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Fisica matematica. — *On wave solutions of unified field equations of Finzi.* Nota (*) di KRISHNA BEHARI LAL e MUSTAQEEM, presentata dal Socio C. CATTANEO.

RIASSUNTO. — Si ottengono onde soluzioni dell'equazione di campo unificato di Finzi in uno spazio-tempo definito dalla metrica (1.6), e si calcola il tensore di campo elettromagnetico secondo quanto proposto da Graiff.

1. INTRODUCTION

Einstein [1] (1) developed a unified field theory based on the geometrical interpretation of gravitation and electromagnetism by using a four dimensional generalized Riemannian space in which the non-symmetric fundamental tensor g_{ij} is equal to $(h_{ij} + f_{ij})$, where h_{ij} is the symmetric part of g_{ij} and coincides with the fundamental tensor of space representing the gravitational potential, while f_{ij} is the skew-symmetric part of g_{ij} and signifies the electromagnetic field.

Einstein [1], Schrödinger [2], Bonnor [3], Buchdahl [4] and many others have taken a priori that the torsion vector Γ_i vanishes, i.e. $\Gamma_i \equiv \Gamma^j_{\check{ij}} = 0$, as a part of their field equations.

Finzi [5], on the other hand, without pre-supposing the vanishing of the torsion vector identically, has given the following field equations:

$$g_{\underset{+-}{ij;k}} \equiv g_{ij,k} - g_{1j}\,\Gamma^1_{ik} - g_{i1}\,\Gamma^1_{kj} = 0\,, \tag{1.1}$$

$$R^*_{\underline{ij}} = 0\,, \tag{1.2}$$

$$\text{rot}\, R^*_{\check{ij}} = R^*_{\check{ij},k} + R^*_{\check{jk},i} + R^*_{\check{ki},j} = 0\,, \tag{1.3}$$

$$\text{div}\, R^{*\check{ij}} = R^{*\check{ij}}{}_{,j} + R^{*\check{ij}}\,\Gamma^s_{\underline{js}} = 0\,, \tag{1.4}$$

where Γ^i_{jk} are obtained from equation (1.1) and the tensor $R^*_{ij} = R_{ij} + \Gamma_{\underset{+}{i;j}}$, R_{ij} being the generalized Ricci tensor given by

$$R_{ij} = \Gamma^s_{ij,s} - \Gamma^s_{is,j} - \Gamma^s_{tj}\,\Gamma^t_{is} + \Gamma^s_{ts}\,\Gamma^t_{ij}\,. \tag{1.5}$$

Here a comma preceding an index i denotes partial differentiation with respect to x^i, a semicolon (;) denotes covariant differentiation with respect

(*) Pervenuta all'Accademia il 28 luglio 1976.

(1) Numbers in brackets refer to the references at the end of the paper.

to Γ^i_{jk}. A + or — sign below an index fixes the position of covariant index k in the connection as $\Gamma^{\cdot}_{\cdot k}$ and $\Gamma^{\cdot}_{k\cdot}$ respectively and a bar and a hook below the indices denote the symmetry and skew-symmetry respectively between the indices.

In this paper we have considered the wave solutions of the unified field equations of Finzi in a space-time given by the metric [6]:

$$(1.6)\qquad ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 + 2\,A\,dz\,dt + 2\,B\,dx\,dy$$

where $A \equiv A(z, t)$ and $B \equiv B(x, y)$ and have also calculated the electromagnetic field, based on the definition of Graiff [7] who also does not presuppose the condition $\Gamma_i = 0$.

In the space time (1.6) the components of the non-symmetric tensor g_{ij} as calculated by Lal and Mustaqeem [8] are given by

$$(1.7)\qquad g_{ij} = \begin{bmatrix} -1 & B & \rho & -\rho \\ B & -1 & \sigma & -\sigma \\ -\rho & -\sigma & -1 & A \\ \rho & \sigma & A & 1 \end{bmatrix}$$

where ρ and σ are functions of $(z - t)$, and the contravariant components g^{ij} are given by

$$(1.8)\qquad g^{ij} = \frac{1}{m}\begin{bmatrix} -(1+A^2+2A\sigma^2) & -(1+A^2)B+2A\rho\sigma & (1+A)(B\sigma+\rho) & (1-A)(B\sigma+\rho) \\ -(1+A^2)B+2A\rho\sigma & -(1+A^2+2A\rho^2) & (1+A)(B\rho+\sigma) & (1-A)(B\rho+\sigma) \\ -(1+A)(B\sigma+\rho) & -(1+A)(B\rho+\sigma) & -(1-B^2)+P & (1-B^2)A+P \\ -(1-A)(B\sigma+\rho) & -(1-A)(B\rho+\sigma) & (1-B^2)A+P & (1-B^2)+P \end{bmatrix}$$

where $m = (1 + A^2)(1 - B^2)$ and $P = 2\,B\rho\sigma + \rho^2 + \sigma^2$.

Equation (1.1) of Finzi's field equations is one of the equations of Einstein's unified field theory which has already been solved by Lal and Mustaqeem [8]. To find the solutions of (1.2), (1.3) and (1.4), we first calculate the tensor R^*_{ij}.

2. The tensor R^{*}_{ij}

Using the values of Γ^{i}_{jk} from [8], the components of Γ_i are given by

$$\Gamma_1 = \Gamma_2 = 0 \qquad \text{and} \qquad -\Gamma_3 = \Gamma_4 = \beta(\mu + \nu)\,, \tag{2.1}$$

where

$$\beta = (\rho + B\sigma)\left(1 + \frac{2\,A\rho^2}{(1+A^2)}\right)\Big/\left\{\left(1 + \frac{2\,A\sigma^2}{(1+A^2)}\right)\cdot\right.$$

$$\left.\cdot\left(1 + \frac{2\,A\rho^2}{(1+A^2)}\right) - \left(-B + \frac{2\,A\rho\sigma}{(1+A^2)}\right)^2\right\},$$

$$\mu = -B_1/(1 - B^2) \qquad \text{and} \qquad \nu = -B_2/(1 - B^2)\,.$$

Putting $\Gamma_{\underset{+}{i;k}} = T_{ik}$, and using the values of Γ_i from (2.1) and Γ^{i}_{jk} from [8] the components of T_{ik} are given by

$$\begin{aligned}
T_{31} &= -(\beta\mu)_1 - (\beta\nu)_1 - 2\,AM\,\mu\beta(\mu + \nu)/(1 + A^2)\,,\\
T_{32} &= -(\beta\mu)_2 - (\beta\nu)_2 - 2\,AL\,\nu\beta(\mu + \nu)/(1 + A^2)\,,\\
T_{33} &= -(\mu + \nu)\,\beta_3 - \beta(\mu + \nu)(1 - A)\,\psi\,,\\
T_{34} &= -(\mu + \nu)\,\beta_4\,, \; T_{43} = (\mu + \nu)\,\beta_3\,,\\
T_{41} &= (\beta\mu)_1 + (\beta\nu)_1 + 2\,AM\,\mu\beta(\mu + \nu)/(1 + A^2)\,,\\
T_{42} &= (\beta\mu)_2 + (\beta\nu)_2 + 2\,AL\,\nu\beta(\mu + \nu)/(1 + A^2)\,,\\
T_{44} &= (\mu + \nu)\,\beta_4 - \beta(\mu + \nu)(1 + A)\,\varphi\,, \qquad \text{other} \quad T_{ik} = 0\,,
\end{aligned} \tag{2.2}$$

where the suffixes 1 and 2 denote partial differentiation with respect to x and y respectively and

$$L = \rho\alpha + \sigma\beta\;,\; M = \sigma\alpha + \rho\beta\;,\; \psi = A_3/(1 + A^2)\;,\; \varphi = A_4/(1 + A^2)$$

and

$$\alpha = -(\rho + B\sigma)\left(-B + \frac{2\,A\rho\sigma}{(1+A^2)}\right)\Big/\left\{\left(1 + \frac{2\,A\sigma^2}{(1+A^2)}\right)\cdot\right.$$

$$\left.\cdot\left(1 + \frac{2\,A\rho^2}{(1+A^2)}\right) - \left(-B + \frac{2\,A\rho\sigma}{(1+A^2)}\right)^2\right\}.$$

Substituting the values of R_{ij} from [8] and that of $\Gamma_{\underset{+}{i;k}}$ from equation (2.2) we find that the symmetric components of $R^*_{\underline{ij}}$ are given by

$$R^*_{\underline{11}} = \mu_2 + B\mu\nu + \frac{2A}{(1+A^2)}(M\mu)_1 - \frac{4A^2M^2\mu^2}{(1+A^2)^2} - \frac{2A\mu(BM\mu + L\nu)}{(1+A^2)},$$

$$R^*_{\underline{22}} = \nu_1 + B\mu\nu + \frac{2A}{(1+A^2)}(L\nu)_1 - \frac{4A^2L^2\nu^2}{(1+A^2)^2} - \frac{2A\nu(M\mu + BL\nu)}{(1+A^2)},$$

$$R^*_{\underline{33}} = \psi_4 + A\varphi\psi + I + H + 2T - (\mu+\nu)\{\beta_3 + \beta(1-A)\psi\},$$

$$R^*_{\underline{44}} = -\varphi_3 - A\varphi\psi + I - H + 2T + (\mu+\nu)\{\beta_4 - \beta(1+A)\varphi\},$$

$$R^*_{\underline{12}} = -(B\mu)_2 - \mu\nu + \frac{A}{(1+A^2)}\{(M\mu)_2 + L\nu)_1\} - \frac{4A^2LM\mu\nu}{(1+A^2)},$$

(2.3)
$$R^*_{\underline{13}} = -\frac{1}{2}\{(\beta\mu)_1 + (\beta\nu)_1\} - MN\mu - AM\mu\beta\frac{(\mu+\nu)}{(1+A^2)} - \left(\frac{M\mu}{1+A^2}\right)_3 - \left(\frac{(1-A)M\mu)}{(1+A^2)}\right)_4,$$

$$R^*_{\underline{14}} = \frac{1}{2}\{(\beta\mu)_1 + (\beta\nu_1)\} + MS\mu + Q + AM\mu\beta(\mu+\nu)/(1+A^2),$$

$$R^*_{\underline{23}} = -\frac{1}{2}\left\{(\beta\mu)_2 + (\beta\nu)_2 + W + \left(\frac{(1-A)L\nu)}{(1+A^2)}\right)_3 + \left(\frac{(1-A)L\nu}{(1+A^2)}\right)_4\right\} - LN\nu - \frac{AL\nu\beta(\mu+\nu)}{(1+A^2)},$$

$$R^*_{\underline{24}} = \frac{1}{2}\left\{(\beta\mu)_2 + (\beta\nu)_2 + W + \left(\frac{(1+A)L\nu}{(1+A^2)}\right)_3 + \left(\frac{(1+A)L\nu}{(1+A^2)}\right)_4\right\} + LS\nu + \frac{AL\nu\beta(\mu+\nu)}{(1+A^2)},$$

$$R^*_{\underline{34}} = -(A\psi)_4 + \frac{1}{2}(\mu+\nu)(\beta_3 - \beta_4) + \varphi\psi - I - AH,$$

while the skew-symmetric components of $R^*_{\check{ij}}$ are given by

$$R^*_{\check{12}} = \frac{A}{(1+A^2)}\{(M\mu)_2 - (L\nu)_1\},$$

(2.4)
$$R^*_{\check{13}} = \frac{1}{2}\{3(\beta\mu)_1 + (\beta\nu)_1\} + (\alpha\mu)_2 - (1-B)\alpha\mu\nu + \frac{AM\mu\beta(\mu+\nu)}{(1+A^2)} + \left(\frac{AM\mu}{(1+A^2)}\right)_3,$$

$$R^*_{\check{14}} = -\frac{1}{2}\{3(\beta\mu)_1 + (\beta\nu)_1\} - (\alpha\mu)_2 + (1-B)\alpha\mu\nu - \frac{AM\mu\beta(\mu+\nu)}{(1+A^2)},$$

$$\underset{\vee}{R}^*_{23} = \frac{1}{2}\left\{3(\beta\nu)_2 + (\beta\mu)_2 + W - \left(\frac{(1-A)\,L\nu}{(1+A^2)}\right)_3 - \left(\frac{(1-A)\,L\nu}{(1+A^2)}\right)_4\right\} +$$

$$(2.4) \qquad + (\alpha\nu)_1 - (1-B)\,\alpha\mu\nu + \frac{AL\nu\beta\,(\mu+\nu)}{(1+A^2)}\,,$$

$$\underset{\vee}{R}^*_{24} = \frac{1}{2}\left\{-2(\alpha\nu)_1 - 3(\beta\nu)_2 - (\beta\mu)_2 + 2(1-B)\,\alpha\mu\nu - W + \right.$$

$$\left. + \left(\frac{(1+A)\,L\nu}{(1+A^2)}\right)_3 + \left(\frac{(1+A)\,L\nu}{(1+A^2)}\right)_4 - \frac{2AL\nu\beta\,(\mu+\nu)}{(1+A^2)}\right\}\,,$$

