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On the contribution of heat flux to the propagation velocity of relativistic shock waves in thermo-elastic bodies

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Fisica matematica. — *On the contribution of heat flux to the propagation velocity of relativistic shock waves in thermo-elastic bodies* (*). Nota II di ALDO BRESSAN, presentata (**) dal Corrisp. G. GRIOLI.

RIASSUNTO. — Si studiano onde d'urto termomeccaniche, (precisamente T–η–onde d'urto) in corpi elastici (o fluidi non viscosi) in una teoria di relatività ristretta o generale, includente il tensore termodinamico di C. Eckart (cfr. [2]). La velocità di propagazione V di queste è calcolata in vari casi, almeno a meno di termini d'ordine 2 (rispetto a $1/c$ ove c è la velocità della luce nel vuoto). A questo scopo è essenziale usare, per esempio, un certo postulato di carattere generale, il quale è compatibile con un'ipotesi di solito fatta implicitamente. Nel caso più generale V dipende da certi rapporti fra parametri di discontinuità e loro derivate. Questi rapporti spariscono in casi speciali importanti concernenti i solidi, e in ogni caso riguardante i fluidi. In particolare è posta in evidenza la dipendenza di V dal flusso di calore.

6. THERMO-MECHANIC SHOCK WAVES IN NON-VISCOUS FLUIDS

We now consider a non-viscous fluid $\mathscr{C}$. Hence for it

$$(6.1)\qquad \begin{cases} w = w(k,\eta) \quad , \quad q_\rho = \mathscr{H}\cdot(\mathrm{T}_{/\rho} + \mathrm{T}\mathrm{A}_\rho) \quad , \quad \mathscr{H} = \mathscr{H}(k,\eta)\,, \\ \mathrm{X}^{\rho\sigma} = p\overset{1}{g}{}^{\rho\sigma} \quad , \quad p = p(k,\eta) = k^2\, w'_k(k,\eta) \quad , \quad \mathrm{T} = w'_\eta\,. \end{cases}$$

By (6.1), (5.13) holds. Then by $(6.1)_{1,5\text{-}7}$ (whence $k^2\,[w] = p\,[k] + k^2\,\mathrm{T}\,[\eta]$)

$$(6.2)\qquad \left\{p'_k + \left(1 - \frac{\mathrm{V}^2}{c^2}\right)\frac{2\,q^3}{ck}\,\mathrm{V} - \left(1 + \frac{w}{c^2} + \frac{p}{c^2 k}\right)\mathrm{V}^2\right\}[k] + \\ + \left(p'_\eta - \frac{k\mathrm{T}}{c^2}\,\mathrm{V}^2\right)[\eta] = \boxed{6}$$

where $\boxed{6} = 0$ *for* $q^\rho \parallel \mathrm{N}^\rho$, (3.6) and $(5.11)_2$ being understood.

POST 6.1. Across a T-η-*shock wave travelling in a viscous fluid*

$$(6.3)\qquad [\eta] = \rho_{5,6}\,[k]$$

where $\rho_{5,6}$ *is a constitutive function of* $k, \eta, q^\rho, \mathrm{N}^\rho$, *and possibly also of the way in which the shock wave is produced.*

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(**) Nella seduta del 14 giugno 1979.

The (only) important thing is that we are able to know $\rho_{5,6}$ before the wave occurs. Experiments say that (6.3) holds satisfactorily for $\rho_{5,6}$ very small or even vanishing.

We can replace $[\eta]$ by $\rho_{5,6}[k]$ in (6.2). The requirement that the coefficient of $[k]$ in the resulting equation should vanish constitutes an equation in V of the third degree (for $\boxed{6}$ negligible):

$$(6.4)\quad \begin{cases} p'_k + \left(1 - \frac{V^2}{c^2}\right)\frac{2q^3}{ck}V - \left(1 + \frac{w}{c^2} + \frac{p}{kc^2}\right)V^2 + \rho_{5,6}\left(p'_\eta - \frac{kT}{c^2}V^2\right) = \boxed{6} \\ \textit{where} \quad \boxed{6} = 0 \quad \textit{for} \quad q^\alpha \parallel N^\alpha . \end{cases}$$

