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On a Kählerian space with recurrent holomorphic projective curvature tensor

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Geometria differenziale. — *On a Kählerian space with recurrent holomorphic projective curvature tensor.* Nota [(*)] di S. S. Singh, presentata dal Socio E. Bompiani.

Riassunto. — In un lavoro precedente [2] [(1)] l'Autore ha discusso le proprietà di uno spazio proiettivo Kähleriano simmetrico e/o ricorrente. Qui sono date ulteriori proprietà degli spazi proiettivi Kähleriani ricorrenti.

1. Introduction

An $n\,(=2m)$ dimensional Kählerian space K^n is a Riemannian space which admits a tensor field $\varphi_\lambda^{\;k}$ satisfying

$$\varphi_\alpha^{\;\lambda}\,\varphi_\mu^{\;\alpha} = -\,\delta_\mu^{\;\lambda}\,, \tag{1.1}$$

(2)

$$\varphi_{\lambda\mu} = -\,\varphi_{\mu\lambda} \quad , \quad (\varphi_{\lambda\mu} = g_{\mu\alpha}\,\varphi_\lambda^{\;\alpha}) \tag{1.2}$$

and

$$\varphi_\lambda^{\;\mu}{}_{,k} = 0\,, \tag{1.3}$$

where the comma followed by an index denotes the operator of covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

The holomorphically projective curvature tensor is given by

$$P_{\lambda\mu\nu}^{\;\;\;\;\;k} = R_{\lambda\mu\nu}^{\;\;\;\;\;k} + \frac{1}{n+2}\,(R_{\lambda\nu}\,\delta_\mu^{\;k} - R_{\mu\nu}\,\delta_\lambda^{\;k} + S_{\lambda\nu}\,\varphi_\mu^{\;k} - S_{\mu\nu}\,\varphi_\lambda^{\;k} + 2\,S_{\lambda\mu}\,\varphi_\nu^{\;k})\,, \tag{1.4}$$

where $R_{\lambda\mu\nu}^{\;\;\;\;\;k}$ and $R_{\lambda\mu} = R_{\alpha\lambda\mu}^{\;\;\;\;\;\alpha}$ are the Riemannian curvature tensor and the Ricci tensor respectively. The tensor $S_{\mu\nu}$ which is defined by

$$S_{\mu\nu} = \varphi_\mu^{\;\alpha}\,R_{\alpha\nu} \tag{1.5}$$

satisfies

$$S_{\mu\nu} = -\,S_{\nu\mu}\,. \tag{1.6}$$

2. Kählerian projective recurrent space

Definition 2.1. A Kählerian space satisfying

$$P_{\lambda\mu\nu}^{\;\;\;\;\;k}{}_{,\varepsilon} = a_\varepsilon\,P_{\lambda\mu\nu}^{\;\;\;\;\;k} \tag{2.1}$$

for some non-zero vector a_ε, has been called a Kähler space with recurrent holomorphic projective curvature.

(*) Pervenuta all'Accademia il 26 settembre 1973.

(1) The numbers in square brackets refer to the references given at the end of the paper.

(2) As to the notations we follow K. B. Lal and S. S. Singh [4] and S. S. Singh [2, 3].

DEFINITION 2.2. A Kähler space is called a space of constant holomorphic sectional curvature if the tensor $U_{\lambda\mu\nu}{}^{k}$ defined by

$$(2.2)\quad U_{\lambda\mu\nu}{}^{k} = R_{\lambda\mu\nu}{}^{k} + \frac{R}{n(n+2)}(g_{\lambda\nu}\,\delta_{\mu}^{k} - g_{\mu\nu}\,\delta_{\lambda}^{k} + \varphi_{\lambda\nu}\,\varphi_{\mu}{}^{k} - \varphi_{\mu\nu}\,\varphi_{\lambda}{}^{k} + 2\,\varphi_{\lambda\mu}\,\varphi_{\nu}{}^{k})$$

vanishes identically.

We shall prove the following theorems:

THEOREM 2.1. *If a Kählerian projective recurrent space is an Einstein one, then it reduces to a space of constant holomorphic sectional curvature or the recurrence vector of the space is null.*

Proof. An Einstein space is characterized by

$$(2.3)\quad R_{\lambda\mu} = R g_{\lambda\mu}/n\,.$$

Using (1.2) and (2.2) in (1.5) we have

$$(2.4)\quad S_{\lambda\nu} = R\varphi_{\lambda\nu}/n.$$

Therefore, in an Einstein space the holomorphically projective curvature tensor reduces to

$$(2.5)\quad P_{\lambda\mu\nu}{}^{k} = R_{\lambda\mu\nu}{}^{k} + \frac{R}{n(n+2)}(g_{\lambda\nu}\,\delta_{\mu}^{k} - g_{\mu\nu}\,\delta_{\lambda}^{k} + \varphi_{\lambda\nu}\,\varphi_{\mu}{}^{k} - \varphi_{\mu\nu}\,\varphi_{\lambda}{}^{k} + 2\,\varphi_{\lambda\mu}\,\varphi_{\nu}{}^{k})\,.$$

Now, as the scalar curvature is constant, we have

$$(2.6)\quad P_{\lambda\mu\nu}{}^{k}{}_{,\varepsilon} = R_{\lambda\mu\nu}{}^{k}{}_{,\varepsilon}\,.$$

In view of (2.1) and (2.6), the identity of Bianchi gives

$$(2.7)\quad a_{\mu}\,P_{\lambda\nu\varepsilon}{}^{k} + a_{\nu}\,P_{\lambda\varepsilon\mu}{}^{k} + a_{\varepsilon}\,P_{\lambda\mu\nu}{}^{k} = 0\,.$$

Contracting μ and k and using $P_{\alpha\mu\nu}{}^{\alpha} = P_{\nu\mu\alpha}{}^{\alpha} = 0$, we have

$$a_{k}\,P_{\lambda\mu\nu}{}^{k} = 0\,,$$

which means

$$(2.8)\quad g_{k\eta}\,a^{\eta}\,P_{\lambda\mu\nu}{}^{k} = a^{\eta}\,P_{\lambda\mu\nu\eta} = 0.$$

Transvecting (2.7) with a^{ε} and using (2.8) we have

$$a^{\varepsilon}\,a_{\varepsilon}\,P_{\lambda\mu\nu}{}^{k} = 0.$$

Hence, if the space is projectively non-flat, we obtain $a^{\varepsilon}\,a_{\varepsilon} = 0$, which means that a_{ε} is a null vector. This completes the proof.

Further, in a Kählerian Einstein space, the Bochner curvature tensor $K_{\lambda\mu\nu}{}^{k}$ defined by

$$(2.9)\quad \begin{aligned} K_{\lambda\mu\nu}{}^{k} = {} & R_{\lambda\mu\nu}{}^{k} + \frac{1}{n+4}\{R_{\lambda\nu}\,\delta_{\mu}^{k} - R_{\mu\nu}\,\delta_{\lambda}^{k} + g_{\lambda\nu}\,R_{\mu}{}^{k} - \\ & - g_{\mu\nu}\,R_{\lambda}{}^{k} + S_{\lambda\nu}\,\varphi_{\mu}{}^{k} - S_{\mu\nu}\,\varphi_{\lambda}{}^{k} + \varphi_{\lambda\nu}\,S_{\mu}{}^{k} - \\ & - \varphi_{\mu\nu}\,S_{\lambda}{}^{k} + 2\,S_{\lambda\mu}\,\varphi_{\nu}{}^{k} