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Interaction of shock waves with acoustic waves

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Fisica matematica. — *Interaction of shock waves with acoustic waves* (*). Nota di ANGELO MORRO, presentata (**) dal Socio C. CATTANEO.

RIASSUNTO. — Si studia l'interazione tra onde d'urto e onde acustiche in accordo con la teoria generale sviluppata recentemente da Brun. Esaminata in dettaglio la condizione caratterizzante l'interazione, si considera la trasmissione e la riflessione nel caso di solidi elastici. Si mostra che, nell'approssimazione di onde d'urto deboli, sia l'onda riflessa che l'onda trasmessa hanno un'ampiezza minore di quella dell'onda incidente.

§ 1. INTRODUCTION

The stability of a shock wave is usually considered through the interaction of acoustic waves with the shock wave itself. In this connection we recall the fundamental paper by D'iakov [1]. Successively, several works on the subject were published; among others we mention the recent papers by Swan and Fowles [2] and Van Moorhem and George [3] concerning shock waves in fluids. The problem of the interaction was also extensively considered within the context of hyperbolic systems and waves as it appears in the exhaustive book by Jeffrey [4].

A noteworthy contribution towards a general theory of the interaction has been given recently by Brun [5]. The main features of such a theory are the following ones. First, the condition for the stability of the shock, in the form of an evolutionary condition leads to the Lax inequalities [6]. Second, it is shown that, in general, the shock undergoes an acceleration as a consequence of the interaction. Finally, the theory provides a precise method for a quantitative study of the interaction. Precisely, once the properties of the shock and of the incident perturbation are known, the acceleration undergone by the shock and the amplitudes of the acoustic waves, are determined in a straightforward manner. In view of these promising results, it seems to us that the theory deserves further attention.

The primary purpose of this Note is to show how the amplitudes of the emergent modes (outgoing perturbations) may be obtained. To this end, sect. 2 deals with a detailed re-examination of the procedure followed by Brun in order to obtain the relation characterizing the interaction. In sect. 3 we briefly recall the evolutionary condition introduced by Brun and finally, in sect. 4, we consider the particular case of interaction in elastic solids. In this connection,

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within the approximation of weak shocks, we examine the transmission and the reflection of longitudinal acoustic waves. As a result, we obtain that the amplitude both of the transmitted and of the reflected wave is smaller than the amplitude of the incident wave.

§ 2. THE RELATION CHARACTERIZING THE INTERACTION

Throughout this note we consider plane waves (shock waves and acoustic waves) propagating into the continuum in hand in the direction of the X axis with respect to a suitable reference configuration. Given any quantity ξ, we denote by $[\xi] = \xi^- - \xi^+$ the jump of the quantity ξ, ξ^- and ξ^+ being the limiting values of ξ respectively behind and ahead of the wave front. Let

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \mathbf{F}(\boldsymbol{u})}{\partial \mathrm{X}} = 0 \tag{2.1}$$

be the system of n conservation equations for the n unknown functions $\boldsymbol{u} = (u_1, \cdots, u_n)$ describing the evolution of the continuum. We assume that the quantity $\boldsymbol{u}$ suffers a jump discontinuity across the shock front. Concerning the system (2.1) we introduce the hypothesis that it is strictly hyperbolic; this means that there exist n linearly independent eigenvectors $\mathbf{D}_1, \cdots, \mathbf{D}_n$ defining the directions of the discontinuities $\left[\frac{\partial \boldsymbol{u}}{\partial \mathrm{X}}\right]$ or $\left[\frac{\partial \boldsymbol{u}}{\partial t}\right]$ which propagate into the continuum with any of the wave speeds $c_1, \cdots, c_n$ (distinct or coincident).

Let us consider a shock wave with speed $s > 0$ at a fixed time t. Without loss of generality, we say that s is greater (smaller) than $\mathrm{M}^+(\mathrm{M}^-)$ characteristic speeds ahead of (behind) the shock. On the whole, we have $\mathrm{M} = \mathrm{M}^+ + \mathrm{M}^-$ types of incident (acoustic) perturbations which may interact with the shock. To begin with, we first suppose that one of these incident perturbations is coming from the region ahead of the shock. The speed of the shock separates N^+ speeds of emergent modes (ahead) from N^- speeds of emergent modes (behind), namely

$$\overset{-}{c}_{k_{\mathrm{N}^-}} \leq \cdots \leq \overset{-}{c}_{k_2} \leq \overset{-}{c}_{k_1} < s < \overset{+}{c}_{k_1} \leq \overset{+}{c}_{k_2} \leq \cdots \leq \overset{+}{c}_{k_{\mathrm{N}^+}} . \tag{2.2}$$

The interaction of the incident perturbation with the shock at the time $t^-(\equiv t - 0)$ gives rise, besides a jump of the acceleration of the shock, to the emission of discontinuities behind and ahead of the shock. Of course, such discontinuities belong to the set of the $\mathrm{N} = \mathrm{N}^+ + \mathrm{N}^-$ emergent modes satisfying the condition (2.2).

