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A model for the occurrence of the acceleration of the ground caused by earthquakes

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Sismologia. — *A model for the occurrence of the acceleration of the ground caused by earthquakes.* Nota (*) del Socio MICHELE CAPUTO.

RIASSUNTO. — L'analisi statistica di quindici insiemi di misure di rilassamento di sforzo elastico in quindici differenti insiemi di terremoti (sequenze, sciami, scosse successive o terremoti scelti a caso in una regione) per un totale di 926 eventi in Italia, Giappone, Isole Aleutine, Messico e California, conferma che il rilassamento di sforzo p è una funzione decrescente di p ed è quasi proporzionale a $p^{\alpha-1}$ ($\alpha < 0$) come previsto da Caputo (1976).

Usando questa distribuzione statistica e quella delle dimensioni lineari delle faglie si introduce una nuova formula per il periodo di ritorno $n(a)$ della radice dell'accelerazione quadratica media maggiore di a causata dai terremoti di una regione sismica ad una distanza R; si trova che $n(a)$ è quasi proporzionale $a(\mathrm{R}^{3/2} a)^{-\delta}$ ($\delta = 3.5$).

INTRODUCTION

Earthquakes are described by a number of parameters which are estimated with an accuracy and frequency largely dependent on the complexity of the method used to measure the parameter.

At times the scale of some of these parameters has been changed, which is the case for intensity and magnitude (Kanamori and Hanks, 1979). Although these quantities are measured to a satisfactory accuracy, their use in modern seismic engineering is not of relevant importance because of the lack of their correlation with the acceleration of the ground due to earthquakes (Ambraseys, 1974).

For some of the parameters the analytic form for the density distribution function is also known, which is the case for the linear size of faults l (Caputo, 1976; Hanks, 1979) and also the stress drops p (Caputo, 1976, 1977; Caputo and Console, 1980). But direct information on the statistical distribution of p and l is still scauty, in spite of the importance of this information as regards the occurrence of ground accelerations caused by earthquakes.

In this paper we shall determine the statistical distribution of p and use it to model the occurrence of the acceleration of the ground.

Statistical analysis of the data.

Two recent papers by Caputo (1980) and Caputo and Console (1980) reported the statistical analysis of seven sets of stress drops and one set of fault linear dimension.

(*) Presentata nella seduta dell'8 novembre 1980.

In this note we shall report in Table I the statistical distribution of p observed in fifteen different sets of earthquakes. These events were selected in earthquake sequences, swarms, aftershocks series, or just random events which occurred in Japan, California, Mexico, Italy and the Aleutian Arc. We shall present in Fig. 1 a tentative numerical analysis of the sets of data which are sufficiently numerous.

TABLE I.

Statistical distribution of the stress drops computed by Basili *et al.* (1978) for a series of 24 aftershocks of the 1976 Friuli (Italy) earthquake (1); of the stress drops computed by Lee *et al.* (1979) for a series of 22 aftershocks of the 1979 Coyote Lake (California) earthquake (2); of the stress drops computed by Fletcher (1980) for a series of 26 aftershocks of the 1978 Oroville (California) earthquake (3); of the stress drops computed by Bakun *et al.* (1976) for the 1974 sequence of 26 earthquakes in Central California (4); of the stress drops computed by Munguia *et al.* (1977–78) for a series of 25 aftershorks of the 1978 Oaxaka (Baja California, Mexico) earthquake (5); of the stress drops computed by Correig and Udias (1980) for 12 scattered earthquakes in the Oceanic ridges (6); of the stress drops computed by Archambeau (1980) for 185 scattered earthquakes in the Aleutian and Alaskan regions (7); and for 123 scattered earthquakes in Japan (8).

