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RIASSUNTO. — In questo lavoro si stabiliscono connessioni fra le strutture di approssimazione introdotte in [5], gli spazi di convergenza [1, 2, 3] e le strutture di merotopia [6]. Nelle proposizioni 1–3 sono ottenute condizioni affinché una struttura di approssimazione si generi tramite una struttura di convergenza. Le relazioni (17), (23) stabiliscono la corrispondenza voluta fra le strutture di approssimazione e le merotopie di Katetov.

The purpose of this paper is to establish a connection between the approximation structures introduced in [5], merotopic structures [6] and convergence spaces [3].

Let X and A be two sets and let $\mathscr{R}(A, X)$ be the collection of all relations $\alpha \subset A \times X$.

DEFINITION 1. An approximation relation on X directed by A is a subset

$$\mathscr{A} \subset X \times \mathscr{R}(A, X)$$

such that

$$\forall x \in X \quad \exists \alpha \in \mathscr{R}(A, X) : (x, \alpha) \in \mathscr{A} . \tag{1}$$

We shall say that $\{X, \mathscr{A}, A\}$ is an approximation structure space. We denote

$$\mathscr{A}[x] = \{\alpha \in \mathscr{R}(A, X) : (x, \alpha) \in \mathscr{A}\}, \qquad x \in X. \tag{2}$$

If $\mathscr{M}$ is a collection of subsets of X, we denote by

$$[\mathscr{M}] = \{K \subset X : \exists M \in \mathscr{M}, \ M \subset K\} \tag{3}$$

the prefilter generated by $\mathscr{M}$. For $\alpha \in \mathscr{R}(A, X)$ and $a \in A$ we set

$$r(\alpha, a) = \{z \in X : (a, z) \in \alpha\} \tag{4}$$

and

$$\pi(\alpha) = [\{r(\alpha, a) : a \in A\}] . \tag{5}$$

DEFINITION 2. Let $\mathscr{A}$ and $\mathscr{B}$ two approximation relations on X directed by A and B, respectively. The relation $\mathscr{A}$ is said to be finer than $\mathscr{B}$ ($\mathscr{B} \prec \mathscr{A}$) if

$$\forall x \in X \quad \forall \alpha \in \mathscr{A}[x] \quad \exists \beta \in \mathscr{B}[x] : \pi(\beta) \subset \pi(\alpha) ; \tag{6}$$

$\mathscr{A}$ and $\mathscr{B}$ are said to be equivalent if

$$\mathscr{A} \sim \mathscr{B} \Longleftrightarrow \mathscr{A} \prec \mathscr{B} \quad \text{and} \quad \mathscr{B} \prec \mathscr{A} . \tag{7}$$

(*) Nella seduta dell'8 febbraio 1975.

§ 1. Approximation structures associated with convergence spaces

The convergence spaces introduced by M. Fréchet [4] have been intensively investigated by H. R. Fischer [3], C. H. Cook and H. R. Fischer [2], H. Poppe [7], H. J. Biesterfeldt [1].

In this paragraph we shall establish a relationship between convergence spaces and approximation structures.

Let $\mathbf{F}(X)$ be the collection of all proper filter on the set X.

Definition 3 ([3], [7]):

a) A non empty set X is said to be a convergence space if there is $\mathscr{L} \subset X \times \mathbf{F}(X)$ such that:

$$\forall x \in X \Rightarrow (x, \dot{x}) \in \mathscr{L}, \qquad (\dot{x} = [\{x\}]), \tag{8}$$

$$(x, \varphi) \in \mathscr{L} \wedge \varphi \subset \psi \Rightarrow (x, \psi) \in \mathscr{L}. \tag{9}$$

b) A convergence space $(X, \mathscr{L})$ satisfying

$$(x, \varphi) \in \mathscr{L} \wedge (x, \psi) \in \mathscr{L} \Rightarrow (x, \varphi \cap \psi) \in \mathscr{L}, \tag{10}$$

is called a filtred convergence spaces or L-space by Poppe [7].

c) A non empty set X associated with a subset $\mathscr{L}$ which satisfies (8) and

$$\exists \nu : X \to \mathbf{F}(X) : (x, \varphi) \in \mathscr{L} \Longleftrightarrow \nu(x) \subset \varphi, \tag{11}$$

is called a space with generalized neighbourhoods and $\nu(x)$ is said to be the filter of generalized neighbourhoods of $x \in X$.

d) An (X, ν) space with generalized neighbourhoods is a topological space if

$$\forall x \in X \quad \forall V \in \nu(x) \quad \exists W \in \nu(x) : \forall y \in W \quad V \in \nu(y). \tag{12}$$

Any filter φ such that $(x, \varphi) \in \mathscr{L}$ is said to be convergent to x. By (8) the set $\mathscr{L}[x] = \{\varphi : (x, \varphi) \in \mathscr{L}\}$ is non-empty.