Let $[\mathrm{G}]_k$ denote the jump of the quantity G, across the wave front, due to the k^{th} discontinuity and let $\Delta\mathrm{G} = \mathrm{G}(t^+) - \mathrm{G}(t^-)$ be the whole jump of G between the times t^- and t^+. Thus we have

$$\Delta\mathrm{G}^+ = \mathrm{G}^+(t^+) - \mathrm{G}^+(t^-) = [\mathrm{G}^+]_{\mathrm{e.m.}} + [\mathrm{G}^+]_{\mathrm{inc}}$$

where e.m. and inc are reminders for the contributions of the emergent modes and of the incident perturbation. More explicitly, we write this result in the form

$$\Delta G^+ = \sum_{j=1}^{N^+} [G^+]_{k_j} + [G^+]_{\text{inc}} .$$

According to the hypothesis introduced above, behind the shock we have only emergent modes but not incident perturbation, and then

$$\Delta G^- = [G^-]_{\text{e.m.}} = \sum_{j=1}^{N^-} [G^-]_{k_j} .$$

Thus, the quantity $\Delta [G] \equiv \Delta G^- - \Delta G^+$ can be given the expression

$$\Delta [G] = \sum_{j=1}^{N^-} [G^-]_{k_j} - \sum_{j=1}^{N^+} [G^+]_{k_j} - [G^+]_{\text{inc}} . \tag{2.3'}$$

An analogous expression can be derived in the case when the incident perturbation is coming from the region behind the shock. We have

$$\Delta G^+ = [G^+]_{\text{e.m.}} .$$

while

$$\Delta G^- = [G^-]_{\text{e.m.}} + [G^-]_{\text{inc}} .$$

A subtraction allows to write

$$\Delta [G] = \sum_{j=1}^{N^-} [G^-]_{k_j} - \sum_{j=1}^{N^+} [G^+]_{k_j} + [G^-]_{\text{inc}} . \tag{2.3''}$$

The results (2.3'), (2.3'') are summarized by the relation

$$\Delta [G] = \sum_{j=1}^{N^-} [G^-]_{k_j} - \sum_{j=1}^{N^+} [G^+]_{k_j} \mp [G^\pm]_{\text{inc}} . \tag{2.3}$$

Concerning the system (2.1) we have the following jump relations for the shock

$$[\mathbf{F}] - s\,[\boldsymbol{u}] = 0$$

whence

$$\mathbf{F}^+ - s\boldsymbol{u}^+ = \mathbf{F}^- - s\boldsymbol{u}^- . \tag{2.4}$$

The relation (2.4) holds identically in time and moreover the functions $\mathbf{F}^\pm$, $\boldsymbol{u}^\pm$, s are smooth functions with respect to time. From (2.4) we obtain

$$\dot{\mathbf{F}}^+ - s\dot{\boldsymbol{u}}^+ + \dot{s}\,[\boldsymbol{u}] = \dot{\mathbf{F}}^- - s\dot{\boldsymbol{u}}^- \tag{2.5}$$

where a superimposed dot denotes time derivative with respect to the observer which travels with the velocity of the shock $\left(\dot{} = \frac{\partial}{\partial t} + s \frac{\partial}{\partial X}\right)$. In particular, the relation (2.5) holds at the times t^+, t^-; a subtraction of the corresponding expressions yields

$$\Delta\dot{\mathbf{F}}^+ - s\Delta\dot{\boldsymbol{u}}^+ + \Delta\dot{s}\,[\boldsymbol{u}] = \Delta\dot{\mathbf{F}}^- - s\Delta\dot{\boldsymbol{u}}^-$$

whence

$$\Delta\,[\dot{\mathbf{F}} - s\dot{\boldsymbol{u}}] = \Delta\dot{s}\,[\boldsymbol{u}]\,. \tag{2.6}$$