The classical determination $\overline{V}$ of V is given by (6.4) for $1/c = 0$ (hence $q^3 = 0 = \boxed{6}$); furthermore

$$(6.5)\quad \begin{cases} V = \overline{V} + \frac{q^3}{kc} - \left(w + \frac{p}{k} + \rho_{5,6}\, kT\right)\frac{\overline{V}}{2c^2} + \boxed{4} \\ \textit{with} \quad \overline{V} = \sqrt{p'_k + \rho_{5,6}\, p'_\eta} \qquad (> 0) \end{cases}$$

where $w = 0$ *when the reference state coincides with the actual one.*

Indeed, for $\xi = c^{-2}$ let $f(V, \xi)$ be the left hand side of (6.4). Then

$$\frac{\partial f}{\partial V}(\overline{V}, 0) = -2\overline{V} \quad , \quad \frac{\partial f}{\partial \xi}(\overline{V}, 0) = \frac{2cq^3}{k}\overline{V} - \left(w + \frac{p}{k} + \rho_{5,6}\, kT\right)\overline{V}^2 ;$$

hence $(6.5)_1$ holds in that (6.4) defines a function $V = V(\xi)$ for which $V(0) = \overline{V}$ (cf. $(6.5)_2$) and $dV/d\xi = -(\partial f/\partial\xi)/(\partial f/\partial V)$.

Remark that $(6.5)_1$ affords the first order relativistic corrections to the classical propagation speed $\overline{V}$ *of a* T-η-*shock wave* σ_t *travelling in a non-viscous fluid under a heat flux, within Eckart's theory.* The new parts of this correction are the term in the (ordinary size) heat flux $cq^3 (\equiv cq^\alpha N_\alpha)$ and the contribution due to the coefficient $\rho_{5,6}$ which vanishes in case the shock is isentropic. The correction to $\overline{V}$ due to the heat flux cq^3 is the contribution of cq^3 to V; and *up to terms in* c^{-4}, *it is the speed* $c^{-1}k^{-1}q^3$ *with which the energy* $kc^2\, dC$ (*associated with the local mass* $k\, dC$) *ought to travel along* N_α *to give rise to a flux that equals* cq^3. Thus *the shock wave is algebraically accelerated by the heat flux*; hence *it travels faster towards regions with lower temperatures.*

Of course for $q^\alpha \parallel N^\alpha$ equation (6.4) holds with $\boxed{6} = 0$. Hence *it affords three exact values for* V *in this case*—see remark below (5.13).

Also in the general case better solutions than (6.5) can be found. Indeed, by (6.4) and $(6.5)_2$

$$(6.6)\quad \begin{cases} \alpha V^2 - 2\,\beta V = \overline{V}^2 + \boxed{6} \quad \text{with} \quad \alpha = 1 + \dfrac{w}{c^2} + \dfrac{p}{c^2 k} + \rho_{5,6}\dfrac{kT}{c^2}\,, \\ \beta = \left(1 - \dfrac{\overline{V}^2}{c^2}\right)\dfrac{q^3}{ck} \end{cases}$$

where $q^3 = q^\rho N_\rho$; *hence*

$$(6.7)\qquad V = \frac{\beta \pm \sqrt{\beta^2 + \alpha\overline{V}^2}}{\alpha} + \boxed{6}\,.$$

7. Basic relativistic thermo-mechanic equations for shock waves in elastic solids

Now we consider a thermo-elastic body $\mathscr{C}$ of constitutive equations

$$(7.1)\qquad w = w(y\,,\eta\,,C_{LM}) = w(y\,,\eta\,,\alpha^\rho_L) \quad , \quad T = T(y\,,\eta\,,\alpha^\rho_L) = w'_\eta$$

$$(7.2)\qquad \mathscr{D}X^{\rho\sigma} = K^{\rho L}\alpha^\sigma_L \quad , \quad K^\rho_L = -k^*\frac{\partial w}{\partial \alpha^\rho_L} \qquad (k\mathscr{D} = k^*)$$

$$(7.3)\qquad q^\rho = \mathscr{H}^{\rho\sigma}(T_{/\sigma} + TA_\sigma) \quad , \quad \mathscr{H}^{\rho\sigma} = \mathscr{H}^{\rho\sigma}(y\,,\eta\,,\alpha^\rho_L) = \mathscr{H}^{\rho\sigma} \quad (\mathscr{H}_{\rho\sigma}u^\sigma = 0)\,.$$

By $(7.1)_3$

$$(7.4)\qquad [T] = T'_\eta\,[\eta] + T'_{\alpha^\rho_L}\,[\alpha^\rho_L]\,, \qquad \text{i.e.}\quad A_4 = T'_\eta\,A_5 + T'_{\alpha^\rho_L}\,B^\rho_*\,\mathscr{N}^*_L\,,$$

where $A_4 = [T]$ and $A_5 = [\eta]$ refer to both σ_3 and σ^*_t.