$$\underset{\vee}{R}^*_{34} = -\frac{1}{2}(\mu+\nu)(\beta_3+\beta_4)\,,$$

where

$$N = \frac{A(1-A)\,\varphi + (1+A)\,\psi}{(1+A^2)}\,, \qquad S = \frac{A(1+A)\,\psi - (1-A)\,\varphi}{(1+A^2)}\,,$$

$$Q = \left(\frac{(1+A)\,M\mu}{(1+A^2)}\right)_3 + \left(\frac{(1+A)\,M\mu}{(1+A^2)}\right)_4, \quad W = \left(\frac{(1+A)\,L\nu}{(1+A^2)}\right)_3 + \left(\frac{(1-A)\,L\nu}{(1+A^2)}\right)_4\,,$$

$$I = \beta^2(\mu^2+\nu^2) + 2\alpha^2\mu\nu\,, \qquad H = \frac{4(M^2\mu^2 + 2BML\,\mu\nu + L^2\nu^2)}{(1-B^2)(1+A^2)}\,,$$

$$T = \frac{B\{M\mu^2 + (L+M)\,B\mu\nu + L\nu^2\}}{(1-B^2)} + \left(\frac{M\mu + BL\nu}{(1-B^2)}\right)_1 + \left(\frac{BM\mu + L\nu}{(1-B^2)}\right)_2.$$

3. Solutions of field equations (1.2), (1.3) and (1.4)

Substituting the values of $\underline{R}^*_{ij}$ from equation (2.3) in (1.2), we get

$$(3.1) \qquad L = M = 0\,, \qquad (\mu_1+\nu_1)\,\beta_2 = (\mu_2+\nu_2)\,\beta_1\,,$$

$$(3.2) \qquad A_{34} - AA_3A_4/(1+A^2) = 0\,, \qquad (1-A)\,A_3 + (1+A)\,A_4 = 0\,.$$

Using (3.1) in equation (2.4); the components of $\underset{\vee}{R}^*_{ij}$ are reduced to

$$\underset{\vee}{R}^*_{12} = 0\,,$$

$$\underset{\vee}{R}^*_{13} = -\underset{\vee}{R}^*_{14} = \frac{1}{2}\{3(\beta\mu)_1 + (\beta\nu)_1\} + \alpha_2\,\mu + I\,,$$

$$\underset{\vee}{R}^*_{23} = -\underset{\vee}{R}^*_{24} = \frac{1}{2}\{3(\beta\nu)_2 + (\beta\mu)_2\} + \alpha_1\,\nu + I\,,$$

$$\underset{\vee}{R}^*_{34} = -\frac{1}{2}(\mu+\nu)(\beta_3+\beta_4)$$

where $I = \alpha\nu_1 - (1 - B)\,\alpha\mu\nu$, which when substituted in equation (1.3) gives

$$
\begin{aligned}
&\beta\,(\mu_{12} - \nu_{12}) + \beta_{12}\,(\mu - \nu) + 2\,\nu_1\,(\beta_1 - \beta_2) + \alpha_2\,\mu_2 + \\
(3.3)\qquad &\qquad + \alpha_{22}\,\mu - \alpha_1\,\nu_1 - \alpha_{11}\,\nu - I_1 + I_2 = 0\,, \\
&(\beta_3 + \beta_4)\,(\mu_1 - \nu_2) + \mu\,(\beta_{13} + \beta_{14} + \alpha_{23} + \alpha_{24}) - \\
&\qquad - \nu\,(\beta_{23} + \beta_{24} + \alpha_{13} + \alpha_{14}) = 0\,.
\end{aligned}
$$

Using (1.8) and (2.4) under the condition (3.1), the skew-symmetric contravariant components $\underset{\vee}{R}^{*ij}$ of the tensor $\underset{\vee}{R}^{*}_{ij}$ are given by

