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Gravitational field theory for the continuum: convergence to general relativity

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Teorie relativistiche. — *Gravitational field theory for the continuum: convergence to general relativity* (*). Nota di GIANCARLO SPINELLI, presentata (**) dal Socio C. CATTANEO.

RIASSUNTO. — Si mostra come il metodo iterativo con il quale si può costruire la teoria gravitazionale di campo per il continuo, converga alla relatività generale. Per fare ciò si considera dapprima la formulazione variazionale delle equazioni della relatività generale per il continuo e si ottiene di tale teoria la traduzione al secondo ordine nello spazio pseudoeuclideo. Dal confronto con il secondo ordine dell'approccio in teoria dei campi, si mostra come operi la rinormalizzazione dei metri e degli orologi. Per ottenere la convergenza dell'approccio iterativo, vengono eliminate le difficoltà, dovute ai vincoli che sono presenti nella formulazione variazionale per il continuo, con opportune trasformazioni del potenziale gravitazionale.

1. INTRODUCTION

In a preceding paper [1] the formal bases of a theory for the gravitational field generated by a continuum were given in the pseudo-Euclidean, " unrenormalized " [2] space-time (i.e. in the space-time that would appear to an ideal observer using ideal rods and clocks, unaffected by gravity). In such a space-time gravity is represented by a second rank symmetric tensor potential $\psi_{\alpha\beta}$. This field theoretical approach is iterative and the detailed calculations were given for the field equations to second order.

As Deser says [3], once the iteration is begun it must be continued to all orders. Indeed there is incosistency to each order between the field equations and the equations of motion. The consistency will be reached only when considering the full series.

As to the pure field terms, Deser has shown in a fundamental paper [3] the convergence to general relativity by implementing a linear action integral written in the Palatini form.

As to the matter part, one usually applies the minimal prescription [4]. The action integral of the theory to which the method converges is obtained by substituting into the action integral of the case without gravity the fundamental metric tensor $g_{\alpha\beta}$ of the curved space-time for the fundamental metric tensor $a_{\alpha\beta}$ of the flat space-time, where $g_{\alpha\beta}$ is given by $a_{\alpha\beta} - 2f\psi_{\alpha\beta}$.

Here it will be shown that such a procedure for obtaining the exact action integral relevant to the matter is correct also for the continuum and gives the general relativity action integral. The difference with respect to the case

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of point-like particles is the presence of constraints. In order to obtain the convergence in the presence of such constraints (generally changing to each order) it is convenient to compare the second step of the iteration procedure of the field theoretical approach with the second order approximation of general relativity. Once the identity of the two approximation has been recognized, it can be shown that by a proper transformation (different to each order) of the potentials, the problem can be reduced to the same formulation as for the point-like particles case.

2. GENERAL RELATIVITY

The equations of general relativity for a continuum can be written, in the Riemannian space-time,

$$R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R = - f^2 (\mu_0^* \dot{z}_\alpha^* \dot{z}_\beta^* + S_{\alpha\beta}^*) , \tag{1}$$

where $R_{\alpha\beta}$ is the contracted curvature tensor of the Riemannian space-time, i.e.

$$R_{\alpha\beta} = \partial_\beta \Gamma_{\alpha\lambda}^{*\lambda} - \partial_\lambda \Gamma_{\alpha\beta}^{*\lambda} + \Gamma_{\alpha\gamma}^{*\lambda} \Gamma_{\beta\lambda}^{*\gamma} - \Gamma_{\alpha\beta}^{*\lambda} \Gamma_{\gamma\lambda}^{*\gamma} , \tag{2}$$

$\Gamma_{\alpha\beta}^{*\lambda}$ being the Christoffel symbols of the second kind. $g_{\alpha\beta}$ is the fundamental metric tensor. The asterisk denotes quantities in the Riemannian space-time (if they have the same symbol in the pseudo-Euclidean space-time): μ_0^* is the proper density of proper mass, $S_{\alpha\beta}^*$ is the stress tensor, $z^{*\alpha}$ are the coordinates of the matter element, and $\dot{z}^{*\alpha} = dz^{*\alpha}/ds$ where $ds^* = (g_{\alpha\beta}\, dz^{*\alpha}\, dz^{*\beta})^{1/2}$.

