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Fisica matematica. — *Equilibria with non-rigid motions in a magnetoplasma* (*). Nota (**) di MASSIMO TESSAROTTO (***), presentata dal Socio D. GRAFFI.

RIASSUNTO. — Si studiano soluzioni perturbative dell'equazione cinetica di Fokker-Planck nella approssimazione « di raggio di Larmor piccolo » che corrispondono a stati iniziali in presenza di moti non rigidi del plasma. Quali esempi si considerano, in particolare, configurazioni di equilibrio idromagnetico spazialmente simmetriche.

1. INTRODUCTION

As is well-known, transport theory for quiescent magnetoplasmas (i.e. those in which turbulence is negligible) has been customarily developed only for a class of initial states which are appropriately " near " the local thermodynamic equilibrium and are described by local maxwellian distributions, for each particle species present in the system [1]. On this limiting assumption are founded, in particular, all the present theoretical predictions of collisional transport (as is the case of the so-called " neo-classical " theory [3-5], which concerns, more precisely, toroidal axisymmetric hydromagnetic equilibria). On the other hand, it is well-known that the same type of limitation is shared by most turbulent-transport theories (see for example Ref.s [6-7] and further references therein indicated).

It has rceently been pointed out by the author [1, 2], in the context of a kinetic approach to irreversible thermodynamics for magnetoplasmas, that even retaining a request of linearity of the material fluxes (of particle and kinetic energy, as well as of the electric current density) w.r. to the thermodynamic forces, whichever they may be, a broader class of initial states may exist which are singular enough to persist, before reaching equilibrium, on a time scale comparable to (or even larger than) the shortest collisional relaxation time (i.e., the so-called collisional diffusion time) characterizing the temperature and density of the plasma.

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In this paper an example of this type is pointed out which concerns initial states in which non-rigid particle motions are allowed. As an illustration, the case of an hydromagnetic equilibrium which is spatially symmetric is considered.

The motivations of the present research are twofold. They follow, in particular, from the observation of the present inadequacy of theoretical predictions based on current transport models based on kinetic theory [8] and the realization that this might well require a refinement or a modification of the present mathematical models (and possibly of the mathematical methods of approximation currently adopted).

It is well-known from collisional transport theory [3-5, 9] that in closed magnetic configurations " radial " transport, i.e. the transport process occurring across isobaric surfaces, is essentially produced by collisional friction forces—due to Coulomb two-particle collisions—among the various particle species, which perturb their motion along the magnetic flux lines. Evidently such forces are strongly influenced by the relative average (" bulk ") velocities of such particle species. Therefore, initial states with non-rigid particle motions will correspond, intuitively, to enhanced radial (as well as " parallel ", w.r. to the magnetic field) transport.

It is the purpose of this paper to show that for spatially symmetric systems (i.e. those in which the electromagnetic field and the one-particle distribution function have at least one ignorable spatial coordinate) there exist initial states of this type in the class of " drifted " maxwellian distributions. In particular, it is possible to show that such initial states relax toward thermodynamic equilibrium on a time scale comparable to or even larger than the collisional diffusion time.

An essential feature of the model is that the linearity of the basic equations (i.e. the so-called drift Fokker-Plank equation) is preserved, while additional thermodynamic forces are introduced in the problem. Since such forces —which are evidently related to the presence deviations from the local thermodynamic equilibrium—are still in principle arbitrary, the model seems to possess enough flexibility for actual applications to the development of transport calculations, both for quiescent and weakly turbulent magnetoplasmas.

2. Formulation of the problem

We intend to investigate a plasma magnetically confined and subject to the so-called approximation of " small-Larmor-radius " [9-11]. We look for approximate solutions of the Fokker-Plank kinetic equation:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla + \frac{1}{m_s} \mathbf{F}_s \cdot \frac{\partial}{\partial \boldsymbol{v}}\right) f_s(\boldsymbol{r}, \boldsymbol{v}, t) = \mathrm{C}_s(f|f) \tag{1}$$

where $f_s = f_s(\boldsymbol{r}, \boldsymbol{v}, t)$ is the kinetic distribution function of the s-th species $(s = 1, r)$, $\mathbf{F}_s = e_s \mathbf{E} + m_s \boldsymbol{v} \wedge \boldsymbol{\Omega}_s$ (with $\mathbf{E}$ the electric field and $\boldsymbol{\Omega}_s = e_s \mathbf{B}/m_s c$

