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## Unifying ad hoc Equivariant Cohomology Theories

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**Topologia.** — *Unifying ad hoc Equivariant Cohomology Theories.* Nota di MICHAEL J. CELLINI e YUH-CHING CHEN, presentata (*) dal Corrisp. G. FICHERA.

RIASSUNTO. — In questa Nota noi annunciamo la costruzione di una teoria di coomologia su G–insiemi simpliciali che include come caso speciale la teoria di coomologia singolare di Illman. Il concetto della realizzazione geometrica di G–insiemi simpliciali dà allora la teoria di coomologia equivariante cellulare di Walker ed i suoi « classifying spaces ».

Equivariant cohomology theories have been constructed by Bredon [B1] [B2], Bröcher [Br1], Illman [I1], [I2], [I3], Lee [L1], and Walker [W1] in different contexts. They can be unified under stack cohomology (cfr. [C1]) defined by derived functors of a limit functor $H^0_G$. In this note, we shall announce the construction of a stack cohomology theory on simplicial G-sets which will include Illman's equivariant singular cohomology theory as a special case. This will also provide a simple functorial construction of Illman's theory. Furthermore, the notion of geometrical realization of simplicial G-sets will yield Walker's equivariant cellular cohomology theory and its classifying spaces.

A *simplicial* G-*set* is a simplicial set K on which a group G acts simplicially. This arises naturally from the singular complex $K = S(X)$ of a topological G-space X. We view K as a small category with simplixes as objects and the face operators $d^i : d^i \sigma \to \sigma$, the degeneracy operators $s^j : \tau \leftarrow s^j \tau$, G-actions $g : \sigma \to g\sigma$, and their compositions as morphisms and define a *stack* over K as a functor $A : K \to Ab$ such that $A(s^j)$ is an isomorphism for every $s^j$. The category $Ab^K$ of all stacks over K has enough injectives. Therefore we can define a *stack cohomology*

$$\mathbf{H}^*_G(K ; A) = R^* \mathbf{H}^0_G(K ; A), \qquad \text{where} \quad \mathbf{H}^0_G(K ; A) = \lim A.$$

Let $H^*_G(K ; A)$ be the cohomology defined as usual via an equivariant cochain complex. Then by a standard theorem in [G1], one shows

THEOREM 1. $\mathbf{H}^*_G(K : A) \approx H^*_G(K ; A)$ *for every* $A \in Ab^K$. *As a consequence, Illman's cohomology of a topological* G-*space with a system of coefficients* $l$ *is a stack cohomology once one realizes that the system* $l$ *is in fact a specially chosen stack over* $K = S(X)$.

(*) Nella seduta del 13 dicembre 1975.

Given any stack A over K extending techniques used in [C1] one shows

THEOREM 2. $\mathbf{H}_G^*$ *defines a unique equivariant cohomology theory (with coefficients in* A) *on the category of pairs of simplicial* G-*sets over* K *satisfying all Eilenberg-Steenrod-Milnor axioms.*

The dimension axiom of this theory is modelled in a form to include the case when A is a general " variable " coefficient system. When K is a point, A is a local system and $\mathbf{H}_G^*$ gives Illman's unique theory and therefore other theories mentioned above except Walker's theory which is a " geometrical realization " of $\mathbf{H}_G^*$ to be discussed in the next paragraph.

The first author has shown that the geometrical realization of K as he defined is an equivariant CW-complex in Walker's sense [W1]. Since the cellular theory is equivalent to the singular theory (see [W1], it is essentially a stack cohomology. Moreover, the geometrical realization of generalized (equivariant) Eilenberg-MacLane complexes gives rise to Walker's classifying spaces. The above results will be presented in detail elsewhere.

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