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Michele Caputo

## Statistical analysis and models of stress accumulation and release in the interior of the earth

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**Sismologia.** — *Statistical analysis and models of stress accumulation and release in the interior of the earth.* Nota (*) del Socio MICHELE CAPUTO.

RIASSUNTO. — Si presentano le analisi statistiche di alcuni insiemi di dati sul rilassamento di sforzo $p$ che avviene durante i Terremoti. Queste analisi suggeriscono una densità di distribuzione del tipo $p^{-1+\alpha}$ $(-1 < \alpha < -0{,}4)$ che è in accordo col modello suggerito da Caputo (1976). Si discutono poi alcune ipotesi sulle modalità di accumulo dell'energia elastica all'interno della Terra in funzione del tempo.

## INTRODUCTION

The knowledge of the statistical distribution of stress relaxation associated to the Earthquakes of a seismic regions is of great importance for the information it gives on the time required for the accumulation of stress; this in turn gives information on the mechanism of accumulation of stress.

A physical model for the statistical study of regional seismicity has been recently proposed by Caputo (1976, 1977, 1979).

In the study of Caputo (1979) it has been shown that the following analitic density distribution function of linear size of faults $l$ $(l = \sqrt{\mathrm{A}}$, A area of the fault) and stress drops $p$

$$\mathrm{D} l^{-\nu} p^{-1+\alpha}\, \mathrm{d}l\, \mathrm{d}p \tag{1}$$

where D, $\nu$ and $\alpha$ are real numbers, is the unique distribution which gives the generally accepted empirical density distributions of earthquakes of magnitude M and moment $\mathrm{M}_0$

$$\begin{aligned} \log \bar{n}(\mathrm{M}) &= \bar{\mathrm{A}} - \bar{\mathrm{B}}\mathrm{M} \\ \log \bar{n}_0(\mathrm{M}_0) &= \bar{\mathrm{A}}_0 - \bar{\mathrm{B}}_0 \log \mathrm{M}_0 \,. \end{aligned} \tag{2}$$

It is shown in the study of Caputo (1976) that

$$\bar{\mathrm{B}} = \frac{\nu - 1}{3}\gamma \quad , \quad \bar{\mathrm{B}}_0 = \frac{\nu - 1}{3} \tag{3}$$

where $\gamma$ is the parameter appearing in the following empirical relation established by Richter (1958)

$$\mathrm{E} = 10^{\beta+\gamma \mathrm{M}} \tag{4}$$

(*) Presentata nella seduta del 12 gennaio 1980.

where E is the energy of the Earthquake of magnitude M which has been transformed into elastic waves.

The model has been verified to some extent by Caputo and Console (1977, 1979) with two sets of data on California and Japan reported here in Fig. 1, in a slightly different version.