In Fischer's paper [3] the convergence spaces fulfill (8), (9), (10) i.e. are filtred convergence spaces.

Let $(X, \mathscr{L})$ be any convergence space ((8), (9)) and A a set which is in an one to one correspondence with an ultra filter of X, for example $\dot{x}_0$, $x_0 \in X$.

Consider the following approximation structure on X directed by A

$$(x, \alpha) \in \mathscr{A}_{(\mathscr{L})} \Longleftrightarrow \exists \varphi \in \mathscr{L}[x], \quad \pi(\alpha) = \varphi \quad \alpha \in \mathscr{R}(A, X) \tag{13}$$

where π is defined by (5).

Let $(X, \mathscr{A}, A)$ be an approximation structure such that

(CAS1) $\quad \forall x \in X, \ \exists \alpha \in \mathscr{A}[x] \Rightarrow \pi(\alpha) = \dot{x}$,

(CAS2) $\quad \forall x \in X, \ \forall \alpha \in \mathscr{A}[x] : \pi(\alpha) \in \mathbf{F}(X)$,

$$\forall \beta : \pi(\beta) \in \mathbf{F}(X) \wedge \pi(\alpha) \subset \pi(\beta) \Rightarrow \beta \in \mathscr{A}[x].$$

Then the set $\mathscr{L}_{\mathscr{A}} \subset X \times \mathbf{F}(X)$ defined by

$$\mathscr{L}_{\mathscr{A}} = \{(x, \varphi) : \ \exists \alpha \in \mathscr{A}[x] \ \pi(\alpha) \subset \varphi\} \tag{14}$$

satisfies (8) and (9) i.e. generates a convergence structure on X.

PROPOSITION 1. *The relations* (13)–(14) *define an one-to-one correspondence between the collection of all convergence structures on a nonempty set* X *and the family of equivalence classes generated by the set of approximation structures on* X *satisfying* (CAS1) *and* (CAS2).

Proof. We shall establish that two approximation structures on X generated by (13) and a convergence structure $\mathscr{L}$ on X are equivalent in the sense of Definition 2.

Let $\{X, \mathscr{A}_{(\mathscr{L})}, A\}$ and $\{X, \mathscr{B}_{(\mathscr{L})}, B\}$ be two approximation structures on X generated by $\mathscr{L}$ and let $\alpha \in \mathscr{A}_{(\mathscr{L})}[x]$; this means that $\exists \varphi \in \mathscr{L}[x]$, $\pi(\alpha) = \varphi$ and $\exists \beta \in \mathscr{R}(B, X)$ such that $\pi(\beta) = \varphi$ and $\beta \in \mathscr{B}_{(\mathscr{L})}[x]$ i.e. $\mathscr{B}_{(\mathscr{L})} \prec \mathscr{A}_{(\mathscr{L})}$. Analogously one establishes that $\mathscr{A}_{(\mathscr{L})} \prec \mathscr{B}_{(\mathscr{L})}$ and therefore $\mathscr{A}_{(\mathscr{L})} \sim \mathscr{B}_{(\mathscr{L})}$.

Let $\mathscr{L}$ be a convergence structure on X and let $\mathscr{A}_{(\mathscr{L})}$ be generated by (13). We shall prove that $\mathscr{L}_{\mathscr{B}}$ generated by any approximation structure $\mathscr{B}$ which is equivalent to $\mathscr{A}_{(\mathscr{L})}$ coincides with $\mathscr{L}$.