$$
\begin{aligned}
&\underset{\vee}{R}^{*12} = -\,2A\,(B\rho + \sigma)\,[(1 + A^2 + 2A\sigma^2)\,\underset{\vee}{R}^{*}_{13} + \\
&\qquad + \{(1 + A^2)\,B - 2A\rho\sigma\}\,\underset{\vee}{R}^{*}_{23}]/m^2 + \\
&\qquad + 2A\,(B\sigma + \rho)\,[\{(1 + A^2)\,B - 2A\rho\sigma\}\,\underset{\vee}{R}^{*}_{13} + \\
(3.4)\qquad &\qquad + (1 + A^2 + 2A\rho^2)\,\underset{\vee}{R}^{*}_{23}]/m^2, \\
&\underset{\vee}{R}^{*13} = (1 + A)\,\xi + \varphi',\quad \underset{\vee}{R}^{*14} = (1 - A)\,\xi + \varphi', \\
&\underset{\vee}{R}^{*23} = (1 + A)\,\eta + \psi',\quad \underset{\vee}{R}^{*24} = (1 - A)\,\eta + \psi', \\
&\underset{\vee}{R}^{*34} = -\,\varphi'\,(1 - B^2)/(B\sigma + \rho)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\xi &= [(1 + A^2 + 2A\sigma^2)\,(1 - B^2)\,\underset{\vee}{R}^{*}_{13} + \{(1 + A^2)\,B - 2A\rho\sigma\}\,(1 - B^2)\,\underset{\vee}{R}^{*}_{23} + \\
&\qquad + 2A\,(B\sigma + \rho)^2\,(\underset{\vee}{R}^{*}_{13} + \underset{\vee}{R}^{*}_{23})]/m^2, \\
\eta &= [\{(1 + A^2)\,B - 2A\rho\sigma\}\,(1 - B^2)\,\underset{\vee}{R}^{*}_{13} + (1 + A^2 + 2A\rho^2)\,(1 - B^2)\,\underset{\vee}{R}^{*}_{23} + \\
&\qquad + 2A\,(B\rho + \sigma)^2\,(\underset{\vee}{R}^{*}_{13} + \underset{\vee}{R}^{*}_{23})]/m^2, \\
\varphi' &= (B\sigma + \rho)\,(m + 2AP)\,\underset{\vee}{R}^{*}_{34}/m^2 \quad , \quad \psi' = (B\rho + \sigma)\,(m + 2AP)\,\underset{\vee}{R}^{*}_{34}/m^2.
\end{aligned}
$$

Substituting the values of $\underset{\vee}{R}^{*ij}$ from (3.4) in equation (1.4), and solving them we see that it is satisfied if the relation

$$
(3.5)\qquad \xi = \eta = 0\,,
$$

holds.

Thus the solutions of the field equations of Finzi in the space-time (1.6) are given by (1.7) under the conditions (3.1), (3.2), (3.3) and (3.5).

4. The electromagnetic field

Ikeda [9] in 1954 found a skew-symmetric tensor H_{ij} in terms of a non-symmetric fundamental tensor g_{ij} satisfying the properties (i) that the total rotation of H_{ij} is zero and (ii) that H_{ij} has a non-zero divergence, and called such a tensor as electromagnetic field tensor. In order that the skew-symmetric tensor H_{ij} may satisfy property (i) he assumed $\Gamma^{j}_{\underset{\vee}{ij}} = 0$ and defined the electromagnetic field tensor H_{ij} by the relation

$$H_{ij} = \frac{1}{2} \epsilon_{ijkl} \sqrt{-|g_{ij}|}\, g^{kl}, \tag{4.1}$$

where ϵ_{ijkl} takes the values $+1$ or -1 according as $ijkl$ is even or odd permutation of 1234. Thus (4.1) is valid only in the case of the field equations of Einstein, Bonnor, Schrödinger and Buchdahl. But in case of the field equations of Finzi in which the equation $\Gamma^{j}_{\underset{\vee}{ij}} = 0$ is not necessarily satisfied, H_{ij} given by (4.1) does not necessarily satisfy property (i) and therefore it cannot represent the electromagnetic field tensor in the sense of Ikeda.

In 1955 Graiff [7] gave two possible relations between the non-symmetric tensor g_{ij} and a skew-symmetric tensor F_{ij} each satisfying both properties (i) and (ii) without imposing the condition $\Gamma_{i} = 0$. She gave two forms of the electromagnetic field tensor F_{ij} by the relations

$$F_{ij} = R^{*}_{\underset{\vee}{ij}} - \Gamma_{\underset{\vee}{i,j}}, \tag{4.2}$$

$$F_{ij} = \frac{1}{2} \epsilon_{ijkl} R^{*}_{\underset{\vee}{pq}} g^{kp} g^{lq} - \Gamma_{\underset{\vee}{i,j}}. \tag{4.3}$$

