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An extension of some oscillation theorems of Mawhin and Sedziwy

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Equazioni differenziali ordinarie. — *An extension of some oscillation theorems of Mawhin and Sedziwy.* Nota di ROLF REISSIG, presentata (*) dal Socio G. SANSONE.

RIASSUNTO. — L'Autore estendendo alcune note di J. Mawhin, S. Sedziwy e di altri autori considera un'equazione differenziale vettoriale non autonoma di ordine n contenente un termine di richiamo non lineare. Mediante un'applicazione del teorema del punto fisso di Leray–Schauder l'Autore prova l'esistenza di almeno una soluzione periodica con lo stesso periodo della forza esterna. Le condizioni imposte sul termine non lineare sono abbastanza generali e non implicano la limitatezza di tutte le soluzioni dell'equazione.

Consider the n-th order vector differential equation

$$\boldsymbol{x}^{(n)} + \mathbf{A}_1 \boldsymbol{x}^{(n-1)} + \cdots + \mathbf{A}_{n-k} \boldsymbol{x}^{(k)} + \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{p}(t) \tag{1}$$

where $1 \leqq k \leqq n - 1$ and

$\boldsymbol{x} = \operatorname{col}(x_1, \cdots, x_m) \in \mathbf{R}^m \ (m \geqq 1)$

$\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{p}(t) \equiv \boldsymbol{p}(t + \mathrm{T})$ are continuous functions

$\mathbf{A}_i \ (1 \leqq i \leqq n - k)$ are constant real $m \times m$-matrices.

Let us study the existence of T-periodic solutions. Some results concerning this problem are contained in the papers [1], [2], [3], [4], [5], [7], [8]. In the papers [2], [3] the restoring term $\boldsymbol{f}(\boldsymbol{x}) = \operatorname{col}(f_1(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x}))$ is subject to conditions relative to the components $f_i(\boldsymbol{x})$ whereas the conditions in the papers of Sedziwy [7], [8] are referring to the behavior of $|\boldsymbol{f}(\boldsymbol{x})| = \sqrt{f_1^2 + \cdots + f_m^2}$ and of $(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x})) = x_1 f_1 + \cdots + x_m f_m$ as $|\boldsymbol{x}| \to \infty$. The second type of conditions seems to be more natural. The aim of this note is to prove existence theorems on the basis of such conditions and to give an extension of some previous results of Mawhin, Sedziwy and of other authors.

THEOREM 1. *Assume that*

(*i*) $\varphi(\lambda) \equiv \det(\lambda^{n-k} \mathrm{I}_m + \lambda^{n-k-1} \mathbf{A}_1 + \cdots + \mathbf{A}_{n-k}) = 0 \Rightarrow \lambda \neq r\omega i$ $\left(r \text{ integer}, \ \omega = \frac{2\pi}{\mathrm{T}}\right)$

and

(*ii*) *in case* $k = 1 : \mathbf{A}_{n-1} = \mathbf{A}_{n-1}^*$ (*transposed matrix*).

(*) Nella seduta del 12 aprile 1975.

Assume that

$$(iii) \qquad \int_0^T \boldsymbol{p}(s)\, ds = 0, \text{ i.e. } \mathbf{P}(t) = \int_0^t \boldsymbol{p}(s)\, ds \equiv \mathbf{P}(t+T).$$

Let $\{a_j\}$ *be a positive, monotone-increasing, divergent sequence, and let* $B_j = \{\boldsymbol{x} \in \mathbf{R}^m : a_j \leqq |\boldsymbol{x}| \leqq 2\, a_j\}$.

Finally assume that

$$(iv') \qquad 0 \leqq c < 1 : F(r) = \sup_{|\boldsymbol{x}| \leqq r} |\boldsymbol{f}(\boldsymbol{x})| \leqq F(1 + r^c), \inf_{B_j} \frac{(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))}{|\boldsymbol{x}|^{2c}} \to \to \infty \text{ as } j \to \infty$$

or

$$(iv'') \qquad 0 < c \leqq 1 : F(r) \leqq F + \eta r^c \; (\eta > 0 \text{ small enough})$$

$$\frac{(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))}{|\boldsymbol{x}|^{2c}} \geqq \vartheta\eta \; (0 < \vartheta < 1) \text{ on } B_j \; (j \in \mathbf{N}).$$