Equations (1) can be obtained by the variational principle

$$\delta_g (I^*) = 0 , \tag{3}$$

where the action integral is given by

$$I^* = \int (R - 2 f^2 \mu_0^*)\, d^4 \Omega^*, \tag{4}$$

and $d^4 \Omega^* = \sqrt{-g}\, dx^{*1}\, dx^{*2}\, dx^{*3}\, dx^{*4}$ is the volume element of the Riemannian space ((x^α) being a general coordinate system)). The subscript under δ denotes the quantity which is varied.

Where matter is present we can write $d^4 \Omega^* = ds^*\, dV_0^*$, dV_0^* being the proper three-dimensional volume of the matter element; hence eq. (2) can be splitted into

$$I^* = \int R d^4 \Omega^* - 2 \int f^2 \mu_0^*\, dV_0^*\, ds^*. \tag{5}$$

When varying $g^{\alpha\beta}$ we have a deformation tensor given [5] by $\delta g_{\alpha\beta}/2$. Considering only adiabatic transformations, the energy balance gives that the variation of the energy contained in an infinitesimal proper volume is equal to the work done by the stresses, i.e.

$$\delta_g (\mu_0^* \, dV_0^*) = - \tfrac{1}{2} S_{\alpha\beta}^* \, \delta g^{\alpha\beta} \, dV_0^* . \tag{6}$$

Moreover

$$\delta_g (ds^*) = \tfrac{1}{2} \dot{z}^{*\alpha} \dot{z}^{*\beta} \delta g_{\alpha\beta} \, ds^* , \tag{7}$$

and

$$\delta g_{\alpha\beta} = - \delta g^{\gamma\lambda} g_{\gamma\alpha} g_{\lambda\beta} . \tag{8}$$

Taking into account eqs. (6), (7) and (8) the variational principle (3) with the action integral (5) gives eqs. (1).

3 Second order approximation

Equations (1) and (4) can be translated into the flat, pseudo-Euclidean space-time generalizing the Rosen procedure [6]. We have the following rules:

$$g_{\alpha\beta} = a_{\alpha\beta} - 2 f \tilde{\psi}_{\alpha\beta} , \tag{9}$$

$$z^{*\alpha} = z^{\alpha} , \tag{10}$$

where $a_{\alpha\beta}$ is the fundamental metric tensor of the pseudo-Euclidean space-time and $\tilde{\psi}_{\alpha\beta}$ represents the gravitational potential. As to the proper density of proper mass, if we take the proper mass as invariant (in the translation), i.e.

$$\mu_0^* \, dV_0^* = \mu_0 \, dV_0 , \tag{11}$$

we get

$$\mu_0^* = \frac{dV_0}{dV_0^*} \mu_0 = \frac{d^4 \Omega}{d^4 \Omega^*} \frac{ds^*}{ds} \mu_0 \simeq (1 + f\tilde{\psi} - f\tilde{\psi}_{\rho\nu} \dot{z}^{\rho} \dot{z}^{\nu}) \mu_0 , \tag{12}$$

to second order accuracy (with respect to a parameter ε if wet put $\tilde{\psi}_{\alpha\beta} = \varepsilon \overline{\psi}_{\alpha\beta}$).

As to the translation of $S_{\alpha\beta}^*$ into the flat space-time we are not free to choose it. Indeed, let us recall [1] that in the flat space-time it is

$$\delta_a (\mu_0 \, dV_0) = - \tfrac{1}{2} S_{\alpha\beta} \, \delta a^{\alpha\beta} \, dV_0 , \tag{13}$$

and

$$\delta_{\tilde{\psi}} (\mu_0 \, dV_0) = f S^{\alpha\beta} \, \delta \tilde{\psi}_{\alpha\beta} \, dV_0 . \tag{14}$$

Because of (9) and (11) it has to be

$$\delta_g (\mu_0^* \, dV_0^*) = \delta_a (\mu_0 \, dV_0) + \delta_{\tilde{\psi}} (\mu_0 \, dV_0) . \tag{15}$$

Putting eqs. (6), (13), and (14) into eq. (15) and taking into account (9), gives

$$S^{*\alpha\beta}\, dV_0^* = S^{\alpha\beta}\, dV_0\,, \tag{16}$$

from which, to second order accuracy,

$$S^*_{\alpha\beta} = S_{\alpha\beta}\,(1 - f\tilde{\psi}_{\gamma\lambda}\,\dot{z}^\gamma\,\dot{z}^\lambda + f\tilde{\psi}) - 2\,f\tilde{\psi}^\lambda{}_{(\alpha}\,S_{\beta)\lambda}\,. \tag{17}$$