Since $[cq^\rho]$, which has an ordinary size, substantially occurs in (5.8) also multiplied by Vc^{-2} and we aim at calculating V up to terms in c^{-4}, we can neglect the contributions to $[cq^r]$ of order two in $1/c$:

$$(7.5)\qquad [q^r] = (q^r)'_\eta\,[\eta] + (q^r)'_{\alpha^\rho_L}\,[\alpha^\rho_L] + (q^r)'_{T_{/s}}\,[T_{/s}] + \boxed{3}\,.$$

By equation $(176.2)_1$ in [6], which holds also in general relativity,

$$(7.6)\quad \begin{cases} [T_{/r}] = B_4\,N_r + x_r^{;\mathscr{B}}\,A_{4;\mathscr{B}} \quad ; \quad N^\rho = \delta^\rho_3\,N^3\,, \\ \text{hence} \quad x_3^{;\Gamma} = 0 \quad , \quad B_4 = [T_{/3}]\,. \end{cases}$$

Then, by (7.3), (7.5) with $r = 3$ becomes

$$(7.7)\qquad [q^3] = \mathscr{H}^{33}\,B_4 + \mathscr{H}^{3s}\,x_s^{;\mathscr{A}}\,A_{4;\mathscr{A}} + [\mathscr{H}^{3s}]\,T_{/s} + \boxed{3}$$

so that (for $r = 1, 2$ and trivially for $r = 3$)

$$(7.8)\qquad \boxed{3} + [q^r] = \mathscr{H}^{r3}\,B_4 + \mathscr{H}^{rs}\,x_s^{;\mathscr{A}}\,A_{4;\mathscr{A}} + [\mathscr{H}^{rs}]\,T_{/s} = \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\left([q^3] - [\mathscr{H}^{3l}]\,T_{/l} - \mathscr{H}^{3s}\,x_s^{;\mathscr{A}}\,A_{4;\mathscr{A}}\right) + [\mathscr{H}^{rl}]\,T_{/l} + \mathscr{H}^{rs}\,x_s^{;\mathscr{A}}\,A_{4;\mathscr{A}}\,.$$

By (5.11) this yields

$$(7.9)\qquad \boxed{3} + [q^r] = \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\left(k\,[w]\,\frac{V}{c} - X^{3s}\,[u_s] - [\mathscr{H}^{3l}]\,T_{/l}\right) + [\mathscr{H}^{rl}]\,T_{/l} + \left(\mathscr{H}^{rs} - \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\,\mathscr{H}^{3s}\right) x_s^{;\mathscr{A}}\,A_{4;\mathscr{A}}\,.$$

Then $(5.8)_1$ becomes

$$(7.10)\qquad [X^{r3}] + 2\,q^{(3}\,[u^{r)}] + \boxed{4} = \left\{\rho\,[u^r] + X^{rs}\,[u_s] + \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\left(k\,[w]\,\frac{V}{c} - X^{3s}\,[u_s] - [\mathscr{H}^{3l}]\,T_{/l}\right) + [\mathscr{H}^{rl}]\,T_{/l} + \left(\mathscr{H}^{rs} - \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\,\mathscr{H}^{3s}\right) x_s^{;\mathscr{B}}\,A_{4;\mathscr{A}}\right\}\frac{V}{c}\,.$$

By (5.1), $(5.2)_1$, $(7.1)_{1,2}$, and $(7.2)_1$

$$(7.11)\qquad [X^{r3}] + X^{r3}\,\frac{V_*}{V}\,B_*^s\,N_s = \frac{1}{\mathscr{D}}\,[K^{rl}]\,\alpha_L^3 + \frac{1}{\mathscr{D}}\,K^{rL}\,B_*^3\,\mathscr{N}_L^* \cdot \quad \left(\frac{V_*}{V} = \frac{1}{\gamma}\right).$$