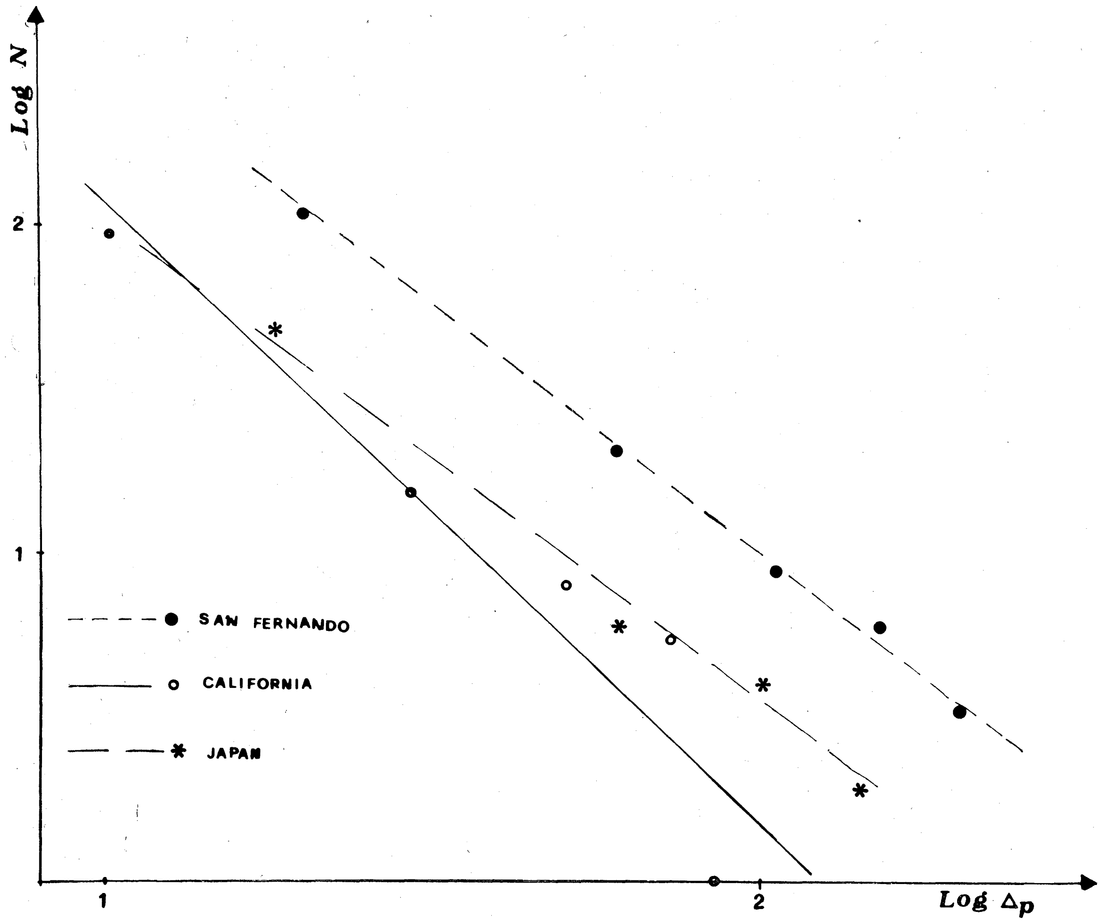


Fig. 1. – Statistical distribution of stress drops in California ($\alpha = -1$) and Japan ($\alpha = -0.4$). The lines represent the best fits and were obtained with the method of maximum likelyhood with $\sigma(\log n)/\sigma(\log \Delta p) = 1$. Nearly the same distributions were obtained computing the $\Delta p$ from (1) and (2).

The $\alpha$ parameter of formula (1) which can be estimated from the statistical analysis of the stress drops of seismic regions is therefore of great importance for the study of mechanism of stress accumulation.

In this note we shall report on data which have recently become available and which seem to give relevant information for the study of the mechanism of stress accumulation.

## THE NEW DATA

The new sets of data concern four series of aftershocks of moderate to large size California Earthquakes: the aftershocks of the 1971 San Fernando Earthquake ($M = 6.9$) in Southern California studied by Tucker and Brune (1973) who computed the linear dimension of the fault, the magnitude and the moment for 155 events; the sequence of Earthquakes of January 15th 1973 in Bear Valley in central California studied by Bakun *et al.* (1976) who computed the linear size of the faults the magnitude and the seismic moment for 25 events; the aftershocks of the Aug. 1st 1979 Oroville Earthquake ($M = 5.7$) in Northern California studied by Fletcher (1979) who computed the magnitude, the linear size of the faults and the seismic moment for 26 events; the aftershocks of the August 6th, 1979 Cayote Lake Earthquake ($M = 5.7$) in central California studied by Lee *et al.* 1979 who computed the magnitude and seismic moment for 21 events and the linear size of the faults for 4 events.

