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Mechanical models of earthquakes and their statistics

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Geofisica. — *Mechanical models of earthquakes and their statistics* (*). Nota (**) del Corrisp. MICHELE CAPUTO.

RIASSUNTO. — Si suggerisce un modello meccanico per la legge empirica di Ishimoto–Iida che fornisce il numero di terremoti $n(\mathrm{M})\,\Delta\mathrm{M}$ nell'intervallo di magnitudo $\mathrm{M}, \mathrm{M}+\Delta\mathrm{M}$

$$\log n(\mathrm{M}) = \bar{a} - b\mathrm{M}$$

con $\bar{a}$ e b costanti per ogni regione sismica.

Il modello mostra che il coefficiente b può variare leggermente con la magnitudo per energie molto piccole o molto grandi, inoltre potrebbe permettere una stima del numero di faglie $\mathrm{N}(l)\,\Delta l$ di dimensione lineare nell'intervallo $l, l+\Delta l$.

The Ishimoto Iida empirical law representing the number of earthquakes $n(\mathrm{M})$ in the interval range of magnitudes $\mathrm{M}, \mathrm{M}+\Delta\mathrm{M}$

$$\log n(\mathrm{M}) = \bar{a} - b\mathrm{M}$$

was suggested in the year 1939. It was proved correct and held valid until our days although no theoretical basis was found for it.

This law was recently supplemented with another empirical law stating that for each magnitude range $\mathrm{M}, \mathrm{M}+\Delta\mathrm{M}$ and for a given time interval sufficiently long, the total area of the faults interested in the earthquakes is nearly independent of M.

In this Note we give a theoretical basis to the Ishimoto Iida law by introducing a mechanical model for the earthquakes and determining the density function of the number of earthquakes as function of the magnitude and the density function, of the number of faults which caused the earthquakes of a given area, as function of the area of the faults. We find also that the $\log n(\mathrm{M})$ ($n(\mathrm{M})$ number of earthquakes per unit area and time in the range $\mathrm{M}, \mathrm{M}+\Delta\mathrm{M}$) is an almost linear decreasing function of M.

Let us set

D/l^{ν}	number of faults with linear dimension, ν is a parameter ($\nu > 1.5$);
$\bar{\eta}k$	maximum and minimum linear dimension of faults geometric factor;
F	threshold of friction in stick slip motion;
α	angle between the direction of the fault and that of the tectonic force;
T_0	time length of the seismic catalogue;
f	coefficient of friction between the faults;
μ	rigidity.
$\mathrm{A} = \mathrm{l}^2$	area of the fault.

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$E = 10^{\beta+\gamma M}$ is the formula relating the magnitude to the elastic energy released; then we have:

$$(1) \qquad t = \frac{F}{\eta\,(\sin\alpha - f\cos\alpha)}$$
$$l = R\,(\sin\alpha - f\cos\alpha)^{2/3}$$
$$R^3 = \frac{E\mu}{\bar{\eta}\bar{k}F^2}$$

$$(2) \qquad n(M) = \frac{T_0 D\eta}{FR^{\nu-1}} \int_{\bar{l}_1/R}^{\bar{l}_2/R} U(u)\,du \quad ; \quad U(u) = u^{(3/2)-\nu}\left[1 + \frac{9}{4}\,\frac{u^3}{1+f-u^3}\right]^{1/2} =$$

$$= \frac{T_0 D\eta}{FR^{\nu-1}} \int_{\bar{\alpha}_1}^{\alpha_2} \varphi(\alpha)\,d\alpha \quad ; \quad \varphi(\alpha) = \frac{[9\,(\sin\alpha - f\cos\alpha)^2 + 4\,(\cos\alpha + f\sin\alpha)^2]^{1/2}}{3\,(\sin\alpha - f\cos\alpha)^{(2/3)(\nu-1)}}\,.$$

The factor D/l^{ν} has been introduced to take into account the number of faults of linear dimension 1 which are present in the system.