Let us now translate eqs. (1) by the rules (9), (10), (12) and (17), to second order accuracy. After some simplifications and substituting, whenever a d'Alembertian multiplied by f appears, the first order reduction of the same equations, we eventually obtain

$$\begin{aligned}
\Box\tilde{\psi}_{\alpha\beta} + \tilde{\psi}_{;\alpha\beta} - \tilde{\psi}_{\gamma(\alpha;\beta)}{}^{\gamma} + a_{\alpha\beta}\,(\tilde{\psi}_{\gamma\lambda}{}^{;\gamma\lambda} - \Box\tilde{\psi}) = \\
= -2\,f\tilde{\psi}^{\rho\gamma}\,(\tilde{\psi}_{\alpha\beta;\rho\gamma} + \tilde{\psi}_{\rho\gamma;\alpha\beta} - \tilde{\psi}_{\rho(\alpha;\beta)\gamma}) + 2\,f\tilde{\psi}^{\rho\gamma}{}_{;\rho}\,(\tilde{\psi}_{\gamma(\alpha;\beta)} - \tilde{\psi}_{\alpha\beta;\gamma}) - \\
- f\tilde{\psi}^{;\gamma}\,(\tilde{\psi}_{\gamma(\alpha;\beta)} - \tilde{\psi}_{\alpha\beta;\gamma}) + 2\,f\tilde{\psi}_\alpha{}^{\rho;\gamma}\,\tilde{\psi}_{\beta\gamma;\rho} - 2\,f\tilde{\psi}_\alpha{}^{\gamma;\rho}\,\tilde{\psi}_{\beta\gamma;\rho} - \\
- f\tilde{\psi}^{\rho\gamma}{}_{;\alpha}\,\tilde{\psi}_{\rho\gamma;\beta} + a_{\alpha\beta}\,f\,(-2\,\tilde{\psi}^{\lambda\gamma}{}_{;\lambda}\,\tilde{\psi}^\rho{}_{\gamma;\rho} + 2\,\tilde{\psi}_{;\lambda}\,\tilde{\psi}^{\lambda\gamma}{}_{;\gamma} - \tfrac{1}{2}\,\tilde{\psi}^{;\lambda}\,\tilde{\psi}_{;\lambda} - \\
- \tilde{\psi}^{\lambda\rho;\gamma}\,\tilde{\psi}_{\lambda\gamma;\rho} + 3/2\,\tilde{\psi}^{\lambda\gamma;\rho}\,\tilde{\psi}_{\lambda\gamma;\rho}) + f^2\,\tilde{\psi}_{\alpha\beta}\,(\mu_0 + S) + \\
+ 2\,f^2\,a_{\alpha\beta}\,\tilde{\psi}^{\rho\gamma}\,(\mu_0\,\dot{z}_\rho\,\dot{z}_\gamma + S_{\rho\gamma}) - f^2\,a_{\alpha\beta}\,\tilde{\psi}\,(\mu_0 + S) + \\
+ f\mu_0\,\dot{z}_\alpha\,\dot{z}_\beta\,(1 + f\tilde{\psi}_{\mu\nu}\,\dot{z}^\mu\,\dot{z}^\nu + f\tilde{\psi}) - 2\,f^2\,\mu_0\,\tilde{\psi}^\rho{}_{(\alpha}\,\dot{z}_{\beta)}\,\dot{z}_\rho + \\
+ fS_{\alpha\beta}\,(1 - f\tilde{\psi}_{\gamma\lambda}\,\dot{z}^\gamma\,\dot{z}^\lambda + f\tilde{\psi}) - 2\,f^2\,\tilde{\psi}^\rho{}_{(\alpha}\,S_{\beta)\rho}\,.
\end{aligned} \tag{18}$$

where semicolons stand for covariant differentiations and parentheses containing two indexes stand for symmetrization.

