumed to be a parabola, its axis lying on the equatorial plane of the disc and the perihelion remaining at the same distance from the sun, after the successive crossings. Furthermore, we consider the path inside the disc as a small portion of the whole orbit, which is true at the initial phases of the process. Consequently, the crossing on a plane (x, y) results as a plane motion perpendicular to the sun-perihelion line.

The resisting force F acting on the incoming body and due to the disc particles is assumed to be

$$\vec{F} = -\frac{\rho C_d \, S V_r^2 \, \vec{u_r}}{2}$$

where ρ is the density of the resisting medium, S the cross section of the incoming body, $\vec{V_r}$ its velocity relative to the disc particles, $\vec{u_r}$ the unit vector along V_r. The coefficient of resistance C_d can be assumed to be about 2.

The modulus of V_r in the adopted reference (fig. 1) results

$$V_r = [(\dot{x} - V_c)^2 + \dot{y}^2]^{\frac{1}{2}}$$

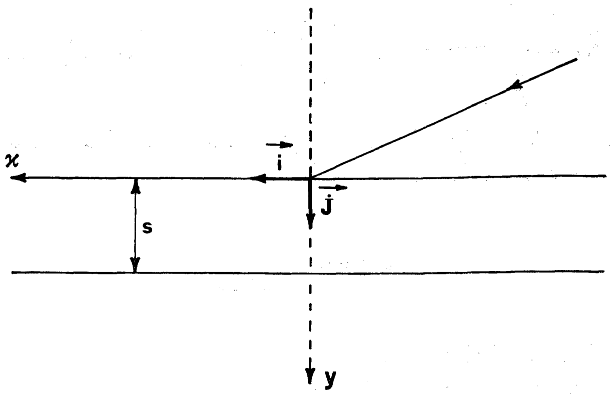

Fig. 1.

V_c being the velocity of the particles forming the disc, each one moving on a keplerian orbit, which is supposed to be circular.

On the other side, the expression of the unit vector is

$$\vec{u}_r = \frac{(x - V_c)\,\vec{i} + y\,\vec{j}}{[(x - V_c)^2 + y^2]^{\frac{1}{2}}}$$

$\vec{i}$ and $\vec{j}$ being the unit vectors according to x and y.

Initially, the relative velocities are

(2)
$$V_{r\parallel} = V_{p\parallel} - V_c = \left(\frac{GM_\odot}{r}\right)^{\frac{1}{2}} (\sqrt{2}\cos i - 1)$$
$$V_{r\perp} = V_{p\perp} = \left(\frac{GM_\odot}{r}\right)^{\frac{1}{2}} \sqrt{2}\ \mathrm{sen}\ i$$

where i is the inclination of the trajectory, G the gravitation constant, $M_\odot$ the mass of the sun and r the distance of the impact point from the sun.

Squared and added, the (2) give

$$V_r^2 = \frac{GM_\odot}{r}(3 - 2\sqrt{2}\cos i) \tag{3}$$

Assuming now for the incoming body a roughly spherical shape, radius R and density δ, its mass m will be

$$m = \frac{4}{3}\pi R^3 \delta \tag{4}$$

During the first crossing, the average acceleration α undergone by the body will be

$$\vec{\alpha}_1 = \frac{\vec{F}}{m} = -0.75 \frac{GM_\odot \rho}{Rr\delta} (3 - 2\sqrt{2}\cos i)\, \vec{u}_r . \tag{5}$$

Therefore, the velocity of the body changes as the product $\alpha\Delta t$. The crossing time at the first impact will be

$$\Delta t_1 = \frac{s}{V_p \operatorname{sen} i}$$

s being the thickness of the disc.

Consequently, the modulus of ΔV_1 is

$$\Delta V_1 = \alpha \Delta t_1 = -A \frac{V_r^2}{V_p \operatorname{sen} i} = -A \left(\frac{GM_\odot}{r}\right)^{\frac{1}{2}} f(i) \tag{6}$$

where

$$A = 0.75 \frac{\rho s}{R\delta} \tag{7}$$

and

$$f(i) = \frac{3 - 2\sqrt{2}\cos i}{\sqrt{2}\operatorname{sen} i} .$$

The function $f(i)$ has a minimum for $i_m = 19°.5$ (fig. 2). This inclination is therefore the less efficient for braking the impacting body. ΔV depends also on the distance r from the sun of the impact, and on the surface density $\sigma = \rho s$ of the disc, its value being small enough to still have an elliptical orbit with very high eccentricity as a consequence of the first crossing.