with **B** the magnetic field) the Lorentz force and $C_s(f \mid f)$ is the Fokker-Plank collision operator in the Landau form, i.e.:

$$(2) \qquad C_s(f \mid f) = \Sigma_{k=1,r}\, q_{sk} \frac{\partial}{\partial \boldsymbol{v}} \cdot \int d^3 v' \frac{\partial^2 u}{\partial \boldsymbol{v}\, \partial \boldsymbol{v}} \cdot \left\{ f_k(\boldsymbol{v}') \frac{\partial}{\partial \boldsymbol{v}} f_s(\mathrm{v}) - \right.$$

$$\left. - \frac{m_s}{m_k} f_s(\boldsymbol{v}) \frac{\partial}{\partial \boldsymbol{v}'}, f_k(\boldsymbol{v}') \right\}$$

where $u = \mid \boldsymbol{v} - \boldsymbol{v}' \mid$, $q_{sk} = 2\,\pi e_s^2 e_k^2 \ln \Lambda_{sk}/m_s^2$, being e_s and m_s the electric charge and the mass of the s-th particle species, while $\ln \Lambda_{sk}$ is the Coulomb logarithm.

We adopt, for this purpose, a perturbative expansion previously described by the author [1, 9, 10], which coincides with the well-known neo-classical model for toroidal and axisymmetric configurations [11]. The relevant expansion parameter is assumed (for each particle species) $\varepsilon_s = r_s/\mathrm{L}$ (with $\varepsilon_s \ll 1$), where $r_s = v_{th,s}/\Omega_s$ is the Larmor radius (as usual it is defined in terms of the thermal velocity $v_{th,s}$ to be introduced later) and L is the smallest " characteristic scale length " of the equilibrium configuration, i.e., $\mathrm{L} = \min \{\mathrm{L}_i\}$ where L_i $(i = 1, 2, \ldots)$ denote, for example, typical dimensions of the system or the " e-folding length " of $\mathbf{E}_0, \mathbf{B}_0, \mathrm{N}_{0,s} = \int d^3 v f_{0,s}$, etc. (namely $\mathrm{L}_i = \mid \mathrm{B}_{0k} \mid / \mid \nabla \mathrm{B}_{0k} \mid ; \mid \mathrm{E}_{0k} \mid / \mid \nabla \mathrm{E}_{0k} \mid ; \mid \mathrm{N}_{0,s} \mid / \mid \nabla \mathrm{N}_{0,s} \mid, \ldots$). Here the notations are standard. Thus $\mathbf{E}_0$ and $\mathbf{B}_0$ are, respectively, the leading-order contributions to **E** and **B** in terms of a perturbative expansion made w.r. to $\varepsilon_1, \ldots, \varepsilon_r$, while $\mathrm{N}_{0,s} = \int d^3 v f_{0,s}(\boldsymbol{r}, \boldsymbol{v}, t)$, with $f_s(\boldsymbol{r}, \boldsymbol{v}, t) = \Sigma_{i=0}\, \varepsilon_s^i f_{i,s}(\boldsymbol{r}, \boldsymbol{v}, t)$.

We shall assume, as in Ref. [9], that $\mathbf{B}_0$ fulfills the hydromagnetic equilibrium equation:

$$(3) \qquad (\nabla \wedge \mathbf{B}_0) \wedge \mathbf{B}_0 = 4\,\pi\, \nabla \pi_0$$

where $\pi_0 = 3\, \Sigma_{s=1,r}\, \mathrm{N}_{0,s} \mathrm{T}_{0,s}$ is the kinetic pressure to leading order in $\varepsilon_1, \ldots \varepsilon_r$ with $\mathrm{T}_{0,s} = \int d^3 v m_s v^2 f_{0,s} / 3\, \mathrm{N}_{0,s}$ the temperature (we identify, thus $v_{th,s} = (2\, \mathrm{T}_{0,s}/m_s)^{1/2}$). Here π_0 is assumed to be of class $C^2(D)$, being D a bounded and connected domain of R^3 with boundary δD, where π_0 assumes a constant value.

In the sequel, we shall limit our analysis to the case in which δD is spatially symmetric, namely it is assumed invariant w.r. to rigid coordinate transformation. It is well known that under such an assumption there exist solutions of Eq. (3) of class $C^1(D)$ which exhibit the same symmetry property [12]. They are denoted as " symmetric hydromagnetic equilibria ". In the cass of " closed " magnetic configurations, i.e. those in which the flux lines belong to a family of bounded and closed surfaces (magnetic surfaces), the only admissible symmetric hydromagnetic equilibrium corresponds to a so-called toroidal and axisymmetric equilibrium, whereas "open" magnetic configurations may exhibit both cylindrical (i.e., rotational), axial (i.e., translational) and helical symmetry. Thus if θ is a spatial coordinate which is ignorable for $\mathbf{B}_0$, in the case of toroidal configurations $\mathbf{B}_0$ will be of the form $\mathbf{B}_0 = \hat{e}_\theta \mathrm{B} + \hat{e}_\chi \mathrm{B}$, with B_θ and $\mathrm{B}_\chi \neq 0$ (where $\mathrm{B}_i = \mathbf{B}_0 \cdot \hat{e}_i$, with $i = \theta, \chi$ and we have introduced an orthogonal and right-handed system of toroidal curvilinear coordinates (θ, χ, ψ), denoting θ a toroidal azimuth); instead, for open configurations one may have both $\mathbf{B}_0 = \hat{e}_\theta \mathrm{B}_\theta$, as well as $\mathbf{B}_0 = \hat{e}_\chi \mathrm{B}_\chi$ (i.e., respectively $\mathrm{B}_\chi = 0$ and $\mathrm{B}_\theta = 0$).

Let us briefly recall [9-10] that in terms of the perturbative expansion previously introduced, a hierarchy of perturbative equations is obtained for $f_s(\mathbf{r}, \mathbf{v}, t)$ from Eq. (1) (and analogous equations follow from the Maxwell's equation for electro-magnetic field). In particular, to the lowest order in ε_s results $f_{0,s} = f_{0,s}(v, \lambda, \boldsymbol{r}, t)$ (where $v = |\boldsymbol{v}|$, $\lambda = v_\perp^2 / \mathrm{B} v^2$, $\boldsymbol{v} = v_{\|} \hat{\boldsymbol{n}} + \boldsymbol{v}_\perp$ with $\hat{\boldsymbol{n}} = \mathbf{B}_0/\mathrm{B}_0$, $\boldsymbol{v}_\perp = |v_\perp| (\hat{b} \cos \zeta + \hat{p} \sin \zeta)$ and $\zeta = \operatorname{arctg} (\boldsymbol{v} \cdot \hat{b}/v \cdot \hat{p})$), while to order $0(\varepsilon_s)$ one gets:

$$(4) \qquad \left(\frac{\partial}{\partial t} + \boldsymbol{v}\nabla \cdot f_{0,s} + \frac{e_s}{m_s} \mathbf{E}_0 \cdot \frac{\partial}{\partial \boldsymbol{v}}\right) f_{0,s} + \boldsymbol{v} \wedge \Omega_{0,s} \cdot \frac{\partial}{\partial \boldsymbol{v}} \tilde{f}_{1,s} = \mathrm{C}_s (f_0 \mid f_0)$$

which delivers by taking the ζ-average:

$$(5) \qquad \left(\frac{\partial}{\partial t} + v_{\|} \hat{\boldsymbol{n}} \cdot \nabla + \frac{e_s}{m_s} \mathbf{E}_0 \cdot \frac{\partial}{\partial \boldsymbol{v}}\right) f_{0,s} = \mathrm{C}_s (f_0 \mid f_0)$$

and $\tilde{f}_{1,s} = \boldsymbol{v} \wedge \hat{\boldsymbol{n}} \cdot \nabla f_{0,s}/\Omega_{0,s}$, where we have defined $f_s = \bar{f}_s + \tilde{f}_s$, with $\bar{f}_s = (2\pi)^{-1} \oint_0^{2\pi} d\zeta f_s$. In general at each order ε_s $(i \geq 1)$ we obtain a perturbative equation for $\tilde{f}_{i,s}$ and, taking the ζ-average, an equation for $f_{i-1,s}$. Thus, in particular, to second order in ε_s, one gets the so called drift Fokker-Planck equation [9-10]:

$$(6) \qquad \left(\frac{\partial}{\partial t} + v_{\|} \hat{\boldsymbol{n}} \cdot \nabla\right) \bar{f}_{1,s} = - \boldsymbol{v}_{\mathrm{D},s} \cdot \nabla f_{0,s} - \frac{e_s}{m_s} v_{\|} \hat{\boldsymbol{n}} \cdot \mathbf{E}_1 \hat{\boldsymbol{n}} \cdot \frac{\partial}{\partial \boldsymbol{v}} f_{0,s} + \mathrm{C}_s (f_0 \mid \bar{f}_1)$$

where $\boldsymbol{v}_{\mathrm{D},s} = \hat{\boldsymbol{n}} \wedge (\mu \nabla \mathrm{B} + v_{\|}^2 \hat{\boldsymbol{n}} \cdot \nabla \hat{\boldsymbol{n}})/\Omega_{0,s}$ is the diamagnetic drift velocity (with $\mu = v_\perp^2/2\,\mathrm{B}$ the magnetic moment per unit mass).

We notice that the choice of the initial conditions for Eq. (1), and hence for Eqs. (5) and (6), as for all higher-order perturbative equations, is not completely arbitrary due to Eq. (3). Current transport theories [1-5, 9-11] assume that $f_{0,s}$ is a local maxwellian distribution, for each particle species, constant on each isobaric surface $(\hat{\boldsymbol{n}} \cdot \nabla f_{0,s} = 0)$ and subject to the condition of temperature equilibration $\mathrm{T}_{0,s} = \mathrm{T}_{0,k}$. In such an hypothesis it is necessary to infer that the average entropy production rate $\dot{S}(f \mid f) = \Sigma_{s=1,r} \langle \int d^3 v \ln f_s C_s (f \mid f) \rangle$ (where the brackets " $< >_S$ " denotes an appropriately weighted average on an isobaric surface S) results of order $0(\varepsilon_s^2)$ (with $s = 1, r$), or:

$$\dot{S}(f \mid f) \sim 0(\varepsilon^2) \quad (\text{with } \varepsilon = \max\{\varepsilon_1, \ldots, \varepsilon_r\}) \tag{7}$$

and, more important, $\dot{S}(f \mid f)$ results, to leading-order w.r. to $0(\varepsilon_s)$, a linear function of the material fluxes, whatever they are (and which can be identified here as the radial particle and kinetic energy fluxes and the electric current density vector J) [1, 2].

The request (7), which is usually denoted as "transport ansatz" may, however, correspond to a broader class of initial conditions for $f_{0,s}$. In particular, since to the lowest order in ε_s results:

$$\begin{aligned} \dot{S}(f \mid f) &\cong \dot{S}(f_0 \mid f_0) = \tfrac{1}{2} \Sigma_{s,k=1,r} \langle q_{sk} m_s^2 \int\!\!\int d^3 v d^3 v' f_{0,s}(\boldsymbol{v}) f_{0,k}(\boldsymbol{v}') u^{-3} \\ &\left\{ \boldsymbol{u} \wedge \left(m_s^{-1} \frac{\partial}{\partial \boldsymbol{v}} \ln f_{0,s} - m_k^{-1} \frac{\partial}{\partial \boldsymbol{v}} \ln f_{0,k} \right) \right\}^2 \rangle_s \quad (\text{with } \boldsymbol{u} = \boldsymbol{v} - \boldsymbol{v}') \end{aligned} \tag{8}$$

a solution $f_{0,s}(\boldsymbol{r}, \boldsymbol{v}, t)$ of Eq. (5) which is compatible with (7) is clearly of the type:

$$f_{0,s}(\boldsymbol{r}, \boldsymbol{v}, t) = f_{s,M}(\boldsymbol{r}, \boldsymbol{v}) + \varepsilon_s f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t) \tag{9}$$

where $f_{s,M}(\boldsymbol{r}, \boldsymbol{v})$ is a local maxwellian distribution ($f_{s,M}(\boldsymbol{r}, \boldsymbol{v}) = \frac{N_{0,s}}{\pi^{3/2} v_{th,s}^3} \exp(-v^2/v_{th,s}^2)$) while $f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t)$ must be compatible with Eq. (3). For this purpose we notice that Eq. (3) can also be written in terms of Ampere's law as:

$$(\nabla \wedge \mathbf{B}_0) \wedge \mathbf{B}_0 = \frac{4\pi}{c} \mathbf{J}_1 \wedge \mathbf{B}_0 \tag{3'}$$

where

$$\mathbf{J}_1 = \Sigma_{s=1,r} e_s \int d^3 v \boldsymbol{v} (\tilde{f}_{1,s} + \tilde{f}_{0,s}^{(1)}) . \tag{10}$$

However, since $\pi_0 = 3 \Sigma_{s=1,r} N_{0,s} T_{0,s}$, it follows from Eqs. (3), (3') and (10) that:

$$\tilde{f}_{1,s} + \tilde{f}_{0,s}^{(1)} = \boldsymbol{v} \wedge \boldsymbol{n} \cdot \nabla f_{M,s} / \Omega_{0,s} . \tag{11}$$

One infers that $f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t)$ is clearly non-unique.

The arbitrariness of $f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t)$ can be used, as previously pointed out [2], to investigate transport problems in "transients" (denoting here as "transient" a state of the plasma in which $\bar{f}_{1,s}$ decays in time on a time scale much smaller than the largest possible collisional diffusion time). However, it is interesting to note that in the case of symmetric hydromagnetic equilibria, non-trivial initial conditions for $f_{0,s}$ can be found such that the perturbation $f_{0,s}^{(1)}$ may persist before reaching "equilibrium" for times comparable to or even larger than the collisional diffusion time.

This situation occurs when:

$$\overline{\boldsymbol{v} \cdot \nabla f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t)} = 0 \tag{12}$$

while letting, in order to avoid the trivial solution:

$$C_s(f_M \mid \bar{f}_0^{(1)}) \neq 0 \qquad \text{(at least for some } s = 1, r). \tag{13}$$

A particular solution of Eq. (12) fulfilling Eq. (11) and the latter condition can be found in the class of drifted maxwellian distributions, namely:

$$\begin{aligned} f_{0,s}^{(1)}(\boldsymbol{r}, \boldsymbol{v}, t) &= \alpha_{1,s} p_{c,s} f_{M,s}(\boldsymbol{r}, \boldsymbol{v}) + \frac{N_{1,s}}{N_{0,s}} f_{M,s}(\boldsymbol{r}, \boldsymbol{v} - \boldsymbol{v}_{0,s}) + \\ &+ h_s(v) f_{M,s}(\boldsymbol{r}, \boldsymbol{v}) \end{aligned} \tag{14}$$

where $p_{c,s} = m_s \boldsymbol{v} \cdot \frac{\partial \boldsymbol{v}}{\partial \theta} + \frac{e_s}{c} \mathbf{A} \cdot \frac{\partial v}{\partial \theta}$ is the conserved canonical momentum, conjugate to θ, with $\mathbf{A}$ the vector potential; furthermore:

$$\hat{n} \cdot \nabla \alpha_{1,s} = \hat{n} \cdot \nabla h_s(v) = 0 \quad ; \quad g = \hat{n} \cdot \frac{\partial \boldsymbol{v}}{\partial \dot{\theta}}$$

$$h_s(v) = \frac{N_{0,s}^{(1)}}{N_{0,s}} + \frac{T_{1,s}}{T_{0,s}} \left(x_s^2 - \frac{3}{2} \right) \tag{15}$$

$$x_s = v / v_{th,s} \quad ; \quad \boldsymbol{v}_{o,s} = \hat{e}_\theta \, v_{o,s}$$

with $\hat{e}_\theta$ denoting the unit vector $\hat{e} = \nabla\theta / \mid \nabla\theta \mid$, being θ an ignorable spatial coordinate for $\mathbf{B}_0$. We notice that Eq. (14) can be formally obtained from a drifted maxwellian distribution of the type $f_{0,s}(\boldsymbol{r}, \boldsymbol{v}t, t) = N_s (2\pi\, T_s/m_s)^{-3/2}$ $\exp\{-(\mathbf{v} - \hat{n} w_s)^2 m_s / 2\, T_s\}$ by a truncated power series expansion w.r. to ε_s.

