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Algebra. — *On subgroups of certain alternating groups.* Nota di RUDY J. LIST, presentata (*) dal Socio G. ZAPPA.

RIASSUNTO. — Siano S_n e A_n rispettivamente il gruppo simmetrico e il gruppo alterno su n lettere, e sia G un sottogruppo di S_n. Per le seguenti coppie (G, n), se $G \subseteq H \subseteq S_n$, si ha che o $H \subseteq \mathrm{Aut}\, G$ o $H \supseteq A_n$.

(*i*) G è il gruppo semplice eccezionale scoperto da Higman e Sims, e $n = 100$;

(*ii*) G è come in (*i*), e $n = 176$;

(*iii*) G è il gruppo semplice eccezionale scoperto da McLaughlin, e $n = 275$;

(*iv*) G è il più piccolo gruppo semplice eccezionale scoperto da Conway, e $n = 276$;

(*v*) G è $PSU_4(3^2)$, e $n = 112$.

1. INTRODUCTION

Let Ω denote a finite set, and let $S(\Omega)$ and $A(\Omega)$ be the symmetric and alternating groups on Ω respectively. A general approach to problems involving the question of maximality of a primitive permutation group G in $A(\Omega)$ or $S(\Omega)$ is to consider whether an overgroup H must be more highly transitive than G. The general idea is to examine the extent to which the orbits on $\Omega - U$ of the stabilizer in G of a subset U of Ω must join together when passing to the stabilizer of U in H. In this note we illustrate some aspects of this approach by examining the pairs (G, Ω) in the following cases:

a) G is the exceptional simple group discovered by Higman and Sims, and $|\Omega| = 100$.

b) G is again the Higman-Sims group, and $|\Omega| = 176$.

c) $G \simeq PSU_4(3^2)$, and $|\Omega| = 112$.

d) G is the simple group discovered by McClaughlin, and $|\Omega| = 275$.

In each case we prove that if $G \subseteq H \subseteq S(\Omega)$, either $H \subseteq \mathrm{Aut}(G)$, or $H \supseteq A(\Omega)$. If G is the McClaughlin group $\mathrm{Aut}(G) \simeq G.2$, and this is the stabilizer of a point in the smallest Conway group when it is represented on 276 points. Hence the smallest Conway group is a maximal subgroup of A_{276}.

If M is a permutation group on a set Λ, and if $\{\alpha, \beta, \cdots, \gamma\} \subseteq \Lambda$, the pointwise stabilizer of $\{\alpha, \beta, \cdots, \gamma\}$ is denoted by $M_{\alpha\beta\cdots\gamma}$, and the setwise stabilizer is denoted by $M_{(\alpha\beta\cdots\gamma)}$. $M \cdot N$ denotes an extension of M by N. When convenient an orbit of length m is denoted by O_m. If there are several orbits of length m, they may be denoted by $O_m^1, O_m^2, \cdots$. If $\Delta \subseteq \Lambda$, $M \mid \Delta$

(*) Nella seduta dell'8 marzo 1980.

denotes the restriction of M to Δ. If m and n are integers, $m \mid n$ means m divides n. Thus $|H| \mid |(G \mid \Delta)|$ means the order of H divides the order of G restricted to Δ.

2. In this section we prove *a*), *b*), *c*), *d*)

a) Higman and Sims construct a graph $\mathscr{I}$ on 100 vertices, and G is a subgroup of index 2 in Aut $(\mathscr{I})$. G is rank-3 on Ω and $G_\alpha \simeq M_{22}$, with subdegrees 1, 22, 77. Hence if $H \nsubseteq$ Aut (G), H is 2-transitive on Ω. O_{22} and O_{77} correspond to the points and blocks of a Steiner system $\mathscr{S} = \mathscr{S}(3, 6, 22)$, and edges of $\mathscr{I}$ may be described in terms of incidence in $\mathscr{S}$. A detailed description of the geometry of $\mathscr{S}$ has been given in [11]. We use results and easy consequences from [11] without further reference to it. If $\beta \in O_{22}$, $\gamma \in O_{77}$ the orbits of $G_{\alpha\beta}$, $G_{\alpha\gamma}$ may be diagrammatically summarized as follows:

$G_{\alpha\beta} : \alpha\ \beta$ |— 21 —|— 21 —|— 56 —|

$G_{\alpha\gamma} : \alpha\ \gamma$ |— 6 —|— 16 —|— 16 —|— 60 —|

Here, for example, |— 21 —|— 21 —|— 56 —| means that $\Omega - \{\alpha, \beta\}$ is the union of three orbits $O^1_{21}, O^2_{21}, O_{56}$. Set $O^2_{21} \cup O_{56} = O_{77}$.