At then n-th impact, the relative velocity will be

$$V_{rn}^2 = V_{n-1}^2 + V_c^2 - 2\, V_{n-1} \cdot V_c \cdot \cos i_{n-1} \tag{8}$$

the variation of $\vec{V}$

$$\overrightarrow{\Delta V}_n = -A \frac{V_{rn}^2\, \vec{u}_{rn}}{V_{n-1} \operatorname{sen} i_{n-1}} \tag{9}$$

and the exit velocity

$$V_n^2 = V_{n-1}^2 + \Delta V_n^2 - 2\, V_{n-1}\, \Delta V_n \cos \beta_n \tag{10}$$

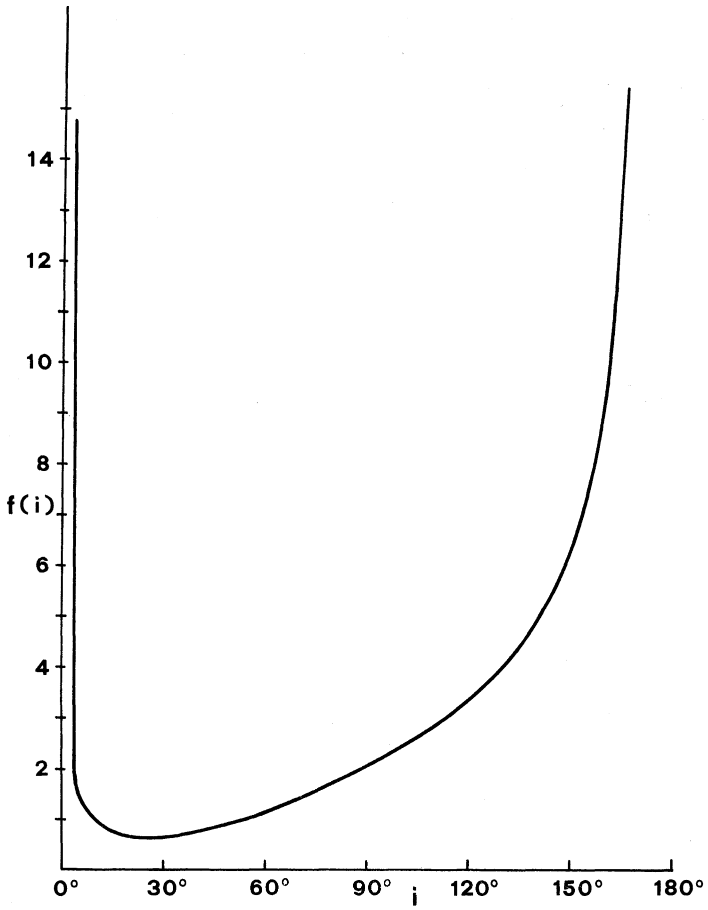

Fig. 2.

where β_n is the angle ACB between the velocity V_{n-1} and its variation ΔV_n (fig. 3), being

$$\text{sen}\,\beta_n = \frac{V_c}{V_{rn}}\,\text{sen}\,i_{n-1}\,. \tag{11}$$

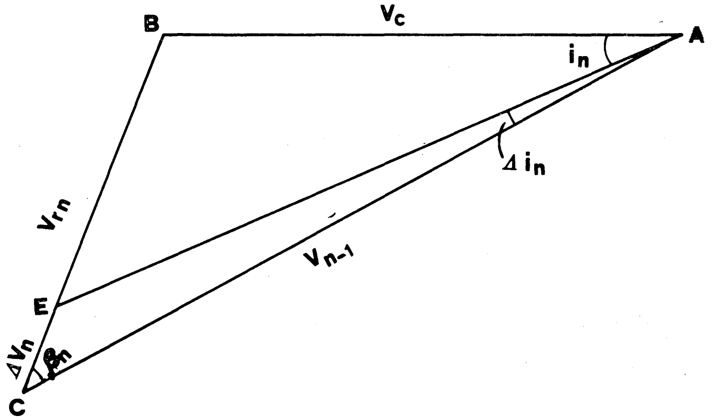

Fig. 3.