Then equation (1) *admits at least one* T*-periodic solution.*

COROLLARY 1. *Let* $\varepsilon_i = +1$ *or* $\varepsilon_i = -1$, *and let* $\mathbf{E} = diag(\varepsilon_1, \cdots, \varepsilon_m)$. *Then Theorem 1. is valid also when* (*ii*) *is replaced by*

$$\mathbf{E}\, \mathbf{A}_{n-1} = \mathbf{A}^*_{n-1}\, \mathbf{E}$$

and when $(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$ *in* (*iv'*)-(*iv''*) *is replaced by*

$$(\mathbf{E}\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x})) = \varepsilon_1 x_1 f_1(\boldsymbol{x}) + \cdots + \varepsilon_m x_m f_m(\boldsymbol{x}).$$

In order to prove the theorem by means of the Leray-Schauder degree we choose a positive constant a (sufficiently small) such that

$$\varphi_a(\lambda) \equiv \det(\lambda^n I_m + \lambda^{n-1} \mathbf{A}_1 + \cdots + \lambda^k \mathbf{A}_{n-k} + a\mathbf{I}_m) = 0 \Rightarrow \lambda \neq r\omega i \quad (r \text{ integer}).$$

This is possible since the roots of $\varphi_a(\lambda) = 0$ are continuous with respect to a and since

$$\varphi_a(0) = a^m > 0, \; \varphi_0(\lambda) = \lambda^{km} \varphi(\lambda) \; [\varphi(\lambda) \text{ according to (i)}].$$

After that we deal with the auxiliary equation

$$(2) \qquad \boldsymbol{x}^{(n)} + \mathbf{A}_1 \boldsymbol{x}^{(n-1)} + \cdots + \mathbf{A}_{n-k} \boldsymbol{x}^{(k)} + a\boldsymbol{x} = \mu \boldsymbol{p}(t) + a\boldsymbol{x} - \boldsymbol{f}_\mu(\boldsymbol{x})$$

where $0 \leqq \mu \leqq 1$ and

$$\boldsymbol{f}_\mu(\boldsymbol{x}) = \frac{1+\mu}{2} \boldsymbol{f}(\boldsymbol{x}) - \frac{1-\mu}{2} \boldsymbol{f}(-\boldsymbol{x}), \qquad \text{i.e. } \boldsymbol{f}_0(-\boldsymbol{x}) \equiv -\boldsymbol{f}_0(\boldsymbol{x}).$$

The T-periodic solutions of equation (2) are determined as the continuous solutions of an integral equation of Hammerstein type:

$$\boldsymbol{x}(t) = \theta_\mu \{\boldsymbol{x}(t)\} \equiv \int_0^T \mathbf{G}(t-s)\{\mu \boldsymbol{p}(s) + a\boldsymbol{x}(s) - \boldsymbol{f}_\mu(\boldsymbol{x}(s))\}\, ds.$$

The Green matrix $\mathbf{G}(t) \equiv \mathbf{G}(t+T)$ is of class C^{n-2} whereas the derivative of order $n-1$ is piecewise continuous: $\mathbf{G}^{(n-1)}(0+) - \mathbf{G}^{(n-1)}(T-) = \mathbf{I}_m$.

Define the Banach space $X = \{\boldsymbol{x}(t) \in C^0(\mathbf{R}) : \boldsymbol{x}(t+T) \equiv \boldsymbol{x}(t),\ \|\boldsymbol{x}\|_X = \sup |\boldsymbol{x}(t)|\}$ and the Banach space $X^* = X \times \mathbf{R}$ with norm $\|\boldsymbol{x}\|_X + |\mu|$. The operator $\theta_\mu : X^* \to X,\ (\boldsymbol{x}, \mu) \to \theta_\mu \boldsymbol{x}$ is completely continuous on each subset $B^* = B \times [0, 1] \subset X^*$, $B \subset X$ bounded. If $S_R = \{\boldsymbol{x} \in X : \|\boldsymbol{x}\|_X < R\}$ is such that $(I - \theta_\mu)\boldsymbol{x} \neq 0$ on ∂S_R for $0 \leqq \mu \leqq 1$, then the Leray-Schauder degree

$$d[I - \theta_\mu, S_R, 0] = d[I - \theta_0, S_R, 0]$$

is odd (i.e. nonzero) since $\theta_0(-\boldsymbol{x}) = -\theta_0 \boldsymbol{x}$. Hence

$$(I - \theta_\mu)\boldsymbol{x} = 0 \quad \text{for at least one} \quad \boldsymbol{x} \in S_R,\ 0 \leqq \mu \leqq 1.$$

Such a ball S_R exists, for example, if the T-periodic solutions of (2) are a priori bounded (independently on μ).

