ATTI ACCADEMIA NAZIONALE DEI LINCEI

CLASSE SCIENZE FISICHE MATEMATICHE NATURALI

RENDICONTI

PATRICK W. DARKO

On cohomology with bounds on complex spaces

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **60** (1976), n.3, p. 189–194.

Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA_1976_8_60_3_189_0>

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Accademia Nazionale dei Lincei, 1976.

RENDICONTI

DELLE SEDUTE

DELLA ACCADEMIA NAZIONALE DEI LINCEI

Classe di Scienze fisiche, matematiche e naturali

Seduta del 13 marzo 1976

Presiede il Presidente della Classe BENIAMINO SEGRE

SEZIONE I

(Matematica, meccanica, astronomia, geodesia e geofisica)

Matematica. — *On cohomology with bounds on complex spaces*

Nota di PATRICK W. DARKO, presentata (*) dal Corrisp. E. MARTINELLI.

RIASSUNTO. — In [3; Teorema 2] R. Narasimhan ha stabilito un teorema di isomorfismo di tipo Leray, in presenza di limitazioni date da una crescita polinomiale. In questa Nota si stabilisce un teorema che generalizza quello accennato, basando su uno schizzo di dimostrazione dato da Narasimhan nel caso da lui considerato.

1. HOLOMORPHIC SECTIONS WITH GROWTH

Let Ω be an open subset of some $\mathbf{C}^n$ and φ a plurisubharmonic function on Ω with the property that there are constants $K_1 > 0$, $K_2 > 0$, $K_3 \geq 1$, $K_4 > 0$ such that $z \in \Omega$ and $|z - \xi| \leq \exp(-K_1 \varphi(z) - K_2)$ imply $\xi \in \Omega$ and $\exp \varphi(\xi) \leq K_3 \varphi(z) + K_4$. The pair (Ω, φ) is then said to satisfy the condition (H). For a bounded Ω in $\mathbf{C}^n$ this condition is almost equivalent to that given in [2], and Ω is then a domain of holomorphy.

Suppose Ω is bounded and the pair (Ω, φ) satisfies the condition (H), $\mathcal{O}$ is the structure sheaf of $\mathbf{C}^n$ and $p > 0$ an integer. A section $f = (f_1, \cdots, f_p) \in \Gamma(\Omega, \mathcal{O}^p)$ is said to be of φ-growth if there are constants $c > 0$, , $\rho \geq 0$ such that

$$|f(z)| = |f_1(z)| + \cdots + |f_p|(z) \leq c \exp(\rho\varphi(z))$$

for all $z \in \Omega$. We then denote all sections of $\mathcal{O}^p$ over Ω of φ-growth by $\Gamma_\varphi(\Omega, \mathcal{O}^p)$.

For the definition of φ-growth for coherent analytic sheaves we require the closure of Ω to have a fundamental system of neighbourhoods each of

(*) Nella seduta del 13 marzo 1976.

which is a domain of holomorphy. If F is a coherent analytic sheaf on a neighbourhood of $\bar{\Omega}$, the by Cartan's Theorem A there is an exact sequence

$$\mathcal{O}^p \xrightarrow{\lambda} \mathrm{F} \to 0$$

of $\mathcal{O}$-homomorphisms in a neighbourhood of $\bar{\Omega}$, where $p > 0$ is some integer, and $f \in \Gamma(\Omega, \mathrm{F})$ is of φ-growth if $f \in \Gamma_\varphi(\Omega, \mathrm{F}) = \lambda\Gamma_\varphi(\Omega, \mathcal{O}^p))$. It is shown in [1] that φ-growth for coherent analytic sheaves as given here is well defined.