1	P_i	0	30	60	90	120	
	N	14	8	1	1		
2	P_i	0	5	10	15	20	
	N	10	7	3	1		
3	P_i	0	4	8	12	16	20
	N	17	5	1	2	1	
4	P_i	0	10	20	30		
	N	16	3	3			
5	P_i	0	100	200	300	400	
	N	13	4	3	2		
6	P_i	0	5	10			
	N	10	2				
7	P_i	0	40	80	120		
	N	113	56	22			
8	P_i	0	100	200	300		
	N	77	32	12			

The analysis is made using, in a first approximation, the empirical laws (Caputo, 1976)

$$\text{(1)} \qquad \log N(p) = c_1 - (1 - \alpha) \log p \quad , \quad \log N(l) = c_2 - \nu \log l$$

where N is the number of stress drops (or linear size of faults) in the range p, $p + dp$ (or l, $l + dl$). c_1, c_2, $1 - \alpha$ and ν are constants to be determined empirically.

It has been shown that ν can be retrieved from the formulae (Caputo, 1976)

$$\text{(2)} \qquad \nu = 1 + \frac{3b}{\gamma} \quad , \quad 10^{\beta + \gamma M} = \eta k \frac{l^3 p^2}{2\mu}.$$

where b is the coefficient of the Gutenberg Richter empirical law for earthquakes statistics, η is the seismic efficiency, k is a geometric factor and μ is the rigidity.

One may immediately verify from Table I that the density distribution of the stress drops is a decreasing function of p: Fig. 1 verifies satisfactorily the validity of (1).

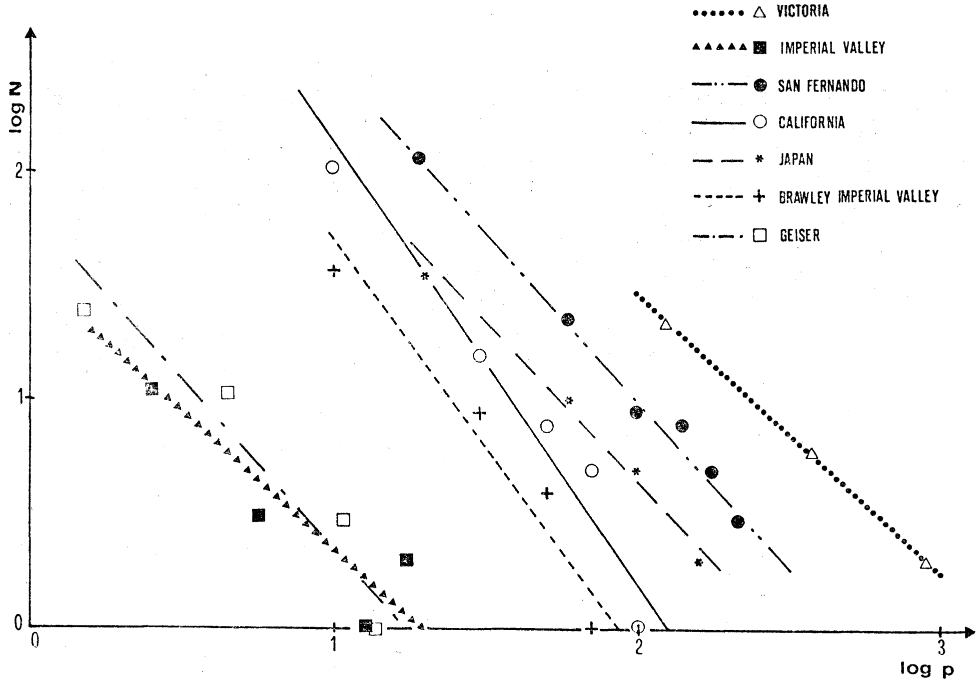

Fig. 1. – Linear regression for the stress drops of the 1971 San Fernando earthquake aftershocks (Tucker and Brune, 1973), of Japan (Mikumo, 1979; Utsu, 1979), of the Brawley–Imperial Valley, 1975 swarm (Hartzell and Brune, 1977), of Southern California (Thatcher and Hanks, 1973), of the Imperial Valley, 1977 swarm (Adair *et al.*, 1977), of scattered earthquakes in the Geiser (California) region (Peppin and Bufe, 1980) and of the 1978 Victoria (Mexico) swarm (Munguia *et al.*, 1978).

It is important to note that the stress drops computed by Archambeau (1980) for the Alaskan-Aleutian region and the Japanese region have been obtained by fitting the observed seismograms with the synthetic ones obtained from source models of the events which have the observed source parameters.