Note 1. Define $F_\mu(r) = \sup\limits_{|\boldsymbol{x}| \leqq r} |\boldsymbol{f}_\mu(\boldsymbol{x})|$; then $F_\mu(r)$ satisfies the estimate (iv′) or (iv″) since

$$\sup_{|\boldsymbol{x}|=r} |\boldsymbol{f}_\mu(\boldsymbol{x})| \leqq \sup_{|\boldsymbol{x}|=r} |\boldsymbol{f}(\boldsymbol{x})|.$$

Taking into account that

$$(\boldsymbol{f}_\mu(\boldsymbol{x}), \boldsymbol{x}) = \frac{1+\mu}{2}(\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{x}) + \frac{1-\mu}{2}(\boldsymbol{f}(-\boldsymbol{x}), -\boldsymbol{x})$$

we obtain

$$\inf_{|\boldsymbol{x}|=r} (\boldsymbol{f}_\mu(\boldsymbol{x}), \boldsymbol{x}) \geqq \inf_{|\boldsymbol{x}|=r} (\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{x});$$

therefore, also the second part of condition (iv′) or (iv″) is valid for $\boldsymbol{f}_\mu(\boldsymbol{x})$.

Now, let $\boldsymbol{x}(t) = \theta_\mu\{\boldsymbol{x}(t)\} \in X$, and let $\|\boldsymbol{x}\|_X = H$, $\inf |\boldsymbol{x}(t)| = h$. The derivative $\boldsymbol{y}(t) = \boldsymbol{x}^{(k)}(t)$ is a solution of the differential equation

$$(3) \qquad \boldsymbol{y}^{(n-k)} + \mathbf{A}_1 \boldsymbol{y}^{(n-k-1)} + \cdots + \mathbf{A}_{n-k}\boldsymbol{y} = \mu \boldsymbol{p}(t) - \boldsymbol{f}_\mu(\boldsymbol{x}(t))$$

which is considered as a nonhomogenous linear one (noncritical case). Consequently, $\boldsymbol{y}(t)$ satisfies an integral equation of Hammerstein type

$$\boldsymbol{y}(t) = \int_0^T \mathbf{G}_k(t-s)\{\mu \boldsymbol{p}(s) - \boldsymbol{f}_\mu(\boldsymbol{x}(s))\}\, ds$$

from which we conclude:

$$y^{(i)}(t) = \int_0^T \mathbf{G}_{k+i}(t-s)\{\mu \boldsymbol{p}(s) - \boldsymbol{f}_\mu(\boldsymbol{x}(s))\}\, ds, \quad 1 \leqq i \leqq n-k-1$$

$[\mathbf{G}_{k+i}(t) = \mathbf{G}_k^{(i)}(t)$ only piecewise continuous in case $i = n-k-1]$. Denoting $\|\boldsymbol{p}\|_X = P_0$ we derive the estimates

$$\|y^{(i)}\|_X = \|\boldsymbol{x}^{(k+i)}\|_X \leqq \rho_{k+i}(P_0 + F_\mu(H)), \quad 0 \leqq i \leqq n-k-1$$

and a corresponding estimate for $y^{(n-k)} = \boldsymbol{x}^{(n)}$ by virtue of equation (3). The coefficients ρ_{k+i} depend upon the matrices $\mathbf{A}_1, \cdots, \mathbf{A}_{n-k}$ and upon the period T.

In case $k > 1$ we have for $\tau \leqq t \leqq \tau + T$

$$|\boldsymbol{x}^{(k-1)}(t) - \boldsymbol{x}^{(k-1)}(\tau)| = \left|\int_\tau^t y(s)\, ds\right| \leqq T\|y\|_X \leqq \rho_k T(P_0 + F_\mu(H)).$$

Each component of the derivative $\boldsymbol{x}^{(k-1)}$ possesses at least two zeros on the interval $[0, T]$. Let $x_i^{(k-1)}(\tau) = 0$; then

$$|x_i^{(k-1)}(t)| \leqq \rho_k T(P_0 + F_\mu(H)).$$

Considering this estimate for $1 \leqq i \leqq m$ we conclude that

$$\|\boldsymbol{x}^{(k-1)}\|_X \leqq \rho_{k-1}(P_0 + F_\mu(H)).$$

Repeating this argument, if necessary, we find

$$\|\boldsymbol{x}^{(i)}\|_X \leqq \rho_i(P_0 + F_\mu(H)), \quad 1 \leqq i \leqq n, \tag{4}$$

ρ_i as described above.