Calculating the values of $\Gamma_{\underset{\vee}{i,j}}$ from equation (2.1), we get

$$\begin{aligned} &\Gamma_{\underset{\vee}{1,2}} = 0 \quad , \quad \Gamma_{\underset{\vee}{3,4}} = -\frac{1}{2}(\mu+\nu)(\beta_3+\beta_4), \\ &\Gamma_{\underset{\vee}{1,3}} = -\Gamma_{\underset{\vee}{1,4}} = \frac{1}{2}\{\beta(\mu+\nu)\}_1, \\ &\Gamma_{\underset{\vee}{2,3}} = -\Gamma_{\underset{\vee}{2,4}} = \frac{1}{2}\{\beta(\mu+\nu)\}_2. \end{aligned} \tag{4.4}$$

Assuming the form (4.2) of the electromagnetic field tensor F_{ij} and substituting in it the values of $R^{*}_{\underset{\vee}{ij}}$ and $\Gamma_{\underset{\vee}{i,j}}$ from equations (2.4) and (4.4), we get

$$F_{12} = 0 \quad , \quad F_{34} = 0 \, ,$$

(4.5)
$$F_{13} = - F_{14} = \frac{1}{2} \{3\,(\beta\mu)_1 + (\beta\nu)_1\} + \alpha_2\,\mu + 1 - \frac{1}{2}\{\beta\,(\mu + \nu)\}_1 \, ,$$

$$F_{23} = - F_{24} = \frac{1}{2} \{3\,(\beta\nu)_2 + (\beta\mu)_2\} + \alpha_1\,\nu + 1 - \frac{1}{2}\{\beta\,(\mu + \nu)\}_2 \, .$$

Assuming the form (4.3) for the electromagnetic field tensor, and inserting the relevant quantities in (4.3), we get

$$F_{12} = -(1 - B^2)\,\varphi'/(B\sigma + \rho) \, ,$$

$$F_{13} = (1 - A)\,\delta - \psi' - \frac{1}{2}\{\beta\,(\mu + \nu)\}_1 \, ,$$

$$F_{14} = -(1 + A)\,\delta + \psi' + \frac{1}{2}\{\beta\,(\mu + \nu)\}_1 \, ,$$

(4.6)
$$F_{23} = (1 - A)\,\delta' + \varphi' - \frac{1}{2}\{\beta\,(\mu + \nu)\}_2 \, ,$$

$$F_{24} = -(1 + A)\,\delta' - \varphi' + \frac{1}{2}\{\beta\,(\mu + \nu)\}_2 \, ,$$

$$\begin{aligned} F_{34} = & - 2A\,[(1 + A^2 + 2A\sigma^2)\,(B\rho + \sigma) + \\ & + \{-(1 + A^2)\,B + 2A\rho\sigma\}\,(B\sigma + \rho)]\,\underset{\vee}{R}{}^{*}_{13}/m^2 + \\ & + 2A\,[(1 + A^2 + 2A\rho^2)\,(B\sigma + \rho) + \\ & + \{-(1 + A^2)\,B + 2A\rho\sigma\}\,(B\rho + \sigma)]\,\underset{\vee}{R}{}^{*}_{23}/m^2 + \\ & - \frac{1}{2}(\mu + \nu)\,(\beta_3 + \beta_4) \, , \end{aligned}$$

where

$$\begin{aligned} \delta = & -[\{(1 + A^2)\,B - 2A\rho\sigma\}\,(1 - B^2) + 2A\,(B\rho + \sigma)\,(B\sigma + \rho)]\,\underset{\vee}{R}{}^{*}_{13}/m^2 - \\ & - [2A\,(B\rho + \sigma)^2 + (1 + A^2 + 2A\rho^2)\,(1 - B^2)\,\underset{\vee}{R}{}^{*}_{23}/m^2 , \\ \delta' = & [(1 + A^2 + 2A\sigma^2)\,(1 - B^2) + 2A\,(B\sigma + \rho)^2]\,\underset{\vee}{R}{}^{*}_{13}/m^2 + \\ & + [\{(1 + A^2)\,B - 2A\rho\sigma\}\,(1 - B^2) + 2A\,(B\rho + \sigma)\,(B\sigma + \rho)]\,\underset{\vee}{R}{}^{*}_{23}/m^2 . \end{aligned}$$

Hence the unified field theory of Finzi in the space-time (1.6) gives two different electromagnetic fields which are given by (4.5) and (4.6).

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