Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

Edgar R. Lorch, Hing Tong

On Automorphisms of the Group of Baire Equivalences of a Complete Separable Metric Space

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **60** (1976), n.6, p. 749–750.

Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA_1976_8_60_6_749_0>

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Accademia Nazionale dei Lincei, 1976.

Teoria dei gruppi. — *On Automorphisms of the Group of Baire Equivalences of a Complete Separable Metric Space.* Nota di EDGAR R. LORCH e HING TONG, presentata (*) dal Corrisp. G. FICHERA.

RIASSUNTO. — Viene annunciato il seguente Teorema: Sia E uno spazio completo separabile metrico di cardinalità c. Sia G il gruppo degli automorfismi di Baire di E. (Cioè dire che $g \in G$ equivale a dire che $g : E \to E$ è una biiezione e che per ogni insieme di Baire M, sia $g(M)$ che $g^{-1}(M)$ sono insiemi di Baire). Sia Φ un automorfismo di G. Allora Φ è interno cioè esiste $x \in G$ tale che per ogni $g \in G$, $\Phi(g) = x^{-1}gx$.

In this Note, we announce the following:

THEOREM. *Let* E *be a complete separable metric space of cardinality* c. *Let* G *be a the group of Baire automorphisms of* E. (*Thus to say that* $g \in G$, *is equivalent to saying that* $g : E \to E$ *is a bijection such that for any Baire set* M, *both* $g(M)$ *and* $g^{-1}(M)$ *are Baire sets*). *Let* Φ *be an automorphism of* G. *Then* Φ *is inner.*

The proof is given in three steps.

PROPOSITION 1. *Let* E *be a set of any cardinality* $n \neq 6$. *Let* G *be any group of permutations of the elements of* E *which contains all transpositions. Let* Φ *be any automorphism of* G. *Then there exists a bijection* $x : E \to E$ *such that for any* $g \in G$, $\Phi(g) = x^{-1}gx$.

This theorem in disguised form can be found in [1]. A proof of the theorem has also been given by the present authors [2].

Proposition 1 is applicable to the proof of the theorem since the group of all Baire automorphisms contains all transpositions. Thus we may assume as given the bijection $x : E \to E$, such that for all $g \in G$, $\Phi(g) = x^{-1}gx$. Let (E, τ_0) be the set E with the given complete separable metric topology τ_0. Let τ_1 be the topology which makes $x : (E, \tau_0) \to (E, \tau_1)$ a homeomorphism. The τ_1-Baire sets are the sets $x(M)$ where M is a τ_0-Baire set; the group of τ_1-Baire automorphisms is the group $x^{-1}Gx$. Thus the hypothesis of the theorem gives that $\Phi(G) = x^{-1}Gx = G$. To prove that $x \in G$ it is necessary to show that if M is a τ_0-Baire set then $x(M)$ and $x^{-1}(M)$ are also τ_0-Baire sets. Hence it must be shown that the family $\mathscr{B}_0$ of τ_0-Baire sets coincides with the family $\mathscr{B}_1$ of τ_1-Baire sets. This is done in two steps.

PROPOSITION 2. *There exists a set* M *of cardinality* c *and cocardinality* c *such that* $M \in \mathscr{B}_0 \cap \mathscr{B}_1$.

(*) Nella seduta del 10 giugno 1976.

The proof is delicate and requires a series of constructions. Note that for any set M of denumerable or codenumerable cardinality, $M \in \mathscr{B}_0 \cap \mathscr{B}_1$.

PROPOSITION 3. $\mathscr{B}_0 = \mathscr{B}_1$.

The proof here uses Proposition 2 plus the classic fact that any two Baire sets of the same cardinality in complete separable metric spaces are Baire equivalent [3].

The theorem presented here remains valid if E is any Baire set (in a complete separable metric space) of cardinality c.

BIBLIOGRAPHY

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