Since the tensor potential is not observable, any other function $\psi_{\alpha\beta}$ biunivocally related to $\tilde{\psi}_{\alpha\beta}$ can be chosen in order to describe the gravitational potential. The expression

$$\tilde{\psi}_{\alpha\beta} = \psi_{\alpha\beta} - 2\,f\psi_{\alpha\gamma}\,\psi_\beta{}^\gamma + f\psi\psi_{\alpha\beta} + 1/4\,f\psi\psi a_{\alpha\beta} + 1/2\,f\psi_{\rho\nu}\,\psi^{\rho\nu}\,a_{\alpha\beta}\,, \tag{19}$$

transforms eqs. (18) into the second order equations of the iterative procedure in the field theoretical approach [see eqs. (19) and (23) of Ref. [1]]. Thus one can say that the latter equations can be " renormalized ", to second order accuracy, to eqs. (1) by the metric

$$g_{\alpha\beta} = a_{\alpha\beta} - 2\,f\psi_{\alpha\beta} + 4\,f^2\,\psi_{\alpha\gamma}\,\psi_\beta{}^\gamma - 2\,f^2\,\psi\psi_{\alpha\beta} - \tfrac{1}{2}f^2\,\psi\psi a_{\alpha\beta} - f^2\,\psi_{\rho\nu}\,\psi^{\rho\nu}\,a_{\alpha\beta}\,. \tag{20}$$

That is, by using clocks and rods deformed by (20), the effect of the gravitational potential $\psi_{\alpha\beta}$ is absorbed in the geometry of the space-time, and (to second order) general relativity is reobtained.

The motion of the matter element is obtained, in both procedure, by taking the divergence of the energy-momentum tensor (the source of the field equations) and equating it to zero. In general relativity this gives rise to the equations of motion

$$\mu_0^* \frac{D\dot{z}_\alpha^*}{ds} = -S^*_{\alpha\beta}{}^{;\beta} - \dot{z}_\alpha^* S_{\gamma\beta}{}^{*;\beta} \dot{z}^{*\gamma} . \tag{21}$$

The matter elements deviate from a geodesic owing to the stresses $S^*_{\alpha\beta}$ only. Even for such equations of motion (21), translating them by the use of eqs. (20), (10), (12), and (17) one obtains the second order equations of motion of the field theoretical approach.

As shown by Thirring [2] real rods and clocks (made out of atoms) are just the ones by which one has a metric given by $ds^2 = g_{\alpha\beta}\, dz^\alpha\, dz^\beta$. Therefore, by real rods and clocks deformed (in the unrenormalized picture) by $f\psi_{\alpha\beta}$ one can, assuming them as uninfluenced by gravity (in the renormalized picture), eliminate $\psi_{\alpha\beta}$ and describe the motion of the matter in a Riemannian space-time whose fundamental metric tensor is given by eq. (20). Only if one considers the real rods and clocks as deformed and, by calculations, he comes back to ideal instruments unaffected by gravity, he can judge the space-time as flat and gravity represented by a tensor potential.

4. Convergence to general relativity

The main aim of the two preceding sections was to show, through the comparison of the second order approximations of both the general relativity and of the field theoretical approach, how the "renormalization" works even in this case of the continuum.

Now the problem of the convergence of the matter terms in the iterative field theoretical [1] approach can be tackled.

Let $I_M^{(n)}$ be the matter action integral of the n-th step approximation. We can write it in one of the following ways

$$I_M^{(n)} = \int L_M^{(n)} \sqrt{-a}\, d^4 x = \int \mu_0 \bar{L}_M^{(n)}\, ds\, dV_0 = \int \mu_0 \hat{L}^{(n)}\, dt\, dV_0 , \tag{22}$$

where $L_M^{(n)} = \mu_0 \bar{L}_M^{(n)}$ is the n-th order Lagrangean density, and

$$\hat{L}^{(n)} = \bar{L}_M^{(n)} \left(a_{\alpha\beta} \frac{dz^\alpha}{dt} \frac{dz^\beta}{dt} \right)^{1/2} , \tag{23}$$

t being an auxiliary integration variable. The last expression of (22) with (23) will be convenient in the following because in this form the dependence of ds on $a_{\alpha\beta}$ is transferred to $\hat{L}^{(n)}$.

The matter part (i.e. the right hand side) of the n-th order field equations obtained by varying $\psi_{\alpha\beta}$ in (22) is given by [1]

$$-\mu_0 \frac{\delta \bar{L}_M^{(n)}}{\delta \psi_{\alpha\beta}} - \bar{L}_M^{(n)} D^{(n)\alpha\beta} . \tag{24}$$

The second term of (24) is due to the n-th order continuity equation; it is therefore $D^{(1)\alpha\beta} = f S^{\alpha\beta}$ because of eq. (14). The expression of $D^{(n)\alpha\beta}$ for $n \neq 1$ is not a priori known. The only thing we can notice is that the second term of (24) will contain, when made explicit, terms of order higher than the n-th one (since the lowest order of the terms of $D^{(n)\alpha\beta}$ is the first one). Such latter terms have to be cut when obtaining the n-th order field equations.