The orbits of $H_{\alpha\beta}$, $H_{\alpha\gamma}$ are unions of orbits of $G_{\alpha\beta}$, $G_{\alpha\gamma}$ respectively, and since H is 2-transitive, the orbit diagrammes of $H_{\alpha\beta}$ and $H_{\alpha\gamma}$ are equivalent. This can only happen if H is 3-transitive.

Take $\rho \in O^1_{21}$, $\delta \in O_{56}$. From the geometry of $\mathscr{S}$ the following situation occurs:

$G_{\alpha\beta\rho}$, $\beta \in O_{22}$, $\rho \in O^1_{21} : \alpha\ \beta\ \rho$ |— 5 —|— 16 —|— 16 —|— 20 —|— 40 —|

$G_{\alpha\beta\delta}$, $\beta \in O_{22}$, $\delta \in O_{56} : \alpha\ \beta\ \delta$ |— 6 —|— 6 —|— 10 —|— 15 —|— 15 —|— 45 —|

By 3-transitivity the orbits of $H_{\alpha\beta\rho}$ and $H_{\alpha\beta\delta}$ are equivalent. Clearly the only possibilities are O_{16}, O_{36}, O_{45}, or O_{45}, O_{52}. $(45, 16) = (45, 52) = 1$, so $H_{\alpha\beta}$ is imprimitive by a theorem of Weiss [19; 17.5]. Considering the divisors of 98 this is clearly impossible. Therefore H is 4-transitive and so $H \supseteq A(\Omega)$ [19; 13.9].

b) Ω may be taken to be the cosets of a $P\Sigma U_3(5^2) \subseteq G$. Using the $P\Sigma U_3(5^2)$ located explicitly in G by Magliveras generators of G on Ω were constructed. Much information about G represented as a subgroup of $A(\Omega)$ is contained in [9], and we assemble some of it here.

G is 2-transitive on Ω. If $\alpha, \beta \in \Omega$, $G_{\alpha\beta}$ has orbits $\{\alpha\}, \{\beta\}$, and $\Delta(\beta)$, $\Gamma(\beta), \Sigma(\beta)$ of lengths 12, 72, 90 respectively. $G_{\alpha\beta} \simeq \mathrm{Aut}(S_6)$. Following [9] $\Delta(\beta) = D \cup D^*$, where $D \cap D^* = \varnothing$, $|D| = |D^*| = 6$. Denote $G_{(\{\alpha,\beta\} \cup D)}$ by K. $K \simeq S_8$, and the action of K restricted to $\Omega - (\{\alpha, \beta\} \cup D)$ is imprimitive of block length 6, the blocks of imprimitivity being conjugates of D^* under K. For $\gamma \in D$ the diagrammes of $G_{\alpha\beta\gamma}$ and $G_{(\alpha\beta\gamma)}$ are respectively:

$G_{\alpha\beta\gamma} : \alpha\ \beta\ \gamma$ |5|6|6|6|30|30|30|60|

$G_{(\alpha\beta\gamma)}$: |3|5|18|60|90|

$G_{(\alpha\beta\gamma)}/G_{\alpha\beta\gamma} \simeq S_3$ and acts on the orbits of length 6 and 30.

Take $\sigma \neq 1, \sigma \in H_{\alpha\beta} - G_{\alpha\beta}$. Some conjugate of σ restricts nontrivially to $\Delta(\beta)$, since $\Omega - (\{\alpha, \beta\} \cup D)$ is union of conjugates of D^*. Aut (S_6) is a maximal subgroup of M_{12}. Thus if $\Delta(\beta)$ is an orbit of $H_{\alpha\beta}$, $H_{\alpha\beta} \mid \Delta(\beta)$ contains M_{12}, so $H_{\alpha\beta}$ has an orbit O_i, $i > 12$, $i \mid 11.12$ [19; 17.7]. This is impossible given the subdegrees of $G_{\alpha\beta}$. Thus if H is not 3-transitive, $H_{\alpha\beta} \mid \Omega - \{\alpha, \beta\}$ has orbits (i) $\Delta(\beta) \cup \Gamma(\beta)$, $\Sigma(\beta)$ or (ii) $\Delta(\beta) \cup \Sigma(\beta)$, $\Gamma(\beta)$. In case (i) take $\rho \in \Delta(\beta)$, $\gamma \in \Gamma(\beta)$. Then $|(\Delta(\rho) \cup \Gamma(\rho)) \cap \Sigma(\beta)| = |(\Delta(\gamma) \cup \Gamma(\gamma)) \cap \Sigma(\beta)|$. Taking $\beta = 2$, $\rho = 14$, $\gamma = 13$ and consulting the appendix the cardinalities are 50 and 30 respectively.