Note 2. Let $|\boldsymbol{x}(\tau')| = h$, $|\boldsymbol{x}(\tau'')| = H$ $(0 \leqq \tau', \tau'' \leqq T)$; then $h \geqq H - |\boldsymbol{x}(\tau') - \boldsymbol{x}(\tau'')| > H - \rho_1 T(P_0 + F_\mu(H))$,

$$0 \leqq 1 - \frac{h}{H} < \rho_1 T\left(\frac{P_0}{H} + \frac{F_\mu(H)}{H}\right) \leqq \frac{1}{2} \quad \text{(i.e. } 2h \geqq H) \text{ if } H \geqq H_0 \tag{5}$$

by virtue of condition (iv′) or (iv″) in connection with Note 1. In case $c = 1$ the estimate is ensured if, for instance $4\rho_1 T\eta \leqq 1$.

Subsequently, let us assume that

$$H = 2a_j \geqq H_0 \quad (j \text{ sufficiently large}).$$

As a consequence of (5) we have $h \geqq a_j$ and $\boldsymbol{x}(t) \in B_j$ for all t. From equation (2) we derive:

$$(\boldsymbol{x}^{(n)} + \mathbf{A}_1 \boldsymbol{x}^{(n-1)} + \cdots + \mathbf{A}_{n-k}\boldsymbol{x}^{(k)} - \mu \mathbf{P}', \boldsymbol{x}) + (\boldsymbol{f}_\mu(\boldsymbol{x}), \boldsymbol{x}) = 0;$$

integrating this equation from 0 to T and taking into account that in case $k > 1$

$$(\boldsymbol{x}^{(n)} + \cdots - \mu\mathbf{P}', \boldsymbol{x}) = (\boldsymbol{x}^{(n-1)} + \mathbf{A}_1 \boldsymbol{x}^{(n-2)} + \cdots + \mathbf{A}_{n-k} \boldsymbol{x}^{(k-1)} - \mu\mathbf{P}, \boldsymbol{x})' - (\boldsymbol{x}^{(n-1)} + \cdots - \mu\mathbf{P}, \boldsymbol{x}')$$

we find:

$$(6) \qquad \int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt = \int_0^T (\boldsymbol{x}^{(n-1)} + \mathbf{A}_1 \boldsymbol{x}^{(n-2)} + \cdots + \mathbf{A}_{n-k} \boldsymbol{x}^{(k-1)} - \mu\mathbf{P}, \boldsymbol{x}')\, dt.$$

Hence

$$(7) \qquad \int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt \leqq \rho\,(\mathrm{P}_0 + \mathrm{F}_\mu(\mathrm{H}))^2, \quad \rho = \rho\,(\mathbf{A}_1, \cdots, \mathbf{A}_{n-k}; \mathrm{T}).$$

In case $k = 1$ condition (ii) yields:

$$(\mathbf{A}_{n-1} \boldsymbol{x}', \boldsymbol{x}) = (\boldsymbol{x}', \mathbf{A}_{n-1} \boldsymbol{x}) = \frac{1}{2} (\mathbf{A}_{n-1} \boldsymbol{x}, \boldsymbol{x})';$$

consequently an estimate of type (7) is still valid.

By virtue of (iv′) we deduce from (7) and (5):

$$\inf_{\mathrm{B}_j} \frac{(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))}{|\boldsymbol{x}|^{2c}} \leqq \frac{\rho}{\mathrm{T}} \left(\frac{\mathrm{P}_0}{h^c} + \frac{\mathrm{F}_\mu(\mathrm{H})}{h^c} \right)^2 \leqq \frac{\rho}{\mathrm{T}} \left(\frac{\mathrm{P}_0 + \mathrm{F}}{a_j^c} + 2^c\, \mathrm{F} \right)^2$$

which contradicts condition (iv′) provided that j is large enough. Therefore $\mathrm{H} \neq \mathrm{R} = 2a_j$ for such an index j.