It is well known that, if we accept the empirical relation (4) the parameters $p$ , $l$ , M , $M_0$ are not independent but must satisfy the relations

$$\frac{10^{\beta+\gamma M}}{\eta} = \frac{kl^3 p^2}{2\mu} \tag{5}$$

$$M_0 = \frac{l^3 p}{c} \tag{6}$$

where $K^{-1} = c = 7/16$ for circular faults (Keilis Borok 1959) and $\eta$ is the seismic efficiency. Caputo and Console (1980) estimated $\eta^{-1} 10^{\beta} = 10^{11.1}$ and $\gamma = 1.46$ for California from the data of Thatcher and Hanks (1973). Since the observed parameters are $l$ , M and $M_0$ we can eliminate $l$ in (5) and (6) and obtain

$$10^{11.1+1.46M} = \frac{M_0 p}{2\mu} \tag{6}$$

which would give $p$ in case only M and $M_0$ and not $l$ have been observed. In turn formula (6) can be checked when all the three parameters M , $M_0$ , $l$ have been estimated. This has been done with the data of Tatcher and Hanks, used to estimate $\eta^{-1} 10^{\beta}$ and $\gamma$, with the data of the 1971 San Fernando Earthquake aftershocks (Tucher and Brune 1973), with the data of the 1973 Bear Valley sequence (Bakun *et al.* 1976) and the data of the 1979 Oroville Earthquake (Fletcher 1979).

It was found that there are relevant differences for some of the values of $p$; which in turn implies that the parameters $l$ , M , $M_0$ have not been accurately measured. Discrepancies of the smaller magnitude can be found between the values of $p$ measured from P waves data and those measured from S waves for the same erathquakes. However, inspite of this, the statistical distribution of all sets of data are remarcably stable.

For some sets of events the parameter $p$ has been measured in two different ways for most of the events (e.g. the sets of Fletcher (1979) and Bakun *et al.* (1976)). We report here in Figs. 1, 2, 3, 4 the statistics based on the sets of data which are more numerous.

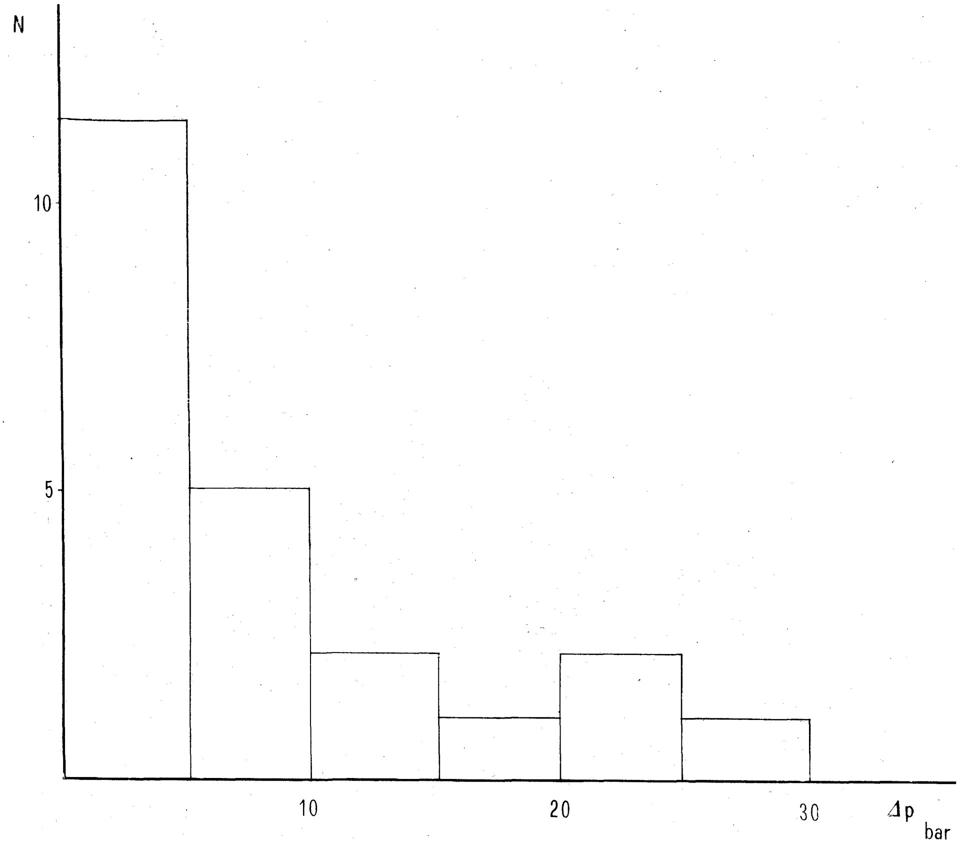


Fig. 2. - Istogramme of the stress drops of the Bear Valley (California) Earthquakes sequence of Jan. 15th 1976 measured from S displacement by Bakun *et. al.* (1976).