The stress drops of the set of data of Mikumo (1979) and Utsu (1979) have been computed by various methods including Brune's (1972) formula. They are mostly different from the method of Archambeau (1979).

The value of $-1 + \alpha$ resulting from the analysis of Archambeau's (1980) data for Japan is -1.2, that resulting from the data of Mikumo (1979) and Utsu (1979) is -1.5. Considering that the amount of data in both sets is relatively small the two values of $-1+\alpha$ are in satisfactory agreement.

In most cases it is difficult to make a good estimate of the errors of the statistical analysis of the sets of data on p and l reported here, but we may say that the errors of log l and log p are of the same order as those of log M_0 and M.

Few authors report the precision of their data. Peppin and Bufe (1980) computed the seismic moment and the stress drop for 41 earthquakes with magnitudes in the range 0.4 to 2.3 recorded in the Geysers geothermal area (California) and the near by active faults of the San Andreas system. They did not find large differences between the stress drops measured from vertical P wave and horizontal S wave spectral corner frequencies. They also computed the moment and stress drop using records of two or three different instruments. The scatter of the data is acceptable for the analysis made here; 65 % of their corner frequencies have a precision better than 10 %.

The values of α determined for the sets of data which are sufficiently numerous to allow a tentative estimate are in the range $-1 < \alpha < 0$ (Fig. 1).

The scattering may be due to the fact that the seismic regions examined are different or may be partly due to the scarcity of data in each set and to the consequently large and varied intervals used in the construction of the histogrames. Varying the size of the interval of the histogrames of each set may cause a scattering of the value of α of almost 10 % in the range -1 to zero. However it seems that $\alpha < 0$.

Since the return period T (p) of the stress drops is inversely proportional to N (p), we have

$$\mathrm{T}(p) = \frac{1}{\mathrm{N}(p)} = 10^{-c_1}\, p^{1-\alpha} \tag{2}$$

from which it results that the time of accumulation of the stress drop p is not proportional to p.

This in turn may have several implication. It is possible that creep takes place in the accumulation of large stresses and/or that there is a particular distribution of the directions of the faults with respect to that of the major tectonic force acting in the region (Caputo, 1977).

The different values of ν imply different fracturation of the crust and may be associated with the length of time during which the crust of the region has been subject to tectonic forces and to the intensity of these forces. A tentative study of this correlation seems to confirm this hypothesis (Schettino, 1980).

In a numerical study Mikumo and Miyatake (1979) have investigated the space and time characteristics of earthquake sequences of a frictional fault model with non uniform strengths and relaxation times which is subject to a time dependent shear stress. They found good linear relations between the log of source area A and the cumulative distribution N

$$\log N = c_3 + c_4 \log A . \tag{3}$$

This is identical to relation (1).

Mikumo and Miyatake (1979) obtain C_4 values ranging from -1.1 to -1.4, which give values of ν in the range from 3.2 to 3.8. The associated values of $b = \frac{\nu - 1}{3} \gamma$ (Caputo, 1976) are somewhat larger than those observed in the world, as noted by Mikumo and Miyatake (1979), but they depend on the value of $\gamma = 1.5$ assumed implicity by Mikumo and Miyatake (1979) but not necessarely valid for the numerical model.

Model of occurrence of the accelerations of the ground.

Let us now apply the results of the preceding sections to the determination of the cumulative distribution of the *rms*-value of the acceleration of the ground caused by a seismic region.

Available seismological evidence suggests that $\omega^{-\varepsilon}$ $(\varepsilon = 2)$ represents the high-frequency spectral decay of the spectrum of the far-field radiation of crustal earthquakes (Hanks, 1979). This high-frequency radiation in the presence of anelastic attenuation Q is integrated as band limited, finite duration, white noise in acceleration; its *rms*-value a, is (Hanks, 1979)

$$a = 0.317 \frac{p}{\rho R} \sqrt{\frac{f_{\max}}{f_0}} \tag{4}$$

where ρ is the density, R the hypocentral distance, f_0 the spectral corner frequency and $f_{\max} = Q\beta/\pi R$ is determined by Q, R and the velocity of the waves β.