The energy E transformed in elastic waves is subject to the limitation imposed by the maximum size of the faults l_2, and by the maximum stress drop $\bar{\bar{p}}$ then

$$E \leq \frac{l_2^3\,\bar{\eta}\bar{k}\,\bar{\bar{p}}^2}{\mu}\,.$$

This limit sets a limit also for $\bar{\alpha}_1$, which is obtained from

$$\eta\bar{\bar{t}} = \bar{\bar{p}} = \frac{F}{\sin\bar{\bar{\alpha}}_1 - f\cos\bar{\bar{\alpha}}_1}\,.$$

If we take into account this limit then we must introduce the energy value

$$E_1 = \frac{l_1^3\,\bar{\eta}\bar{k}\,\bar{\bar{p}}^2}{\mu}\,.$$

If $\alpha_1 \left(0 < \alpha_1 < \frac{\pi}{2}\right)$ is such that

$$\sin\alpha_1 - f\cos\alpha_1 = 0$$

we also have

$$\bar{\bar{\alpha}}_1 > \alpha_1$$

As we shall see later there is another value of E which is relevant to the path of integration; it is:

$$E_0 = \frac{\overline{\eta k}\, l_2^3\, F^2}{\mu} \, .$$

Since $\frac{dl}{d\alpha} > 0$ for $0 < \alpha < \frac{\pi}{2}$, for the integration of (2) one may have four different kind of paths indicated in the Tables and in fig. 1.

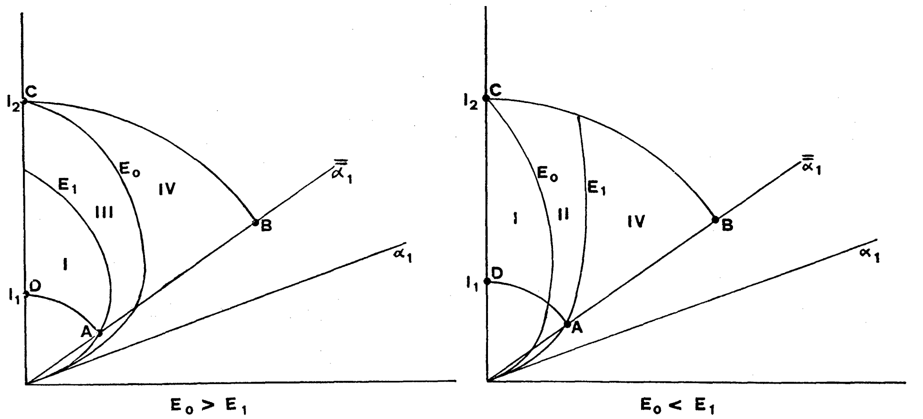

Fig. 1.

	From a Point of	To a Point of	Values of limits of interval of integration	
(I)	AD	CD	$\bar{l}_1 = l_1$	$\frac{\bar{l}_2}{R} = 1$
(II)	AD	CB	$\bar{l}_1 = l_1$	$\bar{l}_2 = l_2$
(III)	AB	CD	$\frac{\bar{l}_1}{R} = \left(\frac{F}{\bar{\bar{p}}}\right)^{2/3}$	$\frac{\bar{l}_2}{R} = 1$
(IV)	AB	CB	$\frac{\bar{l}_1}{R} = \left(\frac{F}{\bar{\bar{p}}}\right)^{2/3}$	$\bar{l}_2 = l_2$

Let $E_0\,(M_0)$ and $E_1\,(M_1)$ be the energies of the curves (1) passing through C and A; for the computation of the integral we must separate the cases $E_0 \lesseqgtr E_1\ (l_2^3\, F^2 \lesseqgtr l_1^3\, p_{max}^2)$; the path of integration for the different values of E (M) is given in the Table:

Value of M	$M < M_0$	$M_0 < M < M_1$	$M_1 < M$
Path of integration	(I)	(II)	(IV)
Value of M	$M < M_1$	$M_1 < M < M_0$	$M_0 < M$
Path of integration	(I)	(III)	(IV)

We are able now to discuss the $\log n(\mathrm{M})$; in case (I) we have:

$$\frac{d\mathrm{R}}{d\mathrm{E}}\frac{d\mathrm{E}}{d\mathrm{M}}=\frac{\mathrm{R}\gamma}{3}\ln 10$$

$$\frac{d\log n(\mathrm{M})}{d\mathrm{M}}=(1-\nu)\frac{\gamma}{3}+\frac{\gamma}{3}\frac{\frac{l_1}{\mathrm{R}}\cup\left(\frac{l_1}{\mathrm{R}}\right)}{\int_{l_1/\mathrm{R}}^{1}\cup(u)\,du}$$

depending on the values of the parameters $\bar{\eta}, \bar{k}, f, \mathrm{F}, \mu, \beta, \gamma$, where M is sufficiently small, this derivative could be positive, but it becomes negative for the larger values of M of the pertinent interval.