Let $(\mathrm{X}, {}_{\mathrm{X}}\mathcal{O})$ be a complex space and Y a relatively compact open subset of X. Y is said to be φ-admissible in X, if there exist a bounded open set Ω in some $\mathbf{C}^n$ and a function φ on Ω so that (i) the pair (Ω, φ) satisfies the condition (H), (ii) $\bar{\Omega}$ has a fundamental system of neighbourhoods which are domains of holomorphy and (iii) there is a neighbourhood Y′ of $\bar{\mathrm{Y}}$ in X and a closed imbedding η of Y′ in a neighbourhood Ω' of $\bar{\Omega}$ such that $\eta^{-1}(\Omega) = \mathrm{Y}$. If F is a coherent analytic sheaf on X and $\eta_*(\mathrm{F})$ is the 0^{th} direct image on Ω' we define $\Gamma_\varphi(\mathrm{Y}, \mathrm{F})$, sections of F on Y of φ-growth by

$$\Gamma_\varphi(\mathrm{Y}, \mathrm{F}) = \{f \in \Gamma(\mathrm{Y}, \mathrm{F}) : \eta_*(f) \in \Gamma_\varphi(\Omega, \eta_*(\mathrm{F}))\}.$$

2. Cohomology with Bounds

In the preceeding, φ was a single function on some open set Ω. In this section it is a collection of functions $\varphi = \{\varphi_i\}_{i \in \mathrm{I}}$, where I is some index set, there is an open bounded subset Ω_i of some $\mathbf{C}^{n_i}$ such that the pair (Ω_i, φ_i) satisfies the condition (H) and $\varphi_i \geq 0$ on Ω_i. By a φ-admissible open covering $\{\mathrm{U}_i\}_{i \in \mathrm{I}}$ of a complex space $(\mathrm{X}, {}_{\mathrm{X}}\mathcal{O})$ we mean an open covering of X such that for each i, U_i is φ-admissible in X.

Note that if $\{\mathrm{U}_i\}_{i \in \mathrm{I}}$ is a φ-admissible open covering of X and $\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2} \cap \cdots \cap \mathrm{U}_{i_p}$ is a finite intersection of open sets of the covering, then there are p functions $\varphi_{i_1, \ldots, i_p}$ such that $\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2} \cap \cdots \cap \mathrm{U}_{i_p}$ is $\varphi_{i_1 \cdots i_p}$-admissible in X. Let η_i be the imbedding of a neighbourhood of $\bar{\mathrm{U}}_i$ into a neighbourhood of the closure of $\eta_i(\mathrm{U}_i)$ in some $\mathbf{C}^{n_i}$. Then, for instance, in the case of the intersection of two open sets $\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2}$, there is a holomorphic map $h_{1,2}$ of a neighbourhood of $\overline{\eta_{i_1}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})}$ into a neighbourhood of $\overline{\eta_{i_2}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})}$ such that $\eta_{i_2} = h_{1,2} \circ \eta_{i_1}$, and a holomorphic map $h_{2,1}$ of a neighbourhood of $\overline{\eta_{i_2}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})}$ into a neighbourhood of $\overline{\eta_{i_1}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})}$ such that $\eta_{i_1} = h_{2,1} \circ \eta_{i_2}$. On $\eta_{i_1}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})$ let φ_{i_1, i_2} be defined by $\varphi_{i_1, i_2}(z) = \max(\varphi_{i_1}(z), \varphi_{i_2}(h_{1,2}(z)))$ and on $\eta_{i_2}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2})$ let $\varphi_{i_2, i_1}(z) = \max(\varphi_{i_1}(h_{2,1}(z)), \varphi_{i_2}(z))$. Then, the pairs $(\eta_i(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2}), \varphi_{i_1, i_2})$, $(\eta_{i_2}(\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2}), \varphi_{i_2, i_1})$ satisfy the condition (H), and $\mathrm{U}_{i_1} \cap \mathrm{U}_{i_2}$ is both φ_{i_1, i_2}-admissible and φ_{i_2, i_1}-admissible in X.