It may be noticed that for $\cos \beta_n > 0$, the velocity decreases after every impact. This process goes on steadily for a retrograde impact, while the velocity cannot diminish below V_e if the impact is direct, namely $i < 90^\circ$.

For $n = 1$, equation (10) becomes

$$V_1 = \left(\frac{GM_\odot}{r}\right)^{\frac{1}{2}} \left[2 + \frac{A^2 (3 - 2\sqrt{2}\cos i)^2}{2 \operatorname{sen}^2 i} - \frac{2A}{h(i)}\right]^{\frac{1}{2}} \tag{12}$$

where (fig. 4)

$$h(i) = \frac{\operatorname{sen} i}{(6 - 10\sqrt{2}\cos i + 11\cos^2 i - 2\sqrt{2}\cos i)^{\frac{1}{2}}} .$$

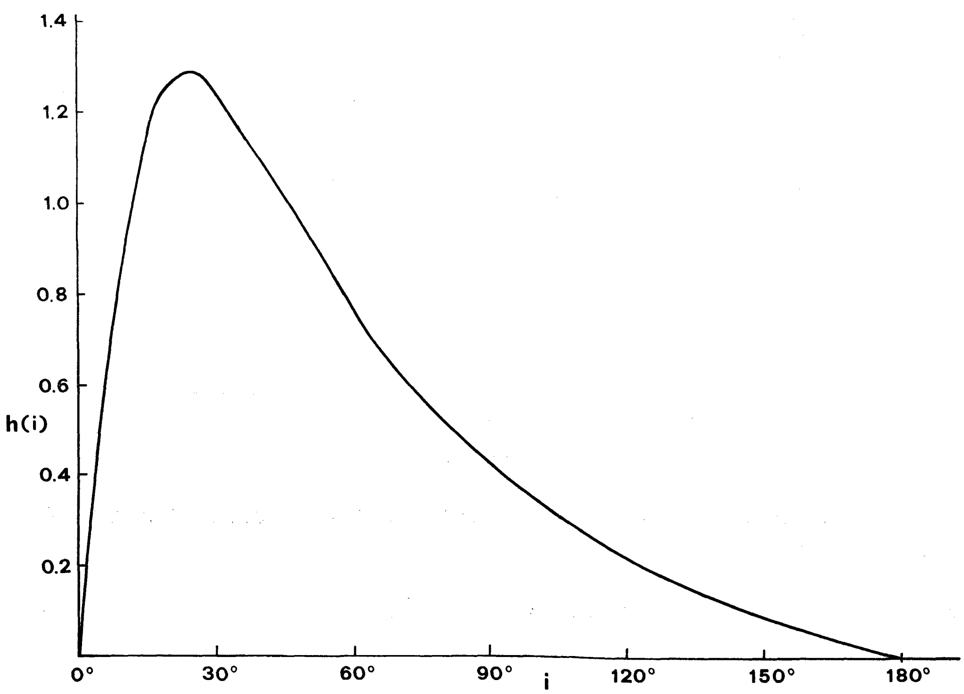

Fig. 4.

If $A \ll 1$, terms of the order of A^2 may be neglected and equation (12) becomes

$$V_1 = \sqrt{2} \left(\frac{GM_\odot}{r}\right)^{\frac{1}{2}} \left[1 - \frac{A}{h(i)}\right]^{\frac{1}{2}} \tag{13}$$

Of course, for $i = 0$ there is not emergency out of the disc, and eq. (13) looses its physical significance.