However the number of data reported in Figs. 2, 3, 4 is not large enough to enable us a determination of $\alpha$ as it was done for the three sets of data reported in Fg. 1. The six sets of data however enable some analysis of the resulting confirmation of the theory ($\alpha < 0$) as well as to infer some geophysical implications of the value of $\alpha \simeq -1$ found for California and Japan; because of the large number of data, the values of $\alpha$ are reliable in spite of the low accuracy of $p$.

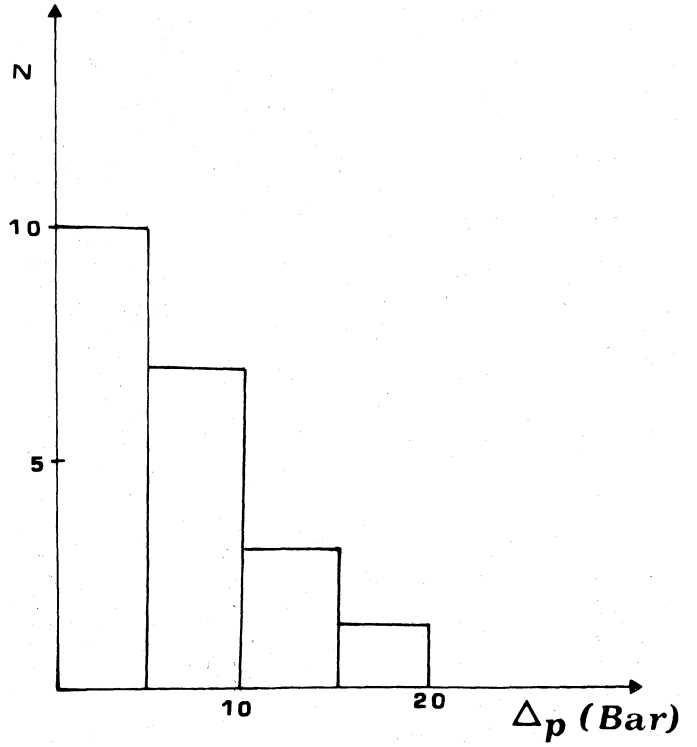


Fig. 3. – Istogramme of the stress drops of the sequence of aftershocks of the 1979 Coyote Lake (California) Earthquake measured from S wave displacement by Lee *et al.* (1979).

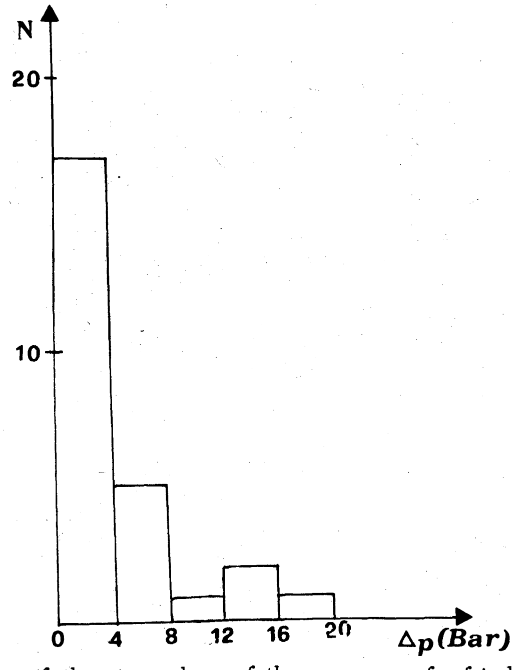


Fig. 4. – Istogramme of the stress drop of the sequence of afetrshocks of the Oroville (California) Earthquake measured from P and S waves displacements by Fletcher (1979).

