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Criterion for Unboundedness in the Constructive Approach to the Stability of Matrices

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Matematica. — *Criterion for Unboundedness in the Constructive Approach to the Stability of Matrices.* Nota di HING TONG, presentata (*) dal Socio G. FICHERA.

RIASSUNTO. — In un lavoro recente [1], R. K. Brayton e C. H. Tong hanno dato un algoritmo costruttivo per determinare la stabilità di un sistema di matrici. In questo, la regione W^* ha un ruolo centrale. Essi danno una condizione sufficiente per la non-limitatezza di W^*. Nella nota presente noi diamo una condizione necessaria e sufficiente perchè W^* sia non-limitata in R^2. È asserita la generalizzazione di questo risultato a R^n e C^n.

In a recent paper [1] R. K. Brayton and C. H. Tong introduced a powerful constructive algorithm which determines the stability of a set of matrices. A set of $n \times n$ complex (real) matrices is stable if for every neighborhood U of the origin in $C^n(R^n)$, there exists a neighborhood of the origin V, such that for each $M \in A'$ (the set of finite products of matrices in A), $MV \subseteq U$. Matrix and Liapounov stability are related. This paper provided a constructive procedure for obtaining Liapounov functions. A dynamical system of differential equation is stable if a corresponding set of matrices is stable. The algorithm was based on the following theorem whose proof was given: Let $A = \{M_0, M_1, \cdots, M_{m-1}\}$ be a set of distinct matrices. Let W_0 be a bounded neighborhood of the origin. Then for integer $k > 0$, define:

$$W_k = \text{convex hull}\left[\bigcup_{i=0}^{\infty} M_{k'}^i, W_{k-1}\right] \quad \text{where } k' \equiv (k-1) \bmod m .$$

Then A is stable if and only if $W^* = \bigcup_{i=0}^{\infty} W_i$ is bounded. Sufficient conditions for the finiteness of constructing W_k from W_{k-1}, and for the stopping of the algorithm for a stable or an unstable set are given. The following sufficient condition was proved.

If there exists a k such that $\partial W_0 \cap \partial W_k = \varnothing$, then W^* is unbounded.

Let $A = \{M_0, M_1, \cdots, M_{m-1}\}$ be a set of stable, real, 2×2 matrices. (M is stable if there exists a K such that $\| M^i \| \leq K$ for all i). Let W_0, W_k, and W^* be as given above and in R^2. We announce here the following thorem:

A necessary and sufficient condition that W^* be unbounded is that either there exists k such that $\partial W_0 \cap \partial W_k = \varnothing$ or there exist M_i, and M_j, distinct members of A, such that in an appropriately chosen coordinate system, the

(*) Nella seduta del 12 aprile 1980.

operators, M_i and M_j, can be put in the form $\begin{pmatrix} 1 & \alpha_1 \\ 0 & -1 \end{pmatrix}$ $\left(\text{or } \begin{pmatrix} -1 & \alpha_1 \\ 0 & 1 \end{pmatrix}\right)$ and $\begin{pmatrix} 1 & \alpha_2 \\ 0 & -1 \end{pmatrix}$ $\left(\text{or } \begin{pmatrix} -1 & \alpha_2 \\ 0 & 1 \end{pmatrix}\right)$ with $\alpha_1, \alpha_2 \neq 0$ and M_i and M_j not negatives of each other.

This result, with appropriate modification, can also be generalized to $W^* \subset C^n (R^n)$.

In the above theorem, we limit ourselves to stable matrices M_i; for it any M_i is not stable, it is readily seen that W^* is unbounded.

Proofs of these results will be presented elsewhere.

REFERENCE

[1] BRAYTON R. K., and TONG C. H. – *Stability of Dynamical Systems: A Constructive Approach*, « IEEE Trans. on Circuits and Systems », Vol. CAS-26, April, 1979, 224–234.