By (14) $(x, \varphi) \in \mathscr{L}_{\mathscr{B}} \Longleftrightarrow \exists \beta \in \mathscr{B}[x] \ \pi(\beta) \subset \varphi$. But $\mathscr{A}_{(\mathscr{L})} \sim \mathscr{B}$ and $\exists \alpha \in \mathscr{A}[x]$ $\pi(\alpha) \subset \pi(\beta)$ and therefore by (13) there is a filter $\psi \in \mathscr{L}[x]$ such that $\pi(\alpha) = \psi$ i.e. $\mathscr{L}_{\mathscr{B}} \subset \mathscr{L}$. Let $(x, \chi) \in \mathscr{L}$. By (13) it follows that there is $\alpha \in \mathscr{A}_{(\mathscr{L})}[x]$ such that $\pi(\alpha) = \chi$ and by $\mathscr{B} \sim \mathscr{A}_{(\mathscr{L})}$ there is $\beta \in \mathscr{B}[x] : \pi(\beta) \subset \pi(\alpha)$. By (14) one has $\chi \in \mathscr{L}_{\mathscr{B}}[x]$ and therefore $\mathscr{L} \subset \mathscr{L}_{\mathscr{B}}$.

Let $\mathscr{A}$ be an approximation structure on X which satisfies (CAS1) and (CAS2). Let $\mathscr{L}_{\mathscr{A}}$ be the convergence structures generated by $\mathscr{A}$ through (14) and let $\mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}$ be the approximation structure generated by $\mathscr{L}_{\mathscr{A}}$ through (13). Obviously, $\mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}$ also satisfies (CAS1) and (CAS2). We shall prove that $\mathscr{A} \sim \mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}$.

Let $\beta \in \mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}[x]$. From (13) there is $\varphi \in \mathscr{L}_{\mathscr{A}}[x] : \pi(\beta) = \varphi$ and by (14) it follows that there exists $\alpha \in \mathscr{A}[x]$ with $\pi(\alpha) = \varphi$. Therefore $\pi(\alpha) \subset \pi(\beta)$ i.e. $\mathscr{A} \prec \mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}$.

Let $\alpha \in \mathscr{A}[x]$. By (14) one has $\varphi = \pi(\alpha) \in \mathscr{L}_{\mathscr{A}}[x]$ and $\beta : \pi(\beta) = \varphi \Rightarrow$ $\Rightarrow \beta \in \mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}[x]$. Thus for every $\alpha \in \mathscr{A}[x]$ there exists $\beta \in \mathscr{B}_{(\mathscr{L}_{\mathscr{A}})}[x]$ with $\pi(\beta) \subset \pi(\alpha)$ so that $\mathscr{B}_{(\mathscr{L}_{\mathscr{A}})} \prec \mathscr{A}$.

The following results are immediate

PROPOSITION 2. *Let* $\mathscr{A}$ *be an approximation structure on* X *which satisfies* (CAS1) *and* (CAS2). $\mathscr{A}$ *generates by* (14) *a filtral convergence structure on* X *if and only if*

(FCAS) $\forall x \in X\,,\ \forall \alpha\,,\ \beta \in \mathscr{A}[x]\ \ \exists \gamma \subset \mathscr{A}[x] : \ \pi(\gamma) \subset \pi(\alpha) \cap \pi(\beta)\,.$

PROPOSITION 3. *An approximation structure* $\mathscr{A}$ *generates by* (14) *a structure with generalized neighbourhoods if and only if*

(GNAS) $\forall x \in X \quad \exists \alpha \in \mathscr{A}[x] : \ \forall a \in A \quad x \in r(\alpha\,,\,a)$

$\forall \beta \in \mathscr{A}[x] \quad \pi(\beta) \in \mathbf{F}(X) : \ \beta \subset \alpha\,.$

§ 2. KATETOV'S MEROTOPY

An important generalization of topological structures is due to M. Katetov [6] which introduces the merotopic structure on a set.

In this paragraph we shall present Katetov's definition of merotopic space, an equivalence relation of merotopies, the concept of factor merotopy and, finally, the merotopic-approximation structures.