It is worth remarking that the condition (2.6) has been derived considering the shock only. However, it must be satisfied also by the acoustic waves at the time t since, at this time, the regions behind and ahead of the shock are the same as the regions behind and ahead of the incident perturbation and the emergent modes. With this in mind, we consider the obvious jump relations

$$[\dot{\mathbf{F}}] = s\left[\frac{\partial \mathbf{F}}{\partial X}\right] + \left[\frac{\partial \mathbf{F}}{\partial t}\right] \tag{2.7}$$

$$[\dot{\boldsymbol{u}}] = s\left[\frac{\partial \boldsymbol{u}}{\partial X}\right] + \left[\frac{\partial \boldsymbol{u}}{\partial t}\right] \tag{2.8}$$

for the incident perturbation and the emergent modes. The continuum in hand satisfies the system (2.1) and then the jump relations

$$\left[\frac{\partial \boldsymbol{u}}{\partial t}\right] + \left[\frac{\partial \mathbf{F}}{\partial X}\right] = 0\,.$$

Substitution into (2.7) yields

$$[\dot{\mathbf{F}}] = -s\left[\frac{\partial \boldsymbol{u}}{\partial t}\right] + \left[\frac{\partial \mathbf{F}}{\partial t}\right]. \tag{2.9}$$

Let c be the speed of an acoustic wave (e.g. the incident perturbation or the emergent modes). If ϕ is any of the quantities $\mathbf{F}$, $\boldsymbol{u}$, we have the compatibility condition

$$\left[\frac{\partial \phi}{\partial t}\right] = -c\left[\frac{\partial \phi}{\partial X}\right] \tag{2.10}$$

Making use of (2.10), eq. (2.9) gives

$$[\dot{\mathbf{F}}] = cs\left[\frac{\partial \boldsymbol{u}}{\partial X}\right] - c\left[\frac{\partial \mathbf{F}}{\partial X}\right] = c\,(s - c)\left[\frac{\partial \boldsymbol{u}}{\partial X}\right].$$

Analogously, eq. (2.8) reads

$$[\dot{\boldsymbol{u}}] = (s - c)\left[\frac{\partial \boldsymbol{u}}{\partial X}\right].$$

Therefore, it follows at once

$$[\dot{\mathbf{F}}] - s\,[\dot{\boldsymbol{u}}] = -(s - c)^2\left[\frac{\partial \boldsymbol{u}}{\partial X}\right]. \tag{2.11}$$

On the other hand, application of the equation (2.3) to (2.6) gives

$$\sum_{j=1}^{N^+} [\dot{\mathbf{F}}^+ - s\dot{\boldsymbol{u}}^+]_{k_j} - \sum_{j=1}^{N^-} [\dot{\mathbf{F}}^- - s\dot{\boldsymbol{u}}^-]_{k_j} + \Delta\dot{s}\,[\boldsymbol{u}] = \mp\,[\dot{\mathbf{F}}^\pm - s\dot{\boldsymbol{u}}^\pm]_{\text{inc}}$$

whence, on account of (2.11),

$$-\sum_{j=1}^{N^+} (c_{k_j}^+ - s)^2\left[\frac{\partial \boldsymbol{u}^+}{\partial X}\right]_{k_j} + \sum_{j=1}^{N^-} (c_{k_j}^- - s)^2\left[\frac{\partial \boldsymbol{u}^-}{\partial X}\right]_{k_j} + \Delta\dot{s}\,[\boldsymbol{u}] = \pm\,(c_{\text{inc}} - s)^2\left[\frac{\partial \boldsymbol{u}^\pm}{\partial X}\right]_{\text{inc}}. \tag{2.12}$$

Since the system (2.1) is hyperbolic, we may introduce the scalar quantities a_{k_j} defined by

$$\left[\frac{\partial \boldsymbol{u}^\pm}{\partial X}\right]_{\text{inc}} = \pm\, a_{\text{inc}}\,\mathbf{D}_{\text{inc}} \quad , \quad \left[\frac{\partial \boldsymbol{u}^\pm}{\partial X}\right]_{k_j} = \mp\, a_{k_j}^\pm\,\mathbf{D}_{k_j}^\pm . \tag{2.13}$$

Substitution of (2.13) into (2.12) produces the relation characterizing the interaction, namely

$$\sum_{k,\pm} a_k\,(c_k - s)^2\,\mathbf{D}_k + \Delta\dot{s}\,[\boldsymbol{u}] = a_{\text{inc}}\,(c_{\text{inc}} - s)^2\,\mathbf{D}_{\text{inc}} \tag{2.14}$$

where $\sum_{k,\pm}$ is shorthand for the sum over the $N = N^+ + N^-$ emergent modes.