Thus is it necessary to interpret the first term on the r.h.s. of Eq. (14) as a contribution due to a first-order drift motion along the direction $\hat{e}$ w.r. to an inertial reference frame (corresponding to $\alpha_{1,s} = v_{0,s} = 0$); similarly the second term represents the contribution due to a zero-order drift motion, in the sense:

(16) $$\frac{v_{0,s}}{v_{th,s}} \sim 0\,(\varepsilon_s)$$

while results

(17) $$\frac{v_{1,s}}{v_{th,s}} \sim 0\,(\varepsilon_s)$$

with the definition $\mathbf{v}_{1,s} = 2\,T_{0,s}\,\alpha_{1,s}\,\frac{\partial \mathbf{v}}{\partial \theta}$. For consistency, we require furthermore in Eq. (14):

(18) $$\frac{N_{1,s}}{N_{0,s}} \sim h_s(v) \sim 0\,(\varepsilon_s) \qquad (\text{where } v \sim v_{th,s})\,.$$

We stress that for generality $v_{0,s} \neq v_{0,k}$ (as well as $v_{1,s} \neq v_{1,k}$) for $s \neq k$ ($s\,,\,k = 1\,,\,r$). The case $v_{0,s} = v_{0,k}$ ($v_{1,s} = v_{1,k}$) is well-known and corresponds to the case of a rigid motion of the plasma usually considered in collisional transport theory. The particular solution (14) also includes, more generally, the effect of a non-rigid drift motion of the plasma along the direction $\hat{e}_\theta$ (notice that $\hat{e}_\theta$ need not be a constant unit vector).

Finally, the third term on the r.h.s. of Eq. (14) represents the contribution due to temperature and density perturbation to the local maxwellian distribution $f_{M,s}(\boldsymbol{r}\,,\,\boldsymbol{v}\,,\,t)$.

The solution (14) fulfills evidently:

(19) $$C_s(f_M \mid f_0^{(1)}) = 0$$

only if $\alpha_{1,s} = \alpha_{1,k}$, $v_{0,s} = v_{0,k}$ and $T_{1,s} = T_{1,k}$ $\forall s\,,\,k = 1\,,\,r$. In order to prove that $f_{0,s}^{(1)}$ fulfills Eq. (12) too, it suffices to notice, for example, that:

(20) $$\int_0^{2\pi} d\zeta\,\boldsymbol{v}_\perp \cdot \nabla\left(m_s\,\boldsymbol{v} \cdot \frac{\partial \boldsymbol{v}}{\partial \theta}\right) = 0\,.$$

Thus the conservation equation for $p_{c,s}$:

(21) $$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla + \frac{e_s}{m_s}\,\mathbf{E} \cdot \frac{\partial}{\partial \boldsymbol{v}} + \boldsymbol{v} \wedge \boldsymbol{\Omega}_s \cdot \frac{\partial}{\partial \boldsymbol{v}}\right) p_{c,s} = 0$$

implies to leading order w.r. to $0(\varepsilon_s)$, after taking the ζ-average:

$$\overline{\boldsymbol{v}\cdot\nabla\left(\alpha_{1,s}\,p_{c,s}\,f_{\mathrm{M},s}(\boldsymbol{r},\boldsymbol{v})\right)}=0 \tag{22}$$

and hence Eq. (12) is obviously fulfilled by the first term on the r.h.s. of Eq. (14). Analogous results can be drawn for the remaining terms.

In conclusion, it follows that whereas Eq. (6) is invariant w.r. to the transformation

$$\overline{f}_{1,s}\rightarrow\overline{f}_{1,s}+\overline{f}^{(1)}_{0,s} \tag{23}$$

if $\alpha_{1,s}=\alpha_{1,k}$, $v_{0,s}=v_{0,k}$ and $\mathrm{T}_{1,s}=\mathrm{T}_{0,k}\,\forall s$, $k=1,r$ (which corresponds, in particular, to a rigid drift motion of the plasma along the direction $\hat{e}_\chi$), this does not occur in general if $\mathrm{T}_{0,s}\neq\mathrm{T}_{0,k}$ at least for some species s and k or $\alpha_{1,s}\neq\alpha_{1,k}$ (or $v_{0,s}\neq v_{0,k}$) (corresponding to a non-rigid drift motion of the plasma). This implies, in particular, that $\overline{f}_{1,s}$ (see Eq. (6)) is a function of $\Delta^{(0)}_{sk}=v_{0,s}-v_{0,k}$. $\Delta^{(1)}_{sk}=v_{1,s}-v_{1,k}$ and, respectively, $\mathrm{T}_{1,s}-\mathrm{T}_{1,k}$ ($s\neq k$ with $s,k=1,r$).