DEFINITION 4. (M. Katetov [6]). Let X be any nonempty set. $\Gamma \subset \mathscr{P}(\mathscr{P}(x))$ is said to be a merotropic structure on X if:

i) $\mathscr{M} \in \Gamma,\ \mathscr{M}_1 \in \mathscr{P}(\mathscr{P}(x)),\ \forall M \in \mathscr{M}\ \ \exists\, M_1 \in \mathscr{M}_1\ \ M_1 \subset M \Rightarrow \mathscr{M}_1 \in \Gamma$;

ii) $\mathscr{M}_1 \cup \mathscr{M}_2 \in \Gamma \Rightarrow (\mathscr{M}_1 \in \Gamma) \vee (\mathscr{M}_2 \in \Gamma)$;

iii) $\forall x \in X : \ \{\{x\}\} \in \Gamma$;

iv) $\varnothing \notin \Gamma$.

Every $\mathscr{M} \in \Gamma$ will be called micromeric.

Let $\Pi(X)$ denote the familly of all prefilters on X. Consider the equivalence relation on $\mathscr{P}(\mathscr{P}(x))$:

$$\mathscr{M} \approx \mathscr{N} \to [\mathscr{M}] = [\mathscr{N}]\,.$$

Obviously, the factor space $\mathscr{P}(\mathscr{P}(x))/\approx$ may be identified with $\Pi(X)$.

Let K be a semilattice $\{K\,,\,\leq\,,\,\cup\}$, i.e. a set with a binary reflexive and transitive relation denoted by $\leq$ and a binary operation $\cup : K \times K \to K$ such that $\forall k_1\,,\,k_2 \in K\ \ k_i \leq k_1 \cup k_2$.

A familly $\varkappa \subset K$ is called an elementary prefilter on K if

$$p \in \varkappa \wedge p \leq r \Rightarrow r \in \varkappa\,;$$

$$p \cup q \in \varkappa \Rightarrow (p \in \varkappa) \vee (q \in \varkappa)\,.$$

We incidentally observe that $\Pi(X)$ is a lattice with $\leq$ defined by set-theoretic inclusion in $\mathscr{P}(\mathscr{P}(x))$.

Denote $[\{x\}]$ by $\dot{x}$.

PROPOSITION 4. *Let* $\tilde{\Gamma} \subset \Pi(X)$ *such that*:

i') $\tilde{\Gamma}$ *is an elementary prefilter on* $\Pi(X)$;

ii') $\forall x \in X : \dot{x} \in \tilde{\Gamma}$;

iii') $\varnothing \notin \tilde{\Gamma}$.

Then there exists an unique merotopy Γ *on* X *such that*

$$(\forall \mathscr{M} \subset \Gamma \Rightarrow [\mathscr{M}] \subset \tilde{\Gamma}) \wedge (\forall \lambda \in \tilde{\Gamma} \Rightarrow \lambda \in \Gamma). \tag{15}$$

Proof. Let λ be fixed in $\tilde{\Gamma}$. We consider the collection of all prefilter basis of λ, i.e. the equivalence class of λ in $\mathscr{P}(\mathscr{P}(X)/\approx$ denoted by $\hat{\lambda}$.

I shall prove that

$$\Gamma = \{\mathscr{M} : \mathscr{M} \in \hat{\lambda} : \lambda \in \tilde{\Gamma}\} \tag{16}$$

is a merotopy.

Let $\mathscr{M} \in \Gamma$ and $\mathscr{M}'$ be such that for every $M \in \mathscr{M}$ there exists $M' \in \mathscr{M}'$ $M' \subset M$. Therefore $[\mathscr{M}] \subset [\mathscr{M}']$.

Since $\tilde{\Gamma}$ is an elementary prefilter on $\Pi(X)$ and $\mathscr{M} \in \Gamma$ it follows $[\mathscr{M}] \in \tilde{\Gamma}$ and $[\mathscr{M}'] \in \tilde{\Gamma}$.

Then, since $\mathscr{M}' \in [\hat{\mathscr{M}}']$, $\mathscr{M}' \in \Gamma$. This proves condition i) of Definition 4.

Let $\mathscr{M}_1, \mathscr{M}_2 \in \mathscr{P}(\mathscr{P}(X))$ be such that $\mathscr{M}_1 \cup \mathscr{M}_2 \in \Gamma$. Then $[\mathscr{M}_1 \cup \mathscr{M}_2] \in \tilde{\Gamma}$. Since $[\mathscr{M}_1 \cup \mathscr{M}_2] = [\mathscr{M}_1] \cup [\mathscr{M}_2]$ it follows $([\mathscr{M}_1] \in \tilde{\Gamma}) \vee ([\mathscr{M}_2] \in \tilde{\Gamma})$ and therefore $(\mathscr{M}_1 \in \Gamma) \vee (\mathscr{M}_2 \in \Gamma)$.