It is worth noticing that the relation (2.14) has been recently obtained also by Boillat and Ruggeri [7] by means of a somewhat different procedure.

§ 3. EVOLUTIONARY CONDITION

The idea of stability of a shock is related to the behaviour in time of transmitted and reflected disturbances. Precisely, one says that a shock is stable to small disturbances if the transmitted and reflected disturbances and the perturbed shock speed remain bounded in time (see, e.g., [1, 8]). Often, however, the stability condition for a shock is assumed in the weaker form of an evolutionary condition [9, 10, 5, 4]. Here we introduce the evolutionary condition in the form proposed by Brun [5].

"Condition d'evolution. Une discontinuité forte est nécessairement telle que le résultat de son interaction avec toute discontinuité ordinaire éventuelle existe et soit unique ".

Let see now how the evolutionary condition can be given an operational form. The properties of the shock—and then the vector $[\boldsymbol{u}]$—are regarded as known for the problem in hand. Thus, the evolutionary condition is satisfied if and only if the amplitudes a_k as well as the discontinuity $\Delta\dot{s}$ exist and are unique for each of the M possible choices for the incident perturbation. This is possible if and only if

i) the N eigenvectors $\mathbf{D}_k$ and the vector $[\boldsymbol{u}]$ are independent,

ii) all the vectors $\mathbf{D}_{\mathrm{inc}}$ belong to the linear space spanned by the N eigenvectors $\mathbf{D}_k$ and the vector $[\boldsymbol{u}]$.

As a consequence of i) we have

$$\mathrm{N} = \mathrm{N}^+ + \mathrm{N}^- = n - 1 . \tag{3.1}$$

The inequalities (2.2) together with the restriction (3.1) constitute the Lax inequalities.

Our concern in the next section will be an application of the statements i), ii) in the case of shocks in elastic solids.

§ 4. Interaction in elastic solids

Following Brun's notations [5], we say that, in the presence of plane waves, the balance equations of an elastic solid can be given the form (2.1) by setting

$$\boldsymbol{u} = (\gamma , \boldsymbol{v} , \mathrm{U} + v^2/2) \quad , \quad -\mathbf{F} = (\boldsymbol{v} , \boldsymbol{b} , \boldsymbol{v}\cdot\boldsymbol{b}) \tag{4.1}$$

where γ is the deformation vector, $\boldsymbol{v}$ the velocity with respect to a suitable frame of reference, U the specific internal energy and $\boldsymbol{b}$ the body force, $\boldsymbol{b} = \partial\mathrm{U}/\partial\gamma$. In view of the balance of linear momentum, an acoustic wave must satisfy the following jump conditions

$$\left\{\begin{aligned} & c^2 \left[\frac{\partial\gamma}{\partial\mathrm{X}}\right] = \mathbf{Q}\left[\frac{\partial\gamma}{\partial\mathrm{X}}\right] + (\nabla_\gamma\,\theta)\left[\frac{\partial\mathrm{S}}{\partial\mathrm{X}}\right] \\ & c\left[\frac{\partial\mathrm{S}}{\partial\mathrm{X}}\right] = 0 \end{aligned}\right. \tag{4.2}$$

where $\mathbf{Q}$ is the acoustic tensor, S is the specific entropy and $\theta = \partial\mathrm{U}/\partial\mathrm{S}$ is the temperature. Therefore the speeds of the acoustic waves are

$$c_0 = 0 \quad , \quad c_i > 0 \quad , \quad c_{-i} = -c_i \tag{4.3}$$

where $c_i\,(i = 1\,,2\,,3)$ are the eigenvalues of the acoustic tensor, namely

$$\mathbf{Q}\boldsymbol{\delta}_i = c_i\,\boldsymbol{\delta}_i \quad , \quad \boldsymbol{\delta}_i = \boldsymbol{\delta}_{-i}\,. \tag{4.4}$$