If $\varphi_{i_1 \cdots i_p}$, $\varphi'_{i_1 \cdots i_p}$ are two of the p functions derived as above such that $\mathrm{U}_\alpha = \mathrm{U}_{i_1} \cap \cdots \cap \mathrm{U}_{i_p}$, $\alpha = (i_1, \cdots, i_p)$ is both $\varphi_{i_1, \ldots, i_p}$-admissible and $\varphi'_{i_1, \ldots, i_p}$-

admissible in X, then it is easy to see that

$$\Gamma_{\varphi_{i_1,\cdots,i_p}}(\mathrm{U}_\alpha\, \mathrm{F}) = \Gamma_{\varphi'_{i_1,\cdots,i_p}}(\mathrm{U}_\alpha\, \mathrm{F}) \qquad \text{for any}$$

coherent analytic sheaf F on X. We shall therefore write $\Gamma_\varphi(\mathrm{U}_\alpha, \mathrm{F})$ for each one of the $\Gamma_{\varphi_{i_1,\cdots,i_p}}(\mathrm{U}_\alpha, \mathrm{F})$.

Now let $(\mathrm{X}, {}_{\mathrm{X}}\mathcal{O})$ be a (paracompact) complex space and F a coherent analytic sheaf on X. Let $\mathcal{U} = \{\mathrm{U}_i\}_{i\in\mathrm{I}}$ be a locally finite open φ-admissible covering of X for some $\varphi = \{\varphi_i\}_{i\in\mathrm{I}}$. As in [3] we define the bounded (alternate) q-cochains of $\mathcal{U}$ with values in F as those Cochains

$$c = (c_\alpha) \in \mathrm{C}^q(\mathcal{U}, \mathrm{F}) = \prod_{\alpha\in\mathrm{I}^{q+1}} \Gamma(\mathrm{U}_\alpha, \mathrm{F}),$$

$\mathrm{U}_\alpha = \mathrm{U}_{i_0} \cap \cdots \cap \mathrm{U}_{i_q}$, $\alpha = (i_0, \cdots, i_q)$, which are alternate and satisfy

$$c_\alpha \in \Gamma_\varphi(\mathrm{U}_\alpha, \mathrm{F}) \qquad \text{for all} \quad \alpha \in \mathrm{I}^{q+1}.$$

We denote by $\mathrm{C}^q_\varphi(\mathcal{U}, \mathrm{F})$ the space of bounded cochains. The coboundary operator

$$\delta : \mathrm{C}^q(\mathcal{U}, \mathrm{F}) \to \mathrm{C}^{q+1}(\mathcal{U}, \mathrm{F})$$

maps $\mathrm{C}^q_\varphi(\mathcal{U}, \mathrm{F})$ into $\mathrm{C}^{q+1}_\varphi(\mathcal{U}, \mathrm{F})$. If $\mathrm{Z}^q_\varphi(\mathcal{U}, \mathrm{F}) = \{c \in \mathrm{C}^q_\varphi(\mathcal{U}, \mathrm{F}) : \delta c = 0\}$ $\mathrm{B}^q_\varphi = \delta \mathrm{C}^{q-1}_\varphi(\mathcal{U}, \mathrm{F})$, then as usual $\mathrm{B}^q_\varphi(\mathcal{U}, \mathrm{F}) \subseteq \mathrm{Z}^q_\varphi(\mathcal{U}, \mathrm{F})$, and we define $\mathrm{H}^q_\varphi(\mathcal{U}, \mathrm{F}) = \mathrm{Z}^q_\varphi(\mathcal{U}, \mathrm{F})/\mathrm{B}^q_\varphi(\mathcal{U}, \mathrm{F})$, and call it the bounded cohomology of $\mathcal{U}$ with values in F. We then have the following

THEOREM. *Let* F *be a coherent analytic sheaf on a complex Space* X *and let* $\mathcal{U}$ *be a locally-finite open* φ*-admissible covering of* X. *Then, for any* $q \geq 0$, *the natural map*

$$\mathrm{H}^q_\varphi(\mathcal{U}, \mathrm{F}) \to \mathrm{H}^q(\mathrm{X}, \mathrm{F})$$

is an isomorphism.

3. PROOF OF THE THEOREM

Since, that $\mathrm{H}^0_\varphi(\mathcal{U}, \mathrm{F}) \to \mathrm{H}^0(\mathrm{X}, \mathrm{F})$ is an isomorphism is clear, we prove the theorem for the case $q \geq 1$.