In case (ii) $H \mid O_{174}$ has orbits O_{72}, O_{102}, and an element of order 17 when restricted to O_{72} has $4 + k.17$ fixed points, $0 \leq k \leq 3$. Clearly $H_{\alpha\beta} \mid O_{72}$ is primitive. Using the fact that $G_{\alpha\beta}$ contains elements of order 5 fixing two points of O_{72} and arguing as in *a*) $H_{\alpha\beta} \mid O_{72}$ is 2-transitive. But 71 is prime. Therefore $H \supseteq A(\Omega)$ [19; 13.10].

Hence H is 3-transitive. From the diagrammes of $G_{\alpha\beta\gamma}$, $G_{(\alpha\beta\gamma)}$ above, either $H_{\alpha\beta} \mid \Omega - \{\alpha, \beta\}$ is primitive or imprimitive with block length 6 and image of imprimitivity in S_{29}. $G_{\alpha\beta\gamma} \supseteq S_5$, so $H_{\alpha\beta}$ is not solvable. Therefore $H_{\alpha\beta}$ acting on the blocks contains A_{29} [1]. Hence a Sylow 17-subgroup of H fixes 74 points, and $H \supseteq A(\Omega)$ [19; 13.10]. If $H_{\alpha\beta} \mid \Omega - \{\alpha, \beta\}$ is primitive $H_{\alpha\beta\gamma}$, $\gamma \in D$, has no O_5 by [19; 17.7]. The possibilities for orbits of $H_{(\alpha\beta\gamma)}$ obtained by joining O_5 to other orbits of $G_{(\alpha\beta\gamma)}$ are O_i, $i = 23, 65, 95, 83, 113, 155, 173$. By the prime factorization of these i, if O_i is an orbit of $H_{(\alpha\beta\gamma)}$ it must also be one of $H_{\alpha\beta\gamma}$. Hence $i = 173$ [19; 17.5], and $H \supseteq A(\Omega)$ [19; 13.9].

c) Ω may be taken to be the set of maximal isotropic subspaces of $V_4(3^2)$ with a unitary geometry. This geometry is classical and we assume familiarity with it. If $\alpha \in \Omega$, let $\Delta(\alpha)$ and $\Gamma(\alpha)$ of lengths 30 and 81 respectively be the nontrivial orbits of G_α. Denote the set of blocks of $G_\alpha \mid \Delta(\alpha)$ by $B(\alpha)$ Take $a_1 \in \Delta(\alpha)$ and let $\{a_2, a_3\} = \Delta(\alpha) \cap \Delta(a_1)$. Set $\{i, j, k\} = \{1, 2, 3\}$. Then $\Delta(a_i) = \{\alpha, a_k, a_j\} \cup O^i_{27}$, where $O^i_{27} = \Gamma(\alpha) \cap \Delta(a_i)$; $O^i_{27} \cap O^j_{27} = \varnothing$, $i \neq j$; $\Delta(a_i) \cap \Delta(a_j) = \{\alpha, a_k\}$; O^t_{27}, $t = 1, 2, 3$ are the orbits of $K_{\alpha a_1 a_2 a_3} \mid \Gamma(\alpha)$ where K is the kernel of G_α acting on $B(\alpha)$. The orbits of a Sylow 3-subgroup P of $G_{\alpha a_1 a_2 a_3}$ are $\{\alpha\}$, $\{a_i\}$, $i = 1, 2, 3$, $\Delta(\alpha) - \{a_1, a_2, a_3\}$, O^i_{27}, $i = 1, 2, 3$.

Aut $(G)/G \simeq D_4$, and Aut $(G)_\alpha \simeq K.(C_2 \times P\Gamma L_2(9))$. The central involution in $C_2 \times P\Gamma L_2(9)$ inverts every element of K.