On account of (4.2) we obtain

$$\left[\frac{\partial \boldsymbol{u}}{\partial \mathrm{X}}\right] = \left(\left[\frac{\partial \gamma}{\partial \mathrm{X}}\right], -c\left[\frac{\partial \gamma}{\partial \mathrm{X}}\right], (\boldsymbol{b} - c\boldsymbol{v})\cdot\left[\frac{\partial \gamma}{\partial \mathrm{X}}\right]\right) \tag{4.5}$$

if $c \neq 0$, and

$$\left[\frac{\partial \boldsymbol{u}}{\partial \mathrm{X}}\right] = \left(-\mathbf{Q}^{-1}(\nabla_\gamma\,\theta)\left[\frac{\partial \mathrm{S}}{\partial \mathrm{X}}\right], 0\,, (-\boldsymbol{b}\cdot\mathbf{Q}^{-1}\nabla_\gamma\,\theta + \theta)\left[\frac{\partial \mathrm{S}}{\partial \mathrm{X}}\right]\right) \tag{4.6}$$

if $c = 0$. In view of (4.4), we are now able to write the characteristic vectors corresponding to (4.5) and (4.6) in terms of the acoustic directions $\boldsymbol{\delta}_i$, namely

$$\begin{aligned} \mathbf{D}_i &= (\boldsymbol{\delta}_i\,, -c_i\,\boldsymbol{\delta}_i\,, (\boldsymbol{b} - c_i\,\boldsymbol{v})\cdot\boldsymbol{\delta}_i\,, \qquad i = \pm 1\,, \pm 2\,, \pm 3\,, \\ \mathbf{D}_0 &= (\mathbf{Q}^{-1}\,\nabla_\gamma\,\theta\,, 0\,, -\theta + \boldsymbol{b}\cdot\mathrm{Q}^{-1}\,\nabla_\gamma\,\theta)\,. \end{aligned} \tag{4.7}$$

For convenience we introduce a new frame of reference such that $\boldsymbol{v} \to \boldsymbol{v}' = \boldsymbol{v} - \dfrac{\boldsymbol{v}^- + \boldsymbol{v}^+}{2}$. In this case we obtain

$$[\boldsymbol{u}] = (\lambda\,, -s\lambda\,, \bar{\boldsymbol{b}}\cdot\lambda) \tag{4.8}$$

$$\mathbf{D}_i^{\pm} = \left(\boldsymbol{\delta}_i^{\pm}, -c_i^{\pm}\,\boldsymbol{\delta}_i^{\pm}, \left(\boldsymbol{b}^{\pm} \mp \frac{1}{2\,s}\,c_i^{\pm}\,[\boldsymbol{b}]\right)\cdot\boldsymbol{\delta}_i^{\pm}\right) \tag{4.9}$$

where $\lambda = -[\boldsymbol{v}]/s$ and $\bar{\boldsymbol{b}} = (\boldsymbol{b}^+ + \boldsymbol{b}^-)/2$. These results enable us to prove the following

PROPOSITION. *In the case of weak shocks, the longitudinal shock travels with supersonic speed with respect to the medium ahead of the front and with subsonic speed with respect to the medium behind.*

Proof. Let us consider a longitudinal shock whose normal $\boldsymbol{n}$ is parallel to the deformation vector γ ahead of the front (and then also behind the front). Setting $\boldsymbol{n} = (1\,,0\,,0)$, the condition (4.8) simplifies to

$$\boldsymbol{u} = (1\,,0\,,0\,,-s\,,0\,,0\,,\bar{b}_1)\,\lambda\,. \tag{4.10}$$

Let us suppose that the incident perturbation is a longitudinal acoustic wave, parallel to the shock, and coming from the region ahead of the front, namely $\mathbf{D}_{\mathrm{inc}} = \mathbf{D}_{-1}^{+}$. As a consequence, we have

$$\mathbf{D}_{-1}^{\pm} = \left(1\,,0\,,0\,,c_1^{\pm},0\,,0\,,b_1^{\pm} \pm \frac{c_1^{\pm}}{2\,s}\,[b_1]\right), \tag{4.11}$$

$$\mathbf{D}_0^{\pm} = \left(\left(\frac{1}{c_1^2}\frac{\partial\theta}{\partial\gamma_1}\right)^{\pm}, 0, 0, 0, 0, 0, \left(-\theta + \frac{1}{c_1^2} b_1 \frac{\partial\theta}{\partial\gamma_1}\right)^{\pm}\right). \tag{4.12}$$