We shall need the following

LEMMA [1; Theorem 2.1]. *Let* $(\mathrm{X}, {}_{\mathrm{X}}\mathcal{O})$ *be a complex space and* Y *a* φ*-admissible subset (where* φ *is a single function), Let* G_1 *and* G_2 *be two coherent analytic sheaves on* X *and* $\lambda : \mathrm{G}_1 \to \mathrm{G}_2$ *a surjective* ${}_{\mathrm{X}}\mathcal{O}$*-homomorphism. Then the induced map*

$$\Gamma_\varphi(\mathrm{Y}, \mathrm{G}_1) \to \Gamma_\varphi(\mathrm{Y}, \mathrm{G}_2) \qquad \textit{is surjective.}$$

Let $\sigma = (U_{\alpha_0}, U_{\alpha_1}, \cdots, U_{\alpha_k})$ be in the nerve of $\mathcal{U}$ and $U_{i_0} \in \mathcal{U}$, $U_{i_1} \in \mathcal{U}, \cdots, U_{iq} \in \mathcal{U}$ with $|\sigma| \cap U_{i_0} \cap U_{i_1} \cap \cdots U_{i_q} \neq \varphi$. There is a holomo-rphic map h_{α_ν} of a neighbourhood of $\eta_{\alpha_0}(U_{\alpha_0} \cap U_{\alpha_\nu})$ into a neighbourhood of $\eta_{\alpha_\nu}(U_{\alpha_0} \cap U_{\alpha_\nu})$ such that $\eta_{\alpha_\nu} = h_{\alpha_\nu} \circ \eta_{\alpha_0}$ $1 \leq \nu \leq k$, and a holomorphich map h_{i_λ} of a neighbourhood of $\eta_{\alpha_0}(U_{\alpha_0} \cap U_{i_\lambda})$ into a neighbourhood of $\eta_{i_\lambda}(U_{\alpha_0} \cap U_{i_\lambda})$ such that $\eta_{i_\lambda} = h_{i_\lambda} \circ \eta_{\alpha_0}$, $0 \leq \lambda \leq$. Let

$$\varphi_{\alpha_1,\alpha_2,\cdots,\alpha_k,i_0\cdots i_q} = \max(\varphi_{\alpha_0}, \varphi_{\alpha_1} \circ h_{\alpha_1}, \cdots, \varphi_{\alpha_k} \circ h_{\alpha_k}, \varphi_{i_0} \circ h_{i_0}, \cdots, \varphi_{i_q} \circ h_{i_q}).$$

On $\eta_{\alpha_0}(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q})$. Then the pair

$$(\eta_{\alpha_0}(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q}), \varphi_{\alpha_1,\cdots,\alpha_k,i_0,\cdots,i_q})$$

satisfies the condition (H). If $\eta_{\alpha_0 *}(F)$ is the 0^{th} direct image we write

$$\Gamma_\varphi(\eta_{\alpha_0}(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q}), \eta_{\alpha_0 *}(F))$$

for

$$\Gamma_{\varphi_{\alpha_1,\cdots,\alpha_k,i_0,\cdots,i_q}}(\eta_{\alpha_0}(|\sigma| \cap U_{i_0} \cap U_{i_1} \cap \cdots \cap U_{i_q}), \eta_{\alpha_0 *}(F)$$

and set

$$\Gamma_\varphi(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q}, F) = \{f \in \Gamma(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q}, F) : \eta_{\alpha_0 *}(f) \in$$
$$\in \Gamma_\varphi(\eta_{\alpha_0}(|\sigma| \cap U_{i_0} \cap \cdots \cap U_{i_q}), \eta_{\alpha_0 *}(F))\}.$$

Let

$$\mathcal{U}/|\sigma| = \{|\sigma| \cap U : U \in \mathcal{U}\}, C^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|) =$$
$$= \{c = (c_\beta) \in C^q(\mathcal{U}/|\sigma|, F/|\sigma|) : c_\beta \in \Gamma_\varphi(|\sigma| \cap U_{\beta_0} \cap \cdots \cap U_{\beta_q}, F)$$

for each

$$\beta = (\beta_0, \beta_1, \cdots, \beta_q)\}, B^q_\beta(\mathcal{U}/|\sigma|, F/|\sigma|) =$$
$$= \delta C^{q-1}_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|), Z^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|) =$$
$$= \{c \in C^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|) : \delta c = 0\}$$

and

$$H^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|) = Z^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|)/B^q_\varphi(\mathcal{U}/|\sigma|, F/|\sigma|)$$

for $q \geq 1$.

