Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

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Extending CR-distributions

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **68** (1980), n.1, p. 42–43.

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<http://www.bdim.eu/item?id=RLINA_1980_8_68_1_42_0>

Geometria. — *Extending* CR-*distributions*. Nota di GILBERTO DINI (*) e CARLA PARRINI (**), presentata (***) dal Socio G. ZAPPA.

RIASSUNTO. — Si danno alcuni teoremi di estensione per CR-distribuzioni su una ipersuperficie reale C^∞ di C^n, $n \geq 2$, e se ne deduce un teorema di singolarità rimovibili per le tracce di funzioni olomorfe.

1. Let M be a real smooth hypersurface of a domain $U \subset C^n$, $n \geq 2$, dividing U in two domains U^+, U^-. Let $\bar\partial_M$ be the tangential Cauchy-Riemann operator (e.g. [2] [4]). Let $\mathcal{D}'$ (M) be the space of distributions on M. We are dealing with the boundary values of holomorphic functions in distribution sense (see [5]). It is well known that such boundary values are CR-*distributions* (i.e. solutions of $\bar\partial_M T = 0$).

For every $z \in M$ we denote by $L_z(\rho)$ the conditioned Levi-form of M at z (ρ being a defining C^∞ function for M at z) ([3]).

2. In a previous paper ([1]) the authors dealt with extendibility and removable singularity sets of CR-distributions on M and boundary values of holomorphic functions, defined on one side of M, when M is a real hyperplane.

In this note we sketch the proof of some results in the case M is a smooth hypersurface.

THEOREM 1. *Suppose that* $L(\rho)$ *has on* M *constant rank* $n - p - 1$, $p > 0$. *If* $f \in \mathcal{O}(U^-)$ *and* f *admits boundary value* $\gamma(f)$ *on* $M - S$, *where* S *is a closed subset with null* $(2p - 1)$-*dimensional Hausdorff measure, then* $\gamma(f)$ *exists on* M.

Proof. By results of Freeman [3] M is locally foliated by complex manifolds of dimension p and these foliations on M can be extended smoothly to neighbourhoods of M. The result is purely local and follows by arguments similar to the proof of theorem 3.1 in [1].

For the definition of CR Levi-flat submanifolds of $\mathbf{C}^n$ see [3].

THEOREM 2. *Let* $S \subset M$ *be a real analytic Levi-flat* CR *submanifold of* $\mathbf{C}^n$, $n \geq 3$, *and* CR-dim $S \neq n - 2$. *If* $T \in \mathcal{D}'(M)$ *is a* CR-*distribution on* $M - S$, *then* T_{M-S} *is locally extendible across* S *as a* CR-*distribution*.

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Proof. To any distribution $T \in \mathcal{D}'(M)$ can be associated a (0, 1)-form $\mu^* T \in \mathcal{D}_M^{(0,1)}(U)$ with distributions as coefficients, and with support on M and $\bar\partial_M T = 0$ if and only if $\bar\partial \mu^* T = 0$ (see [4]). It follows that if $T \in \mathcal{D}'(M)$ is CR on $M - S$; $\bar\partial \mu^* T$ is a (0, 2)—form with support on S and is, of course, $\bar\partial$-closed. If $z \in S$ there exists a ball V in $\mathbf{C}^n$ centred in z and $V \subset U$, such that up to biholomorphisms $S \cap V$ can be assumed to a be a linear submanifold of V. The crucial point in the proof is the following:

PROPOSITION. *There exists* $\omega \in \mathcal{D}_M^{(0,1)}(V)$ *such that* $\bar\partial\omega = \bar\partial\mu^* T$ *and* $\operatorname{supp} \omega \subseteq \operatorname{supp} \bar\partial\mu^* T$.

It follows that $\mu^* T - \omega$ is $\bar\partial$–closed and coincides with $\mu^* T$ away from S. Results of [5] allow to set the requested local extension.

As a corollary we have the following

THEOREM 3. *Suppose* L (ρ) *has at least one positive eigenvalued on* M, *and let* S *be a real analytic Levi-flat* CR *submanifold of* M *with* $\dim_{\mathbf{R}} S \leq 2n - 3$. *If* $f \in \mathcal{O}(U^-)$ *admits a boundary value* $\gamma(f)$ *on* $M - S$ *that extends to a distribution on* M, *then* f *admits a boundary value on* M.

Proof. With the notation of theorem 2 $\mu^* T - \omega = \bar\partial F$, where $F \in \mathcal{O}(V - M) \cap \mathcal{D}'(V)$ and due to the positive eigenvalue of L (ρ) $F^+ = F/V^+$ can be assumed zero (see [5] Corollary II. 1.4). So $F^- = f$ and by corollary I.2.6. of [5], $\gamma(f)$ exists on $V \cap M$ for any z, so $\gamma(f)$ exists on M.

Detailed proofs of Theorems 1, 2, 3 will appear elsewhere.

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