Condition iii') follows immediately by ii') and iv) by iii).

Now, suppose that there exists a meroropy Γ_1 which satisfies (15). Since, in according to condition i) in the definition of merotopy, if $\mathscr{M} \in \Gamma_1$, $\hat{\mathscr{M}} \in \mathscr{P}(\mathscr{P}(X))/\approx$ $\hat{\mathscr{M}}_1 \subset \Gamma_1$ then $[\mathscr{M}] \in \Gamma_1$, and by (15), it follows $[\mathscr{M}] \in \tilde{\Gamma}$. Then, as a consequence of (16), $\mathscr{M} \in \Gamma$ and $\Gamma_1 \subset \Gamma$.

Let $\mathscr{N} \in \Gamma$. Then $[\mathscr{N}] \in \tilde{\Gamma}$ and since Γ_1 satisfies the second part of (15) it follows $[\mathscr{N}] \subset \Gamma_1$. Γ_1 is a merotopy and according to the condition i) it follows $\mathscr{N} \in \Gamma$. Thus $\Gamma = \Gamma_1$ as claimed.

Proposition 4 allows us to define equivalently merotopic structures:

DEFINITION 5. Let X be a set. A collection $\tilde{\Gamma} \subset \Pi(X)$ which satisfies conditions i'), ii') and iii) in Proposition 4 is called a π-merotopic structure on X and its elements are called π-micromerics.

DEFINITION 6. A π-merotopic structure on X is said to be localized if to each π-micromeric λ there exists an $x_\lambda \in X$ such that

$$\lambda \cap \dot{x}_\lambda \in \tilde{\Gamma}.$$

We shall say that the π-micromeric λ is localized in x_λ. Now we shall establish the relation between π-merotopic localized structures and approximation structures.

Let $A = \mathscr{P}(X)$ and $\tilde{\Gamma}$ an localized π-merotopy on X. Consider a map $\alpha : \tilde{\Gamma} \to \mathscr{R}(A, X)$ defined by:

$$\text{for } \tilde{\lambda} \in \tilde{\Gamma} \quad \begin{array}{ll} r(\alpha(\tilde{\lambda}), a) = a & \text{if } a \in \tilde{\lambda}, \\ r(\alpha(\tilde{\lambda}), a) = X & \text{if } a \notin \tilde{\lambda}. \end{array}$$

Since $\tilde{\lambda}$ is a prefilter it follows that $\pi(\alpha(\tilde{\lambda})) = \tilde{\lambda}$.

We define an approximation structure on X associated with $\tilde{\Gamma}$ by:

$$(17) \qquad (x, \alpha) \in \mathscr{A}_{\tilde{\Gamma}} \Longleftrightarrow \exists \tilde{\lambda} \in \tilde{\Gamma} \quad \tilde{\lambda} \cap \dot{x} \in \tilde{\Gamma} \quad \pi(\alpha) = \tilde{\lambda}.$$

For an approximation structure $\mathscr{A}$ on X we set

$$\rho(\mathscr{A}) = \{ \alpha \in \mathscr{R}(A, X) \ : \ \exists x \in X \, (x, \alpha) \in \mathscr{A} \}.$$

Let $\mathscr{A}$ be an approximation structure on X such that

$$(18) \quad \alpha \in \rho(\mathscr{A}) ; \ \mu, \nu \in \Pi(X) \, \pi(\alpha) = \mu \cup \nu \Rightarrow \exists \beta \in \rho(\mathscr{A}) \, (\pi(\beta) = \mu) \vee (\pi(\beta) = \nu);$$

$$(19) \quad \alpha \in \rho(\mathscr{A}) \, \pi(\alpha) \subset \mu \in \Pi(X) \Rightarrow \exists \beta \in \rho(\mathscr{A}) \, \pi(\beta) = \mu ;$$

$$(20) \quad \forall x \in X \, \exists \alpha \in \rho(\mathscr{A}) \, \pi(\alpha) = \dot{x} ;$$

$$(21) \quad \forall \alpha \in \mathscr{A}[x] \, \exists \beta \in \mathscr{A}[x] \, \pi(\alpha) \cap \dot{x} = \pi(\beta).$$

It is immediate that the collection $\{ \pi(\alpha) : \alpha \in \rho(\mathscr{A}) \}$ which satisfies (18), (19), (20), (21) is a localized π-merotopy on X.