Since A_6 cannot be represented reducibly as a subgroup of $GL_4(3)$, $C_{GL_4(3)}(A_6) = Z(GL_4(3)) = C_2$. Also maximal elementary abelian 2-groups of

$GL_4(3)$ are of order 16, and A_6 can be represented in $GL_4(2)$ in just one way; hence if L is a 2-group and $L.A_6 \subseteq GL_4(3)$, $L \simeq C_2$.

Now suppose that H is rank-3. Then $H_\alpha \mid \Delta(\alpha)$ is faithful, since $G_\alpha \mid \Gamma(\alpha)$ is primitive. Suppose $H_\alpha \mid \Delta(\alpha)$ is imprimitive, and let J be the kernel of imprimitivity. $K \mid \Gamma(\alpha)$ is self centralizing in $S(\Gamma(\alpha))$, so $J \mid \Gamma(\alpha) = K \mid \Gamma(\alpha)$. It follows that if σ is of order 3 in $J - K$, $\sigma(\Delta(b)) \neq \Delta(b)$ while $\sigma(b) = b$, for some $b \in \Gamma(\alpha)$. This is impossible. By the remarks concerning embedding A_6 in $GL_4(3)$ and $GL_4(2)$ it now follows that $J = K$ or else $J = K.C_2$, and C_2 inverts each element of K. Hence H_α represented on $B(\alpha)$ contains $A(B(\alpha)) = A_{10}$. But $A_{10} \nsubseteq GL_4(3)$. This is impossible ($K \simeq V_4(3)$). Suppose, therefore, that $H_\alpha \mid \Delta(\alpha)$ is primitive, so that $H_\alpha \mid \Delta(\alpha)$ and $H_\alpha \mid \Gamma(\alpha)$ are both faithful. Considering the orbits of P it follows that $H_\alpha \mid \Delta(\alpha)$ is 2-transitive [19; 13.1]. An element of order 29 fixes at least 25 points of Ω. Therefore $H \supseteq A(\Omega)$ [19; 13.10]. Therefore H is 2-transitive, and $H_\alpha \mid \Omega - \{\alpha\}$ is primitive, since $111 = 3.37$ and $G_\alpha \mid \Gamma(\alpha)$ is primitive. Therefore H is 3-transitive [13]. If $H_{\alpha\beta} \mid \Omega - \{\alpha, \beta\}$ is imprimitive, the block containing $\gamma, \gamma \neq \alpha, \beta$, consists of γ and a union of orbits of P, i.e., blocks must have length 2 or 55. For $\beta \in \Gamma(\alpha)$, $G_{\alpha\beta} \simeq A_6$ has orbit diagramme $\alpha\beta\ \vert\overset{10}{\rule{1em}{0.4pt}}\vert\overset{20}{\rule{3em}{0.4pt}}\vert\overset{20}{\rule{3em}{0.4pt}}\vert\overset{30}{\rule{3em}{0.4pt}}\vert\overset{30}{\rule{3em}{0.4pt}}\vert$ by [4]. Clearly 55 is impossible. Since $A_6 \nsubseteq S_5$, so is 2. Hence $H_{\alpha\beta} \mid \Omega - \{\alpha, \beta\}$ is primitive. Arguing now as in *a*) and *b*) using theorems of Cameron and Weiss [19; 17.5], H is 4-transitive and therefore $H \supseteq A(\Omega)$ [19; 13.9].

d) For $x \in \Omega$, $G_x \simeq PSU_4(3^2)$ with suborbits $\Delta(x)$, $\Gamma(x)$ of lengths 112, 162 respectively. Sylow 3-subgroups of G fix two points and have nontrivial orbits O_3, O_{27}, O_{81}^j, $j = 1, 2, 3$. If $y \in \Gamma(x)$, $G_{xy} \supseteq A_6$ with orbits O_{10}, O_j^1, O_j^2, $j = 20, 30, 36, 45$ [4].

Suppose H is rank–3. By *c*) $[H:G] \mid 8$, and $G \trianglelefteq H$. G contains one class of $PSU_4(3^2)$ and $PSU_3(5^2)$ [4], and each of these has trivial centralizer in S_{275}. Hence H/G is faithfully represented in $Aut(J)/J \simeq C_6, D_4$, for $J \simeq PSU_4(3^2)$, $PSU_3(5^2)$ respectively. Hence $[H:G] \mid 2$ and $H \subseteq Aut(G)$.