The inclination i_n at the n-th impact is (fig. 3) from the triangle $\overset{\Delta}{\text{BAE}}$:

$$\text{(14)} \qquad \text{sen}\, i_n = \frac{V_{rn} - V_n}{V_n} \text{sen}\,(i_{n-1} + \beta_n)$$

and, from the triangle $\overset{\Delta}{\text{ECA}}$:

$$\text{(15)} \qquad \text{sen}\, \Delta i_n = \frac{\Delta V_n \text{ sen}\, \beta_n}{V_n} = - A \frac{V_{rn} \cdot V_c}{V_{n-1} \cdot V_n} \simeq \Delta i_n$$

In conclusion, at the first impact we have

$$\text{(16)} \qquad a_1 = \frac{rh\,(i)}{2\,A} \quad ; \quad P_1 = \left[\frac{rh\,(i)}{2\,A}\right]^{3/2} \quad ; \quad e_1 = 1 - \frac{2\,A}{h\,(i)}$$

Therefore, within the validity of the adopted approximations, a_1 results independent of the mass of the sun and of the gravitation constant, and only depends on the distance of the impact, the inclination of the primitive orbit and mainly on the model of the disc and the structure of the incoming body. The eccentricity of the first ellipse, finally, does not depend on the distance, but only on A and i.

Still assuming $A \ll 1$ and therefore neglecting terms as small as A^2, at the first impact we have

$$\text{(17)} \qquad \Delta i_1 = - \tfrac{1}{2} A \frac{(3 - 2\sqrt{2}\cos i)^{\frac{1}{2}}}{1 - A/h\,(i)}$$

namely, for $A \ll h\,(i)$

$$\Delta i_1 = - \tfrac{1}{2} A\,(3 - 2\sqrt{2}\cos i)^{\frac{1}{2}}$$

$$(\Delta i_1)^{\circ} = - \tfrac{1}{2} A \,.\, 57.3\,(3 - 2\sqrt{2}\cos i)^{\frac{1}{2}} .$$

The trend of a, P and e during the successive impacts can be approximately evaluated supposing that $V_n - V_{n-1}$ remains constant for increasing n's and equal to $V_p - V_1$. Consequently,

$$\text{(18)} \qquad C = V_p - \left[V_p^2 + \Delta V_1^2 - 2\,V_p V_1 \left(1 - \frac{V_c^2}{V_{r1}^2}\text{sen}^2\, i\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

For $A \ll 1$ eq. (13) gives:

$$\text{(19)} \qquad C = \left(\frac{2\,G\,M_{\odot}}{r}\right)^{\frac{1}{2}} \left[1 - \left(1 - \frac{A}{h\,(i)}\right)^{\frac{1}{2}}\right].$$

Consequently

$$a_n = \frac{GM_\odot}{V^2 - (V_p - n\,C)^2}\,. \tag{20}$$

If $A \ll 1$, also $C \ll 1$. Therefore,

$$a_n \simeq \frac{GM_\odot}{2\,n\,V_n\,C} = \frac{1}{n}\,a_1$$

$$P_n \simeq \left(\frac{1}{n}\right)^{3/2} P_1$$

$$e_n \simeq 1 - n\,\frac{r}{a_1}\,.$$

The whole duration P_{tot} of the process is given by $\Sigma \left(\frac{1}{n}\right)^{3/2} \cdot P_1 \leq 3.4142\,P_1$. Consequently, P_{tot} is only slightly larger than P_1, and the orbital parameters resulting after the first impact yield sufficient knowledge about the duration and the aspects of the whole process.

4. Test of the hypothesis with some disc models

On the basis of the σ values summarized in Table I, a numerical test can be made, about the semimajor axes, periods, eccentricities and changes of the inclination after the first crossing.

Since $A = 0.75\,\frac{\rho s}{R\delta}$ depends on the adopted disc model ($\sigma = \rho s$) as well as on the size and density of the incoming body, we may tentatively assume δ ranging between 1 and 3 $g \cdot cm^{-3}$, and 2 R between 1 and 1000 km.

For all models assuming $M \ll M_\odot$ (therefore, excluding Cameron's model for the disc), A actually results much smaller than unity, as supposed in the present paper. Lin's model gives the smallest surface densities σ, while Hayashi's model gives the highest ones. Using consequently eq. (16), and the highest value for $h(i)$, namely $i = 25°$, we obtain the data summarized in Table II.

In Lin's model P_1 increases from 1.29×10^3 to 4.09×10^7 years when 2 R increases from 1 to 1000 km, for $\delta = 1$. For $\delta = 3$, the semimajor axes become three times larger and P_1 range between 6.72×10^3 and 2.13×10^8 years. In both cases, the largest value for 2 R implies a process which lasts too long when compared to the life time of the disc.