Furthermore by $(3.4)_2$, $(7.2)_1$, and $(3.4)_1$

$$(7.12)\qquad \frac{V_*}{V}\,X^{r3} = \frac{g}{g_*}\,X^{rs}\,N_s = \frac{1}{\mathscr{D}}\,K^{rL}\,\alpha_L^s\,N_s\,\frac{g}{g_*} = \frac{1}{\mathscr{D}}\,K^{rL}\,\mathscr{N}_L^*\,,$$

hence (by $(5.11)_2$ $B_*^s\,N_s = B_*^3$)

$$(7.13)\qquad [X^{r3}] = \frac{1}{\mathscr{D}}\,[K^{rL}]\,\alpha_L^3\,.$$

By (7.1) and $(7.2)_{2,3}$

$$(7.14)\qquad \begin{cases} k\,[w] = -\dfrac{1}{\mathscr{D}}\,K_s{}^L\,B_*^s\,\mathscr{N}_L^* - kT\,A_5\,, \\[2ex] [K_r{}^L] = -\,k^*\left(w''_{\alpha_L^r \alpha_M^s}\,B_*^s\,\mathscr{N}_M^* + w''_{\eta\alpha_L^r}\,A_5\right). \end{cases}$$

By (7.13–14) we can turn (7.10) into

$$\beta_{rs}\, B_*^s + \beta_r^5\, B_5 + \beta'_{r\mathscr{A}}\, A_{4;\mathscr{A}} = \boxed{4} \tag{7.15}$$

where ($k^* = k\mathscr{D}$)

$$\left\{\begin{aligned}
\beta_{rs} &= k w''_{\alpha^r_L \alpha^s_M}\, \alpha^3_L\, \mathscr{N}^*_M + \frac{2\,V_*}{c}\, q^{(3}\,\delta_{r)s} - (\rho\delta_{rs} + X_{rs})\,\frac{V_* V}{c^2} + \\
&\quad + \left[(\mathscr{H}^{rl})'_{\alpha^s_L} - \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}}\,(\mathscr{H}^{3l})'_{\alpha^s_L}\right] T_{/l}\, \mathscr{N}^*_L\, \frac{V}{c}\,, \\
\beta'_{r\mathscr{A}} &= \left(\mathscr{H}^{rs} - \frac{\mathscr{H}^{r3}}{\mathscr{H}^{33}}\,\mathscr{H}^{3s}\right) x_{s;\mathscr{A}}\, \frac{V}{c}\,, \\
\beta^5_r &= k w''_{\alpha^r_L \eta}\, \alpha^3_L + \left[(\mathscr{H}^{rl})'_\eta - \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}}\,(\mathscr{H}^{rl})'_\eta\right] T_{/l}\, \frac{V}{c} - \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}}\, kT\, \frac{V^2}{c^2}\,.
\end{aligned}\right. \tag{7.16}$$

To prove this is straightforward when one remarks that the contribution of $(\mathscr{H}^{r3}/\mathscr{H}^{33})\, k\,[w]\, V^2 c^{-2}$ to β_{rs} is the left hand side of

$$\frac{1}{\mathscr{D}}\, K_s{}^L\, \mathscr{N}^*_L\, \frac{V^2}{c^2} = X^3{}_s\, \frac{V V_*}{c^2} \tag{7.17}$$

multiplied by $-\mathscr{H}^3_r/\mathscr{H}^{33}$ and that (7.17) holds by (7.12), so that by $(5.1)_2$ this contribution eliminates the one of $(\mathscr{H}^{r3}/\mathscr{H}^{33})\,(-X^{3s}\,[u_s])\,V/c$.

Of course by (3.4–5) V and N_r, or V_* and $\mathscr{N}^*_L$ can be eliminated from the coefficients (7.16). Furthermore we can obtain covariant expressions for these coefficients by replacing in (7.16) e.g. $\mathscr{H}^{r3}$ with $\mathscr{H}^{r\beta} N_\beta$ and $\mathscr{H}^{33}$ with $\mathscr{H}^{\alpha\beta} N_\alpha N_\beta$.

8. Propagation velocity of relativistic shock waves in elastic solids. Discussion

By $(7.1)_{3-4}$ there is a relation among A_4, A_5, and B^ρ_*, namely

$$A_4 = T'_\eta\, A_5 + T'_{\alpha^\rho_L}\, \mathscr{N}^*_L\, B^\rho_* \qquad (A_4 = A^*_4\,,\ A_5 = A^*_5) \tag{8.1}$$

which can be solved with respect to A_5.