Hence $H \nsubseteq Aut(G)$, and so H is 2-transitive. $274 = 2.137$ and H_x is primitive. Therefore H is 3-transitive [19; 31.1]. If $H \nsubseteq Aut(G)$, then $H \cap A(\Omega) \nsubseteq Aut(G)$, so $H \cap A(\Omega)$ is 3-transitive, and so we may assume that $H \subseteq A(\Omega)$. Then if $|\langle\sigma\rangle| = 137$ and $H \neq A(\Omega)$, σ fixes one point and is self centralizing. If ρ normalizes but does not centralize σ, then, $|\langle\rho\rangle| \mid 136$, ρ fixes exactly 3 points a, b, c of Ω and acts semiregularly on $\Omega - \{a, b, c\} = \Omega'$. Further $|N_H(\sigma)| \neq 2.137$ [10].

Let S be a Sylow 3-subgroup of G_{xy} such that $\{a, b, c\} = O_3$. By 3-transitivity there is an $A_6 \subseteq H_{abc}$ with orbits O_{10}, O_j^1, O_j^2, $j = 20, 30, 36, 45$. Set $H_{(abc)} = M$. $M \supseteq \langle A_6, S, \rho\rangle$, where ρ has order 4 or 17. From the orbits of A_6 and S and the semiregularity of ρ on Ω' it follows that M is transitive on Ω'. If $M \mid \Omega'$ is imprimitive, the orbits of S force block length 2. Then O_{10} is a union of 5 blocks, whereas $A_6 \nsubseteq S_5$. Therefore $M \mid \Omega'$ is primitive. Since

$H_{abc} \triangleleft M$, H_{abc} is transitive on Ω' [19; 8.8], so H is 4-transitive on Ω. Consider M_x, $x \in \Omega'$. By 4-transitivity, there is an element of order 5 fixing $\{a, b, c, x, y\}$. Hence orbits of $M_x \mid \Omega' - \{x\}$ are unions of $\{x\}$, O_{27}, O_{81}^j, $j = 1, 2, 3$, and exactly one has length congruent to 1 (mod 5), all others being congruent to 0 (mod 5). The possibilities are: 1, 270; 190, 81. These both imply that $H \supseteq A(\Omega)$ by arguing as in *a*), *b*), *c*), and using the fact that A_{27} has no proper subgroup of index dividing 190.

APPENDIX

I. *Generators of the Higman–Sims group as a subgroup of* A_{176}.

$a = (1)(i, i+1, i+2, i+3, i+4, i+5, i+6), \quad 2 \le i \le 176, \quad i \equiv 2 \pmod 7$

$b =$ (1,2) (3,9) (4,16) (5,23) (6,30) (7,37) (8,44) (10,25) (11,51) (12,58) (13,65) (14) (15,72) (17,49) (18,79) (19,45) (20,86) (21,93) (22,100) (24,107) (26,108) (27,32) (28,114) (29,121) (31,87) (33,128) (34,77) (35,46) (36,48) (38) (39,135) (40,129) (41,75) (42,54) (43,116) (47) (50,132) (52,59) (53) (55) (56,97) (57,130) (60) (61,142) (62,149) (63,127) (64,92) (66,88) (67,133) (68,156) (69,118) (70,113) (71,163) (73,148) (74,165) (76,81) (78,164) (80,159) (82,106) (83) (84,167) (85,104) (89) (90,168) (91,139) (94,124) (95,105) (96,119) (98) (99,170) (101,162) (102,117) (103,141) (109) (110,160) (111,140) (112,157) (115,154) (120) (122,147) (123,137) (125,150) (126,175) (131,144) (134,171) (136,158) (138) (143,161) (145,176) (146,169) (151) (152,172) (153,155) (166) (173) (174).

II. *Orbits of* $G_{1,2}$; $|\Omega| = 176$.

$A = \{1\}$, $B = \{2\}$, $C = \{14, 35, 38, 43, 46, 83, 102, 116, 117, 136, 151, 158\}$,

$D = \{$3, 4, 7, 8, 9, 12, 15, 16, 19, 21, 24, 26, 28, 29, 31, 34, 37, 39, 40, 41, 44, 45, 52, 56, 58, 62, 63, 67, 69, 71, 72, 75, 76, 77, 81, 84, 85, 87, 91, 93, 94, 97, 101, 104, 107, 108, 112, 114, 118, 121, 124, 127, 129, 131, 133, 134, 135, 139, 144, 149, 152, 153, 155, 157, 162, 163, 167, 171, 172, 176$\}$,

$E = \Omega - (A \cup B \cup C \cup D)$.

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