In Hayashi's model, P_1 ranges between 13.7 and 4.33×10^5 years for $\delta = 1$; between 71.1 and 2.25×10^6 years for $\delta = 3$, and the largest value of 2R is again hardly compatible with the lifetime of the disc, unless the value suggested by Horedt (1982) is accepted, namely 10^9 years, when the mass loss is attributed to EUV solar radiation.

TABLE II

$r = 1$ a.u. $i = 25^{\circ}$	values of a_1 (a.u.)			
2 R	Lin, $\delta = 1$	Lin, $\delta = 3$	Hayashi, $\delta = 1$	Hayashi, $\delta = 3$
1 km	1.19×10^{2}	3.56×10^{2}	5.72×10^{0}	1.716×10^{1}
10 km	$\times 10^{3}$	$\times 10^{3}$	$\times 10^{1}$	$\times 10^{2}$
100 km	$\times 10^{4}$	$\times 10^{4}$	$\times 10^{2}$	$\times 10^{3}$
1000 km	$\times 10^{5}$	$\times 10^{5}$	$\times 10^{3}$	$\times 10^{4}$
	values of P_1 (years)			
1 km	1.29×10^{3}	6.72×10^{3}	1.37×10^{1}	7.11×10^{1}
10 km	4.09×10^{4}	2.13×10^{5}	4.33×10^{2}	2.25×10^{3}
100 km	1.29×10^{6}	6.72×10^{6}	1.37×10^{4}	7.11×10^{4}
1000 km	4.09×10^{7}	2.13×10^{8}	4.33×10^{5}	2.25×10^{6}
	values of eccentricity e			
1 km	0.99158	0.997299	0.825265	0.942814
10 km	.99916	.999730	.982526	.994281
100 km	.999916	.999973	.998253	.999428
1000 km	.999992	.999997	.999825	.999943
	change of inclination — $(\Delta i_1)^{\circ}$			
1 km	1.003×10^{-1}	3.22×10^{-2}	2.08	6.81×10^{-1}
10 km	$\times 10^{-2}$	$\times 10^{-3}$	0.208	$\times 10^{-2}$
100 km	$\times 10^{-3}$	$\times 10^{-4}$	2.08×10^{-2}	$\times 10^{-3}$
1000 km	$\times 10^{-4}$	$\times 10^{-5}$	$\times 10^{-3}$	$\times 10^{-4}$
$r = 5.2$ a.u. $i = 25^{\circ}$	values of a_1 (a.u.)			
1 km	6.172×10^{2}	1.853×10^{3}	2.976×10^{1}	8.92×10^{1}
10 km	$\times 10^{3}$	$\times 10^{4}$	$\times 10^{2}$	$\times 10^{2}$
100 km	$\times 10^{4}$	$\times 10^{5}$	$\times 10^{3}$	$\times 10^{3}$
1000 km	$\times 10^{5}$	$\times 10^{6}$	$\times 10^{4}$	$\times 10^{4}$
	values of P_1 (years)			
1 km	1.533×10^{4}	7.976×10^{4}	1.62×10^{2}	8.42×10^{2}
10 km	4.848×10^{5}	2.522×10^{6}	5.134×10^{3}	2.66×10^{4}
100 km	1.533×10^{7}	7.976×10^{7}	1.62×10^{5}	8.42×10^{5}
1000 km	4.848×10^{8}	2.522×10^{9}	5.134×10^{6}	2.66×10^{7}
	values of eccentricity e_1			
1 km	0.998745	0.999586	0.953933	0.984432
10 km	.999874	.999959	.995393	.998443
100 km	.999987	.999996	.999539	.999844
1000 km	.999999	.999999	.999994	.999984
	change of inclination — $(\Delta i_1)^{\circ}$			
1 km	1.495×10^{-2}	0.2054×10^{-3}	0.549	0.1855
10 km	$\times 10^{-3}$	$\times 10^{-4}$	0.0549	0.0186
100 km	$\times 10^{-4}$	$\times 10^{-5}$	5.49×10^{-3}	1.855×10^{-3}
1000 km	$\times 10^{-5}$	$\times 10^{-6}$	$\times 10^{-4}$	$\times 10^{-4}$

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