Let us say that the (thermo-mechanic) shock wave σ_t is of the *type* $(\beta_{ls}\,, \beta^5_l\,, \beta'^{\mathscr{A}}_l)$ $(l = 4\,, \cdots, 6)$ *at its point* Q if there we have, under condition (3.6)

$$\beta_{ls}\, B^s_* + \beta^5_l\, A_5 + \beta'_{l\mathscr{A}}\, A_4{}^{;\mathscr{A}} = 0 \qquad (l = 4\,, 5\,, 6)\,. \tag{8.2}$$

Then its propagation speed must solve, up to |4|, the polynomial equation

$$\det \| a_{lm} \| = 0 \qquad (l, m = 1, \cdots, 6), \tag{8.3}$$

generally of the 6-th degree, in V_* or V, where (cf. (7.15))

$$a_{lr} = \beta_{lr} \quad , \quad a_{l3+\mathscr{A}} = \beta'_{l\mathscr{A}} \quad , \quad a_{l6} = \beta_l^5 \quad (l = 1, \cdots, 6 ; \mathscr{A} = 1, 2). \tag{8.4}$$

Now remember that discontinuity waves are regarded here, as well in [3] and [4], as small perturbations. Furthermore, in harmony with what is done in [4] (or [3]), remark that by the way in which the usual theory of shock (or acceleration) waves travelling in purely mechanic bodies is applied to real thermo-elastic bodies, a postulate such as $[\eta] = 0$ or $[\eta_{/r}] = 0$ across these waves in quite acceptable, at least for $\mathfrak{c}^{rs}$ or q^r small. In order to have a finer and more general theory, this postulate can be replaced by a constitutive law such as relation (6.3) for fluids:

POST. 8.1. *On* T-η-*shock waves*

$$\beta_\rho B_*^\rho + \beta^5 A_5 + \beta'_{\mathscr{A}} A_4^{;\mathscr{A}} = 0 \qquad (\beta_\rho u^\rho = 0) \tag{8.5}$$

where β_ρ, β^5, and $\beta'_{\mathscr{A}}$ *are functions of* y, η, α_L^ρ, q^ρ, $\mathscr{N}_L^*$, *and possibly the way in which* σ_t *is produced.*

The acceptable first order approximation substantially referred to above on considering $\mathfrak{c}^{rs}$ small, is afforded by (8.5) for $\beta^5 = 1$, $\beta_\rho = 0$, and $\beta'_{\mathscr{A}} = 0$ — which incidentally renders (8.5) a universal relation. In any case (8.5) cannot differ much from the latter relation, so that we can always assume $\beta^5 \neq 0$ and hence $\beta^5 = -1$.

The speeds V and V_* can be calculated by (8.3) again, by identifying (8.5) with (8.2) for $l = 6$. Of course by Post. 8.1 with $\beta^5 = -1$ we can regard the *(local) type* of σ_t as determined by $(\beta_{ls}, \beta'_{l\mathscr{A}})$ $(l = 4, 5; \mathscr{A} = 1, 2)$ in that we can assume (8.2) to hold with

$$\beta_4^5 = \beta_5^5 = 0 \quad , \quad \beta_6^5 = \beta^5 = -1 . \tag{8.6}$$

9. SPECIAL CASES. EXPLICIT EXPRESSION OF V_* IN THEM

We consider the case where, at least locally,

$$A_{5;\mathscr{A}} = 0 \tag{9.1}$$

holds. In special relativity it occurs in particular for (spatially) homogeneous plane (shock) waves and for waves having a spherical symmetry. It is worth while remarking that surely of the first [second] kind are the shock waves produced in problems (including initial and boundary conditions) in which the data are independent of the first two spatial co-ordinates x^1 and x^2 in a given Minkowskian frame $(x^0, \cdots, x^3)$ [the data are independent of the angular co-ordinates φ and ϑ in the spherical co-ordinate system $(x^0, \varphi, \vartheta, r)$ associated with $(x^0, \cdots, x^3)$].

By (9.1) and $(8.6)_{3,4}$ we can replace (8.2) with the only relation

$$A_5 = \beta_s B^*_s \qquad (\text{or} \quad A_5 = \beta_\sigma B^\sigma_* \quad \text{with} \quad \beta_\sigma u^\sigma = 0) \tag{9.2}$$

where, as in the sequel, (3.6) is assumed. Then, by (7.16) equations (7.15) become

$$b_{rs} B^s_* = \boxed{4}, \tag{9.3}$$

where (cf. $(3.4)_{2-4}$ and $(3.5)_1$)