PROPOSITION 5. *Let* $\mathscr{A}$ *be an approximation structure on* X *such that are the conditions* (18), (19), (20) (21) *fulfilled and*

$$(22) \qquad \forall \alpha \in \mathscr{R}(A, X) : \exists \beta \in \rho(\mathscr{A}) \, \pi(\beta) = \pi(\alpha) \cap \dot{x} \Rightarrow \alpha \in \mathscr{A}[x].$$

Let

$$(23) \qquad \tilde{\Gamma}_{\mathscr{A}} = \{ \pi(\alpha) : \alpha \in \rho(\mathscr{A}) \}$$

and $\mathscr{B}_{\tilde{\Gamma}_{\mathscr{A}}}$ *be constructed by* (17). *Then (according to Definition* 2) $\mathscr{A} \sim \mathscr{B}_{\Gamma_{\mathscr{A}}}$.

Proof. Let $\alpha \in \mathscr{A}[x]$. By (23) $\pi(\alpha) \subset \tilde{\Gamma}_{\mathscr{A}}$ and by (21) $\pi(\alpha)$ is localized in x. Then there exists $\beta \in \mathscr{B}_{\Gamma_{\mathscr{A}}}[x]$ such that $\pi(\alpha) = \pi(\beta)$. Thus $\mathscr{B}_{\Gamma_{\mathscr{A}}} < \mathscr{A}$.

Let $\beta \in \mathscr{B}_{\Gamma_{\mathscr{A}}}[x]$, i.e. there exists $\tilde{\lambda} \in \tilde{\Gamma}_{\mathscr{A}}, \tilde{\lambda} \cap \dot{x} \in \tilde{\Gamma}_{\mathscr{A}}$, therefore $\exists \alpha \in \rho(\mathscr{A})$ $\pi(\alpha) = \tilde{\lambda}$ and $\exists \beta \in \rho(\mathscr{A}) \, \pi(\beta) = \pi(\alpha) \cap x$. Hence by (22) $\alpha \in \mathscr{A}[x]$ and $\mathscr{A} \prec \mathscr{B}_{\Gamma_{\mathscr{A}}}$.

It follows immediately the

PROPOSITION 6. *Let* $\tilde{\Gamma}$ *be a localized* π*-merotopy on* X, $\mathcal{A}_{\tilde{\Gamma}}$ *be the approximation structure defined by* (17), *and* $\Gamma_{\mathcal{A}_{\tilde{\Gamma}}}$ *be defined by* (23). *Then* $\tilde{\Gamma} = \Gamma_{\mathcal{A}_{\tilde{\Gamma}}}$.

From Propositions 5 and 6 one obtains

PROPOSITION 7. *The relations* (17)–(23) *establish an one-to-one correspondence between the set of all localized* π*-merotopies on* X *and the set of all equivalence classes of approximation structures on* X *satisfying* (18), (19), (20), (21).

Let $\tilde{\Gamma}$ be a π-merotopy on X such that there exists a π-micromeric $\tilde{\lambda}$ with the property: $\forall x \in X\ \tilde{\lambda} \cap \dot{x} \in \tilde{\Gamma}$. We consider the localized extension of X, denoted by $\overline{X}$, $\overline{X} = X \cup \{x_\infty\}$ such that, if $\tilde{\Gamma}$ is not localized on X, we define on $\overline{X}$ the merotopy:

$$\tilde{\Gamma}^* = \{\tilde{\lambda}^* : \tilde{\lambda}^* = \{M \cup \{x_\infty\}\ M \in \tilde{\lambda}\}\ \tilde{\lambda} \in \tilde{\Gamma}\}.$$

Obviously: $\dot{x}_\infty \in \tilde{\Gamma}^*$ because $\mathcal{P}(X) \in \tilde{\Gamma}$ and $\mathcal{P}(X)^* = \dot{x}_\infty$.

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