As a final point, it is interesting to notice that whereas here the "effect" due to such drift motions has been assumed to be of first order in $0(\varepsilon_s)$ (in fact $f^{(1)}_{0,s}/f_{0,s}\sim 0(\varepsilon_s)$) exact solutions Eq. (1) in the presence of zero-order (rigid) drift motions are well known in the literature [16]. However, since they are non-neutral, in the sense $\Sigma_{s=1,r}\,e_s\,\mathrm{N}_{0,s}\neq 0$, they do not correspond to a true situation of plasma confinement and are therefore not relevant to the present analysis.

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List of symbols

$\mathbf{A}$	vector potential
A_{is}	($i=1,3$; $s=1,r$) thermodynamic forces
$\mathbf{B}$	magnetic field
$\mathbf{B}_0$	equilibrium magnetic field
D	connected domain ($\subset \mathrm{R}^3$) with boundary δD
$\mathbf{E}$	electric field
$\mathbf{E}^{rot}$	inductive electric field
e_s	electric charge
$f_s(\boldsymbol{r},\boldsymbol{v},t)$	distribution function of the species s
$f_{0,s}(\boldsymbol{r},\boldsymbol{v},t)$	equilibrium distribution function
$f_{1,s}$	first-order perturbation of the distribution function

Φ	electrostatic potential
$\Gamma_{is}\,(i=1,2)$	geometrical (radial) material fluxes
L	characteristic scale length of the equilibrium
m_s	mass
$\hat{n}=\mathbf{B}_0/B_0$	
N_s	number density
π_0	kinetic pressure
$p_{c,s}$	canonical momentum conjugated to the cyclic coordinate θ
r_s	Larmor radius
S	average of the local entropy production rate on an isobaric surface ($\pi_0=$ const.)
T_s	temperature
$\boldsymbol{v}_{D,s}=\hat{n}\wedge(\mu\nabla B+v_{\parallel}^2\hat{n}\cdot\nabla\hat{n})/\Omega_s$	diamagnetic drift velocity
$v_{i,s}$	drift velocity along the direction $\hat{e}_\theta\,(i=0,1)$
Ω_s	Larmor frequency

REFERENCES

[1] M. TESSAROTTO (1982) – « N. Cimento », *69B*, 257.
[2] M. TESSAROTTO (June 1982) – In Proc. « IV Seminario Fisico-Matematico », Istituto di Meccanica, Università di Trieste, in press.
[3] M. N. ROSENBLUTH, R. D. HAZELTINE, F. L. HINTON (1972) – « Phys. Fluids », *15*, 116.
[4] S. P. HIRSHMAN, D. J. SIGMAR, J. F. CLARKE (1976) – « Phys. Fluids », *19*, 656.
[5] M. TESSAROTTO (1983) – « N. Cimento », *75B*, 19.
[6] R. D. HAZELTINE, H. R. STRAUSS (1976) – « Phys. Rev. Lett. », *37*, 102.
[7] A. B. RECHESTER, M. N. ROSENBLUTH (1978) – « Phys. Rev. Lett. », *40*, 38.
[8] J. T. HOGAN (1981) – « Nucl. Fusion », *21*, 365.
[9] M. TESSAROTTO (1981) – « Ann. Mat. Pura Appl. », *127*, 253.
[10] M. TESSAROTTO (1982) – « Z. Angew. Math. Mech. », *62* 521.
[11] P. H. RUTHERFORD (1970) – « Phys. Fluids », *13*, 482.
[12] J. W. EDENSTRASSER (1980) – « J. Plasma Phys », *24*, 299.
[13] M. TESSAROTTO (1982) – « Meccanica », *17*, 119.
[14] M. TESSAROTTO (1981) – « Rend. Acc. Lincei », *59*, 165.
[15] R. D. HAZELTINE, P. J. CATTO (1981) – « Phys. Fluids », *24*, 290.
[16] R. C. DAVIDSON (1974) – *Theory of Non-neutral Plasmas.* « Frontiers in Physics », W. A. Benjamin Inc. Reading, Mass. USA.