## ANALYSIS OF THE RESULTS AND IMPLICATIONS ON THE MECHANISM OF STRESS ACCUMULATION.

The density distribution of the number of stress drops according to (1) is

$$n^{-1} = p^{1-\alpha}/C \tag{7}$$

where C and $\alpha$ are parameters characterizing the regions.

The non linearity of stress accumulation expressed by formula (7) with $\alpha \neq 0$, can be explained in more than one way. In the following two simple interpretations are given.

Formula (7) could imply that the time $t$ of accumulation of stress drop is proportional to $n^{-1}$

$$t = \frac{B}{n} = \frac{B}{Cp^{-1+\alpha}} \tag{8}$$

where B is a constant depending on the unit of time. Since $\alpha < 0$, formula (8) implies that the time for the accumulation of stress is a non linear function of the stress.

The first interpretation consists in assuming that creep takes place in the accumulation of stress.

A model for creep is obtained by considering the following modification of Hooke's law

$$\tau = \mu \frac{\partial^z \varepsilon}{\partial t^z}, \qquad \mu > 0 \quad , \quad 0 < z < 1 \tag{9}$$

where $\tau$, $\varepsilon$ and $\mu$ are respectively the stress, the strain and the elastic parameter. Assuming that the strain increases linearly with time

$$\varepsilon = at \tag{10}$$

where a is the rate of strain accumulation, by substituting (10) in (9) we find (e.g. Caputo 1976b)

$$\tau = \frac{\mu a}{(1 - z)!} t^{1-z} \tag{11}$$

or

$$t = \left( \frac{\tau (1 - z)!}{\mu a} \right)^{1/1-z}. \tag{12}$$

By comparing (12) and (8) we find

$$1 - \alpha = \frac{1}{1 - z} \quad , \quad z = \frac{-\alpha}{1 - \alpha} \tag{13}$$

which implies that, for $\alpha < 0$, we have $0 < z < 1$.

In the case of California $\alpha \simeq -1$ we have $z = 0.50$.

This in turn would imply that in case of linearly increasing strain we have from (11) $\tau = \mu a t^{0.50}/(0.50)!$ and consequently a very large creep would take place in the accumulation of stress. This in turn would imply that in California the motions in the crust are much faster and larger than we may estimate from the strain released in Earthquakes. If $\alpha \simeq -1$ also in the other seismic regions of the world, than the geophysical implications would be many.

Another interpretation of the fact that $\alpha \simeq -1$ can be found in the model of Earthquake occurrence of Caputo (1977), in which it is assumed that in the crust there is isotropic orientation in the distribution of the direction $\vartheta$ of the fault planes with respect to the major tectonic force. Assuming a constant rate of strain accumulation $b$ it is found that the return period of stress release P of the fault of direction $\vartheta$ is

$$t = \frac{f_a}{\sin \vartheta \cos \vartheta - f_c \cos^2 \vartheta} = \frac{p}{b} \tag{14}$$

where $f_a$ is a cohesive force between the two sides of the faults and $f_c$ is a friction coefficient. In this model there is linearity also in the stress accumulation but as one may see from formula (14) (Caputo *et al.* 1979) the density distribution function of stress drops but, for small values of $p$, is proportional to $p^{-3}$; for large values of $p$ or non isotropic distribution of fault direction the density distribution is more complex, in general it is

$$\eta \sim \frac{b}{f_a} \left( \frac{\sin 2\vartheta}{2} - f_c \cos^2 \vartheta \right) \frac{d\vartheta}{d\rho}$$

where $d\vartheta$ is a measure of the distribution function of fault direction expressed as function of $p$ by meas of (14). For small values of $p$ it is found

$$n \sim p^{-2} \quad , \quad \alpha = -1$$

while for large values it is

$$n \sim p^{-3} \quad , \quad \alpha = -2 \, .$$

Since large values of $p$ are rare it is not surprising that in short time intervals as those used in California and Japan it is found $\alpha \simeq -1$.

For the aftershock sequences the observed values of $\alpha$ are larger, but this could be a phenomenon typical of regions where a large amount of the stress has been locally released, and in the adjacent areas preferably large adjustment are needed due to the imbalance created by the large shock.

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