A similar argument holds on condition (iv″):

$$\vartheta\eta \leqq \frac{\rho}{\mathrm{T}} \left(\frac{\mathrm{P}_0 + \mathrm{F}}{a_j^c} + 2^c\, \eta \right)^2;$$

it yields a contradiction if j is sufficiently large provided that $\eta < \vartheta \mathrm{T} \rho^{-1} 2^{-2c}$.

When the conditions of Theorem 1. are modified according to Corollary 1. the last part of the proof (behind Note 2.) must be changed in a simple way.

Finally we still discuss the vector differential equation

$$(8) \qquad \boldsymbol{x}^{(n)} + \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{p}(t), \quad n \geqq 1$$

or the auxiliary equation

$$(9) \qquad \boldsymbol{x}^{(n)} + a\boldsymbol{x} = \mu\boldsymbol{p}(t) + a\boldsymbol{x} + \boldsymbol{f}_\mu(\boldsymbol{x}), \quad 0 \leqq \mu \leqq 1.$$

If $0 < a < \omega^n$,

$$\det(\lambda^n \mathrm{I}_m + a\mathrm{I}_m) = (\lambda^n + a)^m = 0 \Rightarrow 0 < |\lambda| < \omega.$$

Let $\boldsymbol{x}(t)$ be a T-periodic solution of equation (9). Proceeding as above we derive the estimate (4) where $\rho_i = \rho_i(T)$. After that equation (6) must be replaced by

$$\int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt = -\mu \int_0^T (\boldsymbol{x}', \mathbf{P})\, dt \text{ in case } n = 2\nu + 1 \ (\nu \geqq 0)$$

or

$$\int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt = (-1)^\nu \int_0^T |\boldsymbol{x}'|^2\, dt - \mu \int_0^T (\boldsymbol{x}', \mathbf{P})\, dt \text{ in case } n = 2\nu + 2 \ (\nu \geqq 0).$$

Instead of (7) we have

$$\int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt \leqq \rho\, P_0 (P_0 + F_\mu(H)) \text{ if } n = 2\nu + 1 \text{ or if } n = 4\nu$$

and

$$\int_0^T (\boldsymbol{x}, \boldsymbol{f}_\mu(\boldsymbol{x}))\, dt \leqq \rho\, (P_0 + F_\mu(H))^2 \text{ if } n = 4\nu + 2.$$

Completing the proof like above we can verify the following theorem.

THEOREM 2. *Assume that*

(i) $$\int_0^T \boldsymbol{p}(t)\, dt = 0.$$

Furthermore, assume that in the cases $n = 2\nu + 1$, $n = 4\nu$,

(ii′) $$0 \leqq c < 1 : F(r) \leqq F(1 + r^c), \ \inf_{B_j} \frac{(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))}{|\boldsymbol{x}|^c}, \to \infty \quad \text{as } j \to \infty$$

or

(ii″) $$0 < c \leqq 1 : F(r) \leqq F + \eta r^c \ (\eta > 0 \ \textit{small enough})$$

$$\frac{(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))}{|\boldsymbol{x}|^c} \geqq q > 0 \quad \textit{on } B_j \ (j \in \mathbf{N}).$$

In case $n = 4\nu + 2$ *the restoring term* $\boldsymbol{f}(\boldsymbol{x})$ *is assumed to fulfill condition (iv′) or (iv″) of Theorem 1.*

Then equation (8) *admits at least one* T-*periodic solution.*

COROLLARY 2. *Theorem 2. remains valid when* $(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$ *is replaced by* $(\mathbf{E}\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$ *where* $\mathbf{E} = \mathit{diag}\,(\varepsilon_1 \cdots, \varepsilon_m)$, $|\varepsilon_i| = 1$.

Note 3. Sedziwy [8] studies equation (1), case $k = 1$, proposing a condition of type (iv″), case $c = 1$, which is partly weaker and partly stronger:

$$|\boldsymbol{f}(\boldsymbol{x})| \leqq \eta |\boldsymbol{x}| \quad \text{for} \quad |\boldsymbol{x}| \geqq \xi \ (\eta > 0 \ \text{small enough})$$

$$(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x})) \geqq |\boldsymbol{x}|\, |\boldsymbol{f}(\boldsymbol{x})| \quad \text{for all } \boldsymbol{x}, \vartheta \inf_{|\boldsymbol{x}| \geqq \xi} |\boldsymbol{f}(\boldsymbol{x})| > P_0.$$

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