Now, because $\mathcal{U}$ is a locally finite covering and the closure of $|\sigma|$ is compact. $\mathcal{U}/|\sigma|$ is a finite covering of $|\sigma|$; $\mathcal{U}/|\sigma| = \{|\sigma| \cap U_1, \cdots, |\sigma| \cap U_l\}$, say. Let

$$\eta_{\alpha_0}(|\sigma|) = V, \eta_{\alpha_0 *}(F) = F', U'_i = \eta_{\alpha_0}(|\sigma| \cap U_i), \qquad 1 \leq i \leq l$$

and

$$\mathcal{U}' = \{U_1', \cdots, U_i'\}.$$

Set

$$C_\varphi^q(\mathcal{U}', F'/V) = \{c = (c_\beta)\, C^q(\mathcal{U}', F'/V) : c_\beta \in \Gamma_\varphi(U_{\beta_0}' \cap \cdots \cap U_{\beta_q}', F')$$

for each

$$\beta = (\beta_0, \cdots, \beta_q)\}, \; B_\varphi^q(\mathcal{U}', F'/V) = \delta C_\varphi^{q-1}(\mathcal{U}' F'/V), \; Z_\varphi^q(\mathcal{U}' F'/V) =$$
$$= \{c \in C_\varphi^q(\mathcal{U}', F'/V) : \delta c = 0\}$$

and

$$H^q(\mathcal{U}', F'/V) = Z_\varphi^q(\mathcal{U}', F'/V)/B_\varphi^q(U', F'/V) \text{ for } q \geq 1.$$

As in Leray's Theorem to prove the above theorem it is enough to show that $H_\varphi^q(\mathcal{U}/|\sigma|, F/|\sigma|) = 0$ for all nerves σ of $\mathcal{U}$ when $q \geq 1$. And because of the imbedding, to prove that $H_\varphi^q(\mathcal{U}/|\sigma|, F/|\sigma|) = 0$ for a particular $\sigma = (U_{\alpha_0}, \cdots, U_{\alpha_k})$ it is enough to prove that $H_\varphi^q(\mathcal{U}', F'/V) = 0$.

Since V is a domain of holomorphy and its closure has a fundamental system of neighbourhoods each of which is a domain of holomorphy, there is a terminating chain of syzygies

$$(1) \qquad 0 \to \mathcal{O}^{P_r} \xrightarrow{\mu_r} \mathcal{O}^{P_{r-1}} \xrightarrow{\mu_{r-1}} \cdots \mathcal{O}^{P_0} \xrightarrow{\mu_0} F' \to 0$$

in a neighbourhood of the closure of V, where $\mathcal{O}$ is the structure sheaf of $\mathbf{C}^{n_{\alpha_0}}$ and r is a natural number:

To prove that $H_\varphi^q(\mathcal{U}', F'/V) = 0$ we use induction on the length r of the terminating chain of syzygies. When $r = 0$ the exact sequence (1) reduces to

$$(2) \qquad 0 \to \mathcal{O}^{P_0} \xrightarrow{\mu_0} F' \to 0;$$

thus, in this case we need only show that $H_\varphi^q(\mathcal{U}', \mathcal{O}^p/V) = 0$ for $q \geq 1$, $p \geq 1$. This is done by induction on p. When $p = 1$, that $H_\varphi^q(\mathcal{U}', \mathcal{O}/V) = 0$ is simple to demonstrate because of the finiteness of the covering of V. When $p > 1$ from the exact sequence of sheaves