The iterative procedure [1] rests on the fact that the same terms given in eq. (24) (i.e. the source of the n-th order field equations) must be equal to the $(n-1)$-th order energy-momentum tensor multiplied by f. Such tensor can be got by the $(n-1)$-th order action integral, by varying in it the fundamental metric tensor, and its part relevant to matter is given by

$$(25) \qquad T_M^{(n-1)\alpha\beta} = 2\,\mu_0 \frac{\delta \bar{L}_M^{(n-1)}}{\delta a_{\alpha\beta}} - (S^{\alpha\beta} + \mu_0\, \dot{z}^\alpha\, \dot{z}^\beta)\, \bar{L}_M^{(n-1)} .$$

Notice that $S^{\alpha\beta}$ is present in eq. (25) because eq. (13) has been taken into account. Moreover eq. (13) holds to any order like the relationship

$$\delta_a (ds) = -\tfrac{1}{2}\, \dot{z}^\alpha\, \dot{z}^\beta\, \delta a_{\alpha\beta}\, ds .$$

Therefore the iterative procedure implies.

$$(26) \qquad -\mu_0 \frac{\delta \bar{L}_M^{(n)}}{\delta \psi_{\alpha\beta}} - D^{(n)\alpha\beta}\, \bar{L}_M^{(n)} = 2 f \mu_0 \frac{\delta \bar{L}_M^{(n-1)}}{\delta a_{\alpha\beta}} - (f S^{\alpha\beta} + f \mu_0\, \dot{z}^\alpha\, \dot{z}^\beta)\, \bar{L}_M^{(n-1)} .$$

It has already been said that the tensor potential is not an observable and thus that the tensor potential can be transformed to another $\tilde{\psi}_{\alpha\beta}$ by a widely arbitrary function if also the relationship between $\psi_{\alpha\beta}$ and the quantity $g_{\alpha\beta}$ of the "renormalized" space is correspondingly transformed. What we are searching for is a Lagrangean density L_M to which the $L_M^{(n)}$ are converging. Even the resulting theory will be renormalized into the observable curved space-time, and then each step of the iterative procedure will be seen as the n-th order approximate translation of the exact theory into the flat space-time. It is obvious that the exact L_M will be much more easily found if the translation rules are the same to any order. Therefore we choose to transform $\psi_{\alpha\beta}$ to any order just into the $\tilde{\psi}_{\alpha\beta}$ linked to $g_{\alpha\beta}$ by the eqs. (9). In this way, eq. (14) holds to any order of approximation, the new $\tilde{D}^{(n)\alpha\beta}$ is equal to $f S^{\alpha\beta}$ to any order and eqs. (26) can be written as

$$(27) \qquad \mu_0 \frac{\delta \tilde{L}_M^{(n)}}{\delta h_{\alpha\beta}} - \tfrac{1}{2}\, S^{\alpha\beta}\, \tilde{L}_M^{(n)} = \mu_0 \frac{\delta \tilde{L}^{(n-1)}}{\delta a_{\alpha\beta}} - \tfrac{1}{2}\, (S^{\alpha\beta} + \mu_0\, \dot{z}^\alpha\, \dot{z}^\beta)\, \tilde{L}_M^{(n-1)} .$$

where $h_{\alpha\beta} = -2 f \psi_{\alpha\beta}$ and where $\tilde{L}_M^{(n)}$ are the $\bar{L}_M^{(n)}$ of eqs. (26) in which the potentials have been transformed.

As said after eqs. (24), the terms of order higher than the n-th one have to be cut when obtaining the n-th order field equations. In passing from (26) to (27) a division by f has been made. Hence in eqs. (27) the terms whose order is higher than the $(n-1)$-th order have to be cut. After this cutting the second term of the LHS of eqs. (27) reads $S^{\alpha\beta}\,\tilde{L}_M^{(n-1)}$ and is simpliefied by the same term in the RHS. Moreover it is convenient to pass from $\tilde{L}_M^{(n)}$ to

$$\hat{L}^{(n)} = \tilde{L}_M^{(n)} \left(a_{\alpha\beta} \frac{dz^\alpha}{dt} \frac{dz^\beta}{dt} \right)^{1/2}$$