Substituting (Brune, 1970)

$$f_0 = 2.34 \, \beta/l$$

in (4) we find

$$a = Kp \sqrt{l} \quad ; \quad K = \frac{0.52}{\rho R} \sqrt{\frac{f_{\max}}{\beta}} = \frac{0.52}{\rho R^{3/2}} \sqrt{\frac{Q}{\pi}} . \tag{5}$$

The cumulative distribution function $n(a)$ of a can be determined from the density distributions (1) of l and p as follows. If p_1, p_2, l_1, l_2 are minimum and maximum values of p and l of the seismic region following the method of Caputo (1976), by integrating in the domain of the plane l, p defined by (5) and by the minimum and maximum values of p and l, we find that the cumulative distribution of a for $a > kp_1 \sqrt{l_2}$ (or $a > kp_2 \sqrt{l_1}$ whichever is larger) is

$$n(a) = \frac{D}{1 - \nu} \left\{ \frac{p_2^{\alpha} l_2^{1-\nu}}{\alpha} - \frac{(2\nu - 2) \, l_2^{1-\nu-(\alpha/2)} \, a^{\alpha}}{\alpha (2\nu + \beta - 2) \, k^{\alpha}} - \frac{p_2^{2\nu+\alpha-2} \, a^{2-2\nu}}{k^{2-2\nu} (2\nu + \alpha - 2)} \right\} . \tag{6}$$

In the crust it is reasonable to assume $\rho = 2{,}8$ and $Q = 300$ for the seismic waves of interest, this gives $K = 1.02\ R^{-3/2}$. If we measure a in units of $kp_2 \sqrt{l_2}$ and set $a^* = a/kp_2 \sqrt{l_2}$ formula (6) is very simple

$$(7) \qquad n(a) = \frac{D p_2^{\alpha}\, l_2^{1-\nu}}{\nu - 1} \left(- \frac{1}{\alpha} + \frac{2\nu - 2}{\alpha(2\nu + \alpha - 2)}\, a^{*\alpha} + \frac{a^{*\,2-2\nu}}{2\nu + \alpha - 2} \right).$$

Since $\alpha < 0$, $\nu > 2$, it is seen that $n(a)$ is an increasing function of p_2 and decreasing function of a and R.

For small values of a in the range $p_1 k \sqrt{l_2}$ to $p_2 k \sqrt{l_1}$ like those considered here the second term of (7) is small relative to the third. Since $\nu \simeq 2.8$, $n(a)$ is nearly proportional to $(aR^{3/2})^{-\delta}$ $(\delta \simeq 3.5)$.

It is important to note that, considering formula (5) and the density distributions of p and l given by (1), a given pair of values of a and R correspond to an infinite number of values of M and that, of all the values of M associated with this pair, the minimum value of M is that which has the highest probability of occurrence. In turn this implies that the return periods of the acceleration of the ground caused by earthquakes are much longer than previously estimated through empirical relations between M and a.

If we want to take into account that the seismicity is not concentrated in a point at distance R but covers an area which we assume here for simplicity to be a sector of a circular ring defined by the angles ϑ_1, ϑ_2 and by the radii R_1 and R_2, then $n(a)$ is

$$n(a) = \frac{D p_2^{\alpha}\, l_2^{1-\nu}}{1 - \nu} \left\{ \frac{1}{\alpha} - \frac{4(\nu - 1)\,(a/p_2 \sqrt{l_2})^{\alpha}\,(R_3^{(3/2)\alpha+2} - R_1^{(3/2)\alpha+2})}{(3/2\,\alpha + 2)\,\alpha\,(2\nu + \alpha - 2)\,(R_2^2 - R_1^2)} - \right.$$

$$\left. - \frac{2\,(a/p_2 \sqrt{l_2})^{2(1-\nu)}\,(R_3^{5-3\nu} - R_1^{5-3\nu})}{(2\nu - 2 + \alpha)\,(5 - 3\nu)\,(R_2^2 - R_1^2)} \right\}$$

where R_3 is egual to R_2 or $(p_2 \sqrt{l_2})^{2/3}$, whichever is smaller.