Certainly, $\mathbf{D}_{-1}^{-}$ and $\mathbf{D}_0^{-}$ are emergent modes which actually occur. On the other hand the determinant $(\mathbf{D}_{-1}^{-}, \mathbf{D}_0^{-}, [\boldsymbol{u}])$ is given by

$$(\mathbf{D}_{-1}^{-}, \mathbf{D}_0^{-}, [\boldsymbol{u}]) = -\theta(c_1^{-} + s)\lambda\left(1 - \frac{s^2}{\theta c_1^{-}(c_1^{-} + s)}\left(\frac{\partial\theta}{\partial\gamma_1}\right)^{-}\lambda\right) \tag{4.13}$$

where the jump relation $[\boldsymbol{b}] = s^2\lambda$ has been taken into account [1]. At least in the case of weak shocks (λ small), according to (4.13) we have $(\mathbf{D}_{-1}^{-}, \mathbf{D}_0^{-}, [\boldsymbol{u}]) \neq 0$. Therefore $\mathbf{D}_{-1}^{-}$, $\mathbf{D}_0^{-}$, and $[\boldsymbol{u}]$ constitute a basis in terms of which $\mathbf{D}_{-1}^{+}$ may be represented in a unique way. In view of i) and ii), the emergent modes $\mathbf{D}_1^{-}$ and $\mathbf{D}_1^{+}$ cannot occur and then.

$$c_1^{+} < s < c_1^{-}. \tag{4.14}$$

We have thus obtained the well known result according to which if the incident (longitudinal) wave is coming from the region ahead we have transmission but not reflection whilst if the incident wave is coming from the region behind we have reflection but not transmission. □

Now we consider the amplitudes of the transmitted and reflected waves. First, let $\mathbf{D}_{\text{inc}} = \mathbf{D}_{-1}^{+}$; then the relation (2.14) reads

$$a_{-1}^{-}(c_1^{-} + s)^2 \mathbf{D}_{-1}^{-} + a_0^{-} s^2 \mathbf{D}_0^{-} + \Delta\dot{s}\,[\boldsymbol{u}] = a_{-1}^{+}(c_1^{+} + s)^2 \mathbf{D}_{-1}^{+}.$$

Taking into account (2.13) and (4.5), a straightforward calculation yields

$$\tau = \frac{\left[\dfrac{\partial\gamma_1^{-}}{\partial X}\right]_{-1}}{\left[\dfrac{\partial\gamma_1^{+}}{\partial X}\right]_{\text{inc}}} = \frac{a_{-1}^{-}}{a_{-1}^{+}} = \frac{(c_1^{+} + s)^3}{(c_1^{-} + s)^3}\,\frac{1 - \dfrac{s^3}{\theta(c_1^{-})^2(c_1^{+} + s)}\left(\dfrac{\partial\theta}{\partial\gamma_1}\right)^{-}\lambda}{1 - \dfrac{s^2}{\theta c_1^{-}(c_1^{-} + s)}\left(\dfrac{\partial\theta}{\partial\gamma_1}\right)^{-}\lambda}. \tag{4.15}$$

In the case of reflection, $\mathbf{D}_{\text{inc}} = \mathbf{D}_1^{-}$, the amplitudes must satisfy the condition

$$a_{-1}^{-}(c_1^{-} + s)^2 \mathbf{D}_{-1}^{-} + a_0^{-} s^2 \mathbf{D}_0^{-} + \Delta\dot{s}\,[\boldsymbol{u}] = a_1^{-}(c_1^{-} - s)^2 \mathbf{D}_1^{-}.$$

We obtain

$$r = \frac{\left[\dfrac{\partial\gamma_1^{-}}{\partial X}\right]_{-1}}{\left[\dfrac{\partial\gamma_1^{-}}{\partial X}\right]_{\text{inc}}} = -\frac{a_{-1}^{-}}{a_1^{-}} = \frac{(c_1^{-} - s)^2}{(c_1^{-} + s)^3}\,\frac{c_1^{-} - s - \dfrac{s^2}{\theta c_1^{-}}\left(\dfrac{\partial\theta}{\partial\gamma_1}\right)^{-}\lambda}{1 - \dfrac{s^2}{\theta c_1^{-}(c_1^{-} + s)}\left(\dfrac{\partial\theta}{\partial\gamma_1}\right)^{-}\lambda}.$$

(1) It is worth noticing that the expression (4.13) is the correct form of the analogous relation (15.13), given in [5], which suffers from an error in sign concerning the expression for $\mathbf{D}_i$.

First of all, the results (4.15) and (4.16) show that, for weak shocks, both the transmitted and the reflected wave have an amplitude smaller than that of the incident one. Moreover, it is immediately seen that, in the limiting case $\lambda \rightarrow 0$, we have a complete transmission ($\tau = 1$) without reflection ($r = 0$).

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