$$b_{rs} = b_{rs0} + b_{rs1} V_* + b_{rs2} V^2_* \tag{9.4}$$

with

$$\left\{\begin{aligned} b_{rs0} &= k\,(w''_{\alpha^r_L \alpha^s_M})\,\alpha^3_L \mathscr{N}^*_M + k\,(w''_{\alpha^s_L \eta})\,\alpha^3_L \beta^s , \\ b_{rs1} &= T_{/l} \left\{ \left[(\mathscr{H}^{rl})'_{\alpha^s_L} - \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}} (\mathscr{H}^{3l})'_{\alpha^s_L} \right] \mathscr{N}^*_L + \left[(\mathscr{H}^{rl})'_\eta - \right.\right. \\ &\qquad \left.\left. - \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}} (\mathscr{H}^{rl})'_\eta \right] \beta_s \right\} \frac{\Upsilon}{c} + \frac{2}{c} q^{(3}\, \delta_{r)s} , \\ -\,b_{rs2} &= \left(\rho \delta_{rs} + X_{rs} + \frac{\mathscr{H}^3_r}{\mathscr{H}^{33}} \gamma k\, T \beta_s \right) \frac{\Upsilon}{c^2} . \end{aligned}\right. \tag{9.5}$$

Of course (9.3) has a proper solution B^s_* iff

$$\det \| b_{rs} \| = \boxed{4} . \tag{9.6}$$

In other words (9.3) *holds for some spatial unit vector* B^r_* (*depending on* $1/c$) *and some determination of* $\boxed{4}$ *iff* $\det \| b_{rs} \|$ *is* (a *function of*) $1/c$, *infinitesimal of the* 4^{th} *order*.

It is interesting to identify the reference configuration with the actual one and to assume, besides (3.6) and $N_3 = -1$, the identies $y^L \equiv \delta^L_\rho x^\rho$. Then

$$\left\{\begin{array}{llll} \alpha^r_L = \delta^r_L , & \mathscr{D} = 1 = \gamma , & X^s_r = K^s_r , & \\ \mathscr{N}^*_L = \delta^3_L , & k^* = k , & V_* = V & \qquad (N^3 = 1 , y^L \equiv x^L) , \end{array}\right. \tag{9.7}$$

so that relations (9.5) simplify into

$$
(9.5') \quad \begin{cases}
b_{rs0} = kw''_{\alpha_3^r \alpha_3^s} + kw''_{\alpha_3^r \eta}\, \beta_s\,, \\[2ex]
cb_{rs1} = \mathrm{T}_{/l} \left\{ (\mathcal{H}^{rl})'_{\alpha_3^r} - \dfrac{\mathcal{H}^3_r}{\mathcal{H}^{33}} (\mathcal{H}^{3l})'_{\alpha_3^s} + \left[(\mathcal{H}^{rl})'_{\eta} - \right. \right. \\[1ex]
\qquad\qquad \left. \left. - \dfrac{\mathcal{H}^3_r}{\mathcal{H}^{33}} (\mathcal{H}^{rl})'_{\eta} \right] \beta_s \right\} + 2\, q^{(3}\, \delta_{r)s}\,, \\[2ex]
-c^2\, b_{rs2} = \rho \delta_{rs} + \mathrm{X}_{rs} + \dfrac{\mathcal{H}^3_r}{\mathcal{H}^{33}}\, k\mathrm{T}\, \beta_s\,.
\end{cases}
$$

If $\mathscr{C}$ cannot conduct heat ($\mathcal{H}^{rs} \equiv 0$), all terms in $\mathcal{H}^{rl}$ (, $\mathcal{H}^3_r/\mathcal{H}^{33}$,) and q^3 vanish, as can be seen partly directly on the basis of (9.5′) and partly by remembering that the terms in $\mathcal{H}^3_r/\mathcal{H}^{33}$ arise from the expression (7.9) for $[q^r]$ ($\equiv 0$). We conclude that *the propagation speed* V *of the shock waves travelling in a thermo-elastic body* $\mathscr{C}$ *uncapable of conducting heat are the solutions of the secular equation*

$$
(9.8) \qquad \det \| w''_{\alpha_L^r \alpha_M^s} \mathcal{N}^*_{\mathrm{L}} \mathcal{N}^*_{\mathrm{M}} - (\rho\delta_{rs} + \mathrm{X}_{rs}) \frac{\mathrm{V}^2}{kc^2} \| = 0 \qquad (y^{\mathrm{L}} \equiv \delta^{\mathrm{L}}_{\rho}\, x^{\rho})\,.
$$

As is known, these solutions are also the possible propagation speeds of relativistic acceleration waves in $\mathscr{C}$ (1).

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(1) This can be checked e.g. by means of [2, § 66], based on [1].