In California $\nu = 2.78$, $\alpha = -1$ (Caputo and Console, 1980) a reasonable value of p_2 is 500 bar as shown from the data of Thatcher and Hanks (1973); we may tentatively consider a maximum linear dimension of a fault $l_2 = 5 \cdot 10^6$. The value of D can be deduced using the formula (15) of Caputo (1976) with $p_2 = 5 \cdot 10^8$, $p_1 = 5 \cdot 10^5$ and the data of Epstein and Lomnitz (1966) on Northern California earthquakes: one obtains $D = 10^{16{,}35}$. For $a = 50$ gal , $R_2 = 8\ R_1 = 8.10^6$, formula (7) gives $n(50) = (17)^{-1}$ or a return period of 17 years.

In Fig. 2 we may also see that an earthquake with $M = 5$ could cause a *rms* acceleration of 50 gal at 10 km distance but it would have to be an earthquake with a stress drop of 500 bar and linear dimension of fault of 100 m, which is extremely rare, which is why the return period estimated above is so long.

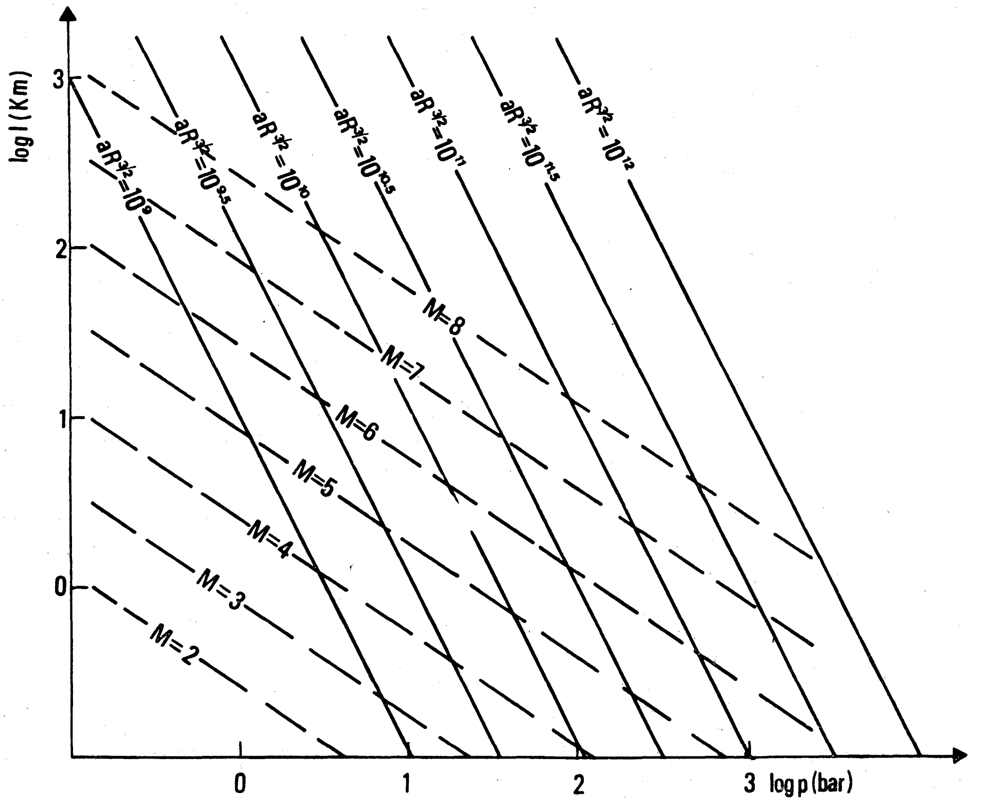

Fig. 2. – Isolines of magnitude and *rms* acceleration a in the plane of the linear dimension of faults l and stress drops p.

The estimate of ν and α allows also to estimate the spectrum of the energy density; for intermediate values of E the energy density is proportional to $CE^{1-\nu/3}\,dE$, for large values of E it is proportional to $(K_1\,E^{\alpha/2} + K_2\,E^{-(\nu-1)/3})\,dE$, K_1, K_2 and C are parameters depending on the seismic region. It is important to note that, although most of the elastic energy of the crust is released in the large Earthquakes, the energy density release is larger in small Earthquakes.

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