as in eq. (23). From here we omit the subscript M for simplicity. Equations (27) imply

$$\frac{\delta \hat{L}^{(n)}}{\delta h_{\alpha\beta}} = \frac{\delta \hat{L}^{(n-1)}}{\delta a_{\alpha\beta}}, \tag{28}$$

where $\hat{L}^{(n)}$ depends on $a_{\alpha\beta}$ and on $h_{\alpha\beta}$ but not on their derivatives. Obviously there are the additional conditions

$$\hat{L}^{(0)}(a_{\alpha\beta}, h_{\alpha\beta}) = \hat{L}^{(0)}(a_{\alpha\beta}), \tag{29}$$

$$\hat{L}^{(n)}(a_{\alpha\beta}, 0) = \hat{L}^{(0)}(a_{\alpha\beta}). \tag{30}$$

As seen in a preceding paper [4], the conditions (28), (29), and (30) imply

$$\hat{L}^{(n)}(a_{\alpha\beta}, h_{\alpha\beta}) = \frac{1}{n!} \sum_0^n{}_j \left(h_{\alpha\beta} \frac{\partial}{\partial a_{\alpha\beta}} \right)^{(j)} \hat{L}^{(0)}(a_{\alpha\beta}). \tag{31}$$

If the sequence $\{\hat{L}^{(n)}\}$ uniformily converges in a domain $\mathscr{D}$ to a function $\hat{L}$, it is

$$\hat{L}(a_{\alpha\beta}, h_{\alpha\beta}) = \hat{L}^{(0)}(a_{\alpha\beta} + h_{\alpha\beta}). \tag{32}$$

In our case of a neutral continuum it is

$$\hat{L}^{(0)}(a_{\alpha\beta}) = \left(a_{\alpha\beta} \frac{dz^\alpha}{dt} \frac{dz^\beta}{dt} \right)^{1/2}. \tag{33}$$

Hence eq. (31) gives

$$\hat{L}^{(n)} = \left(a_{\alpha\beta} \frac{dz^\alpha}{dt} \frac{dz^\beta}{dt} \right)^{1/2} \sum_0^n{}_j \binom{1/2}{j} (h_{\alpha\beta}\, \dot{z}^\alpha \dot{z}^\beta)^j. \tag{34}$$

The sequence $\{\hat{L}^{(n)}\}$ uniformily converges if $|\, h_{\alpha\beta}\, \dot{z}^\alpha \dot{z}^\beta \,| \leq 1$. Under this condition it is

$$\hat{L} = \left[(a_{\alpha\beta} + h_{\alpha\beta}) \frac{dz^\alpha}{dt} \frac{dz^\beta}{dt} \right]^{1/2}. \tag{35}$$

If we make the hypothesis that the exact $\hat{L}$ is an analytic function of $h_{\alpha\beta}$ we can make an analytic continuation for $h_{\alpha\beta}\, \dot{z}^{\alpha}\, \dot{z}^{\beta} \geq -1$. By the identity principle for the analytic function, $\hat{L}$ will have the same expression in all the domain $h_{\alpha\beta}\, \dot{z}^{\alpha}\, \dot{z}^{\beta} \geq -1$.

Now the theory is renormalized in the Riemannian space-time whose fundamental tensor is given by eqs. (9). Equation (35) put into eq. (22), taking into account eq. (11), gives

$$\mathrm{I}_{\mathrm{M}}^{*} = -\int \mu_0^{*}\, d^4\, \Omega^{*}\,. \tag{36}$$

The latter is just the second term of eq. (4) up to the coefficient $2f^2$. This coefficient too comes out when the pure field terms [3] are obtained (i.e. for them one obtains $R/2f^2$ in the integrand).

The condition $h_{\alpha\beta}\, \dot{z}^{\alpha}\, \dot{z}^{\beta} \geq -1$ corresponds to $ds^{*2}/ds^2 \geq 0$, i.e. to having speeds lower than the light speed in the Riemannian space-time if the same happens in the pseudo-Euclidean space-time.

Thus general relativity is obtained to all orders.

As to the stress tensor, in the iterative procedure it has been considered as a given field; i.e. the problem of the link between stresses and strains has been postponed in order to treat it in the exact theory only. It has here been shown that the latter is the general theory of relativity and for such theory the overmentioned problem has already been solved by Cattaneo [7].

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