In case (II) we have:

$$\frac{d\log n(\mathrm{M})}{d\mathrm{M}}=(1-\nu)\frac{\gamma}{3}+\frac{\gamma}{3}\frac{\frac{l_1}{\mathrm{R}}\cup\left(\frac{l_1}{\mathrm{R}}\right)-\frac{l_2}{\mathrm{R}}\cup\left(\frac{l_2}{\mathrm{R}}\right)}{\int_{l_1/\mathrm{R}}^{l_2/\mathrm{R}}\cup(u)\,du}$$

which is negative for all values of M pertinent to the interval.

In case (III) we have

$$\frac{d\log n(\mathrm{M})}{d\mathrm{M}}=(1-\nu)\frac{\gamma}{3}<0$$

which implies that $\log n(\mathrm{M})$ is a straight line for $\mathrm{M}_1<\mathrm{M}<\mathrm{M}_0$.

Finally in case (IV) we obtain:

$$\frac{d\log n(\mathrm{M})}{d\mathrm{M}}=(1-\nu)\frac{\gamma}{3}+\frac{\gamma}{3}\frac{-\frac{l_2}{\mathrm{R}}\cup\left(\frac{l_2}{\mathrm{R}}\right)}{\int_{(\mathrm{F}/\bar{p})^{2/3}}^{l_2/\mathrm{R}}\cup(u)\,du}$$

which is negative for all the values of M pertinent to the interval.

In case II, when l_1 is sufficiently small $\frac{d^2\log n(\mathrm{M})}{d\mathrm{M}^2}>0$ which implies that the b of the Ishimoto Iida law is a decreasing function of M in the same interval.

In the cases (I) and (IV) the $\frac{d^2}{d\mathrm{M}^2}\log n(\mathrm{M})$ is negative.

The minimum and maximum values of M permitted by the geophysical parameters of the systems of faults are:

$$M_m = \frac{1}{\gamma}\left[\log \frac{l_1^3\, F^2\, \bar{\eta} \bar{k}}{\mu} - \beta\right]$$

$$M_m = \frac{1}{\gamma}\left[\log \frac{l_2^3\, \bar{\bar{p}}^2\, \bar{\eta} \bar{k}}{\mu} - \beta\right]$$

for M in this range, $\log n$ (M) is very nearly a straight line; it is exactly a straight line in the interval (III). The deviations of $\log n$ (M) from the straight line are very significant to the discussion of the geophysical parameters of the system of faults.

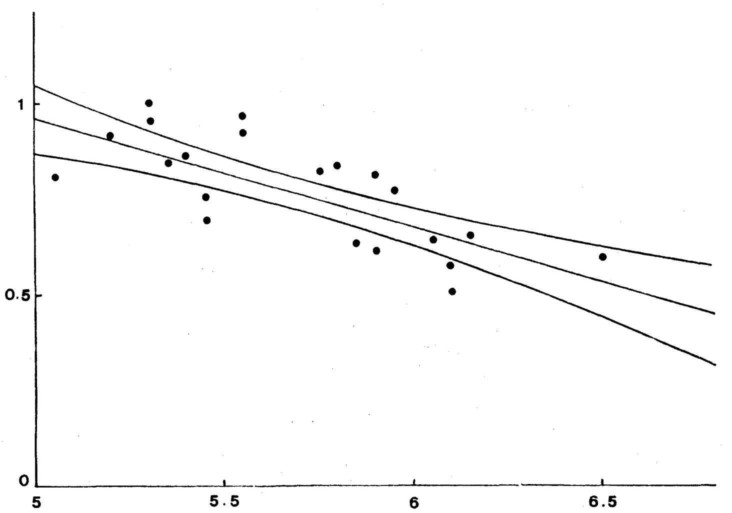

Fig. 2.

This theory has been checked fig. 2 for the Mediterranean area, Europe and the Middle East for which it has been found that the *b* parameter of the Ishimoto Iida law is a decreasing function of M thus confirming that for this area $M_0 < M_1$. The theory is also checked with the seismic data of the Appennines for which it was estimated that $M_0 \simeq 5$; the tentative estimate of the number of faults of this area as function of *l* gives reasonable results. More details on the mechanical model can be found in the communication: "Mechanical models of earthquakes and their statistics", presented at the symposium on eathquakes risk for nuclear power plant and printed in the R. Neth. Met. Inst. publ. 153.

The present version is more complete and detailed mathematically.