$$0 \to \mathcal{O} \to \mathcal{O}^p \to \mathcal{O}^{p-1} \to 0$$

we get for each $\beta = (\beta_0, \cdots, \beta_q)$ an exact sequence

$$(3) \qquad 0 \to \Gamma_\varphi(U_\beta', \mathcal{O}) \to \Gamma_\varphi(U_\beta', \mathcal{O}^p) \to \Gamma_\varphi(U_\beta', \mathcal{O}^{p-1}) \to 0$$

having used the lemma in the exactness of (3) and where

$$U_\beta' = U_{\beta_0}' \cap \cdots \cap U_{\beta_q}.$$

From (3) we get the exact sequence

$$0 \to C^q_\varphi(\mathcal{U}', \mathcal{O}/V) \to C^q_\varphi(\mathcal{U}', \mathcal{O}^p/V) \to C^q_\varphi(\mathcal{U}, \mathcal{O}^{p-1}/V) \to 0. \tag{4}$$

And from (4) we get a long exact sequence of bounded cohomology groups

$$\cdots H^q_\varphi(\mathcal{U}', \mathcal{O}/V) \to H^q_\varphi(\mathcal{U}', \mathcal{O}^p/V) \to H^q_\varphi(\mathcal{U}', \mathcal{O}^{p-1}/V) \to \to H^{q+1}_\varphi(\mathcal{U}', \mathcal{O}/V) \to \cdots \tag{5}$$

$H^q_\varphi(\mathcal{U}', \mathcal{O}/V) = H^{q+1}_\varphi(\mathcal{U}', \mathcal{O}/V) = 0$, hence

$$H^q_\varphi(\mathcal{U}', \mathcal{O}^p/V) \cong H^q_\varphi(\mathcal{U}', \mathcal{O}^{p-1}/V);$$

thus by induction

$$H^q_\varphi(\mathcal{U}', \mathcal{O}^p/V) = 0 \qquad \text{for} \quad q \geq 1, p \geq 1.$$

To conclude the proof of $H^q_\varphi(\mathcal{U}', F/V) = 0$ for all $q \geq 1$ assume that $H^q_\varphi(\mathcal{U}', G) = 0$ for all $q \geq 1$ when G is a coherent analytic sheaf on V which has a terminating chain of syzygies of lenght $\leq r - 1$. The exact sequence (1) can be reduced to the two shorter exact sequences

$$0 \to \mathcal{O}^{p-1} \xrightarrow{\mu_r} \mathcal{O}^{p_{r-1}} \xrightarrow{\mu_{r-1}} \cdots \to \mathcal{O}^{p_1} \xrightarrow{\mu_1} R \to 0, \tag{6}$$

$$0 \to R \to \mathcal{O}^{p_0} \xrightarrow{\mu_0} F' \to 0,$$

where R is the kernel of μ_0. By the induction hypothesis $H^q_\varphi(\mathcal{U}', R/V) = 0$ for all $q \geq 1$. From the short exact sequence in (6), using the lemma again, we get a long sequence of bounded cohomology groups

$$\cdots \to H^q_\varphi(\mathcal{U}', \mathcal{O}^{p_0}/V) \to H^q_\varphi(\mathcal{U}', F'/V) \to H^{q+1}_\varphi(\mathcal{U}', R/V) \to \cdots \tag{7}$$

Since, also $H^q_\varphi(\mathcal{U}', \mathcal{O}^{p_0}/V) = 0$ for all $q \geq 1$, we get that $H^q_\varphi(\mathcal{U}', F'/V) = 0$ for all $q \geq 1$. Q.E.D.

The author wishes to thank Prof. R. Narasimhan from whose comments he got the idea of the proof.

References

[1] P. W. Darko (1975) – *Sections of Coherent Analytic Sheaves with growth on Complex Spaces*, « Annali di Matematica Pura ed applicata (IV) », *104*, 283–295.

[2] L. Hörmander (1967) – *Generators for some rings of analytic functions*, « Bull. Amer. Math. Soc. », *73*, 943–949.

[3] R. Narasimhan (1970) – *Cohomology with bounds on Complex Spaces*, « Lecture Notes in Mathematics », *155*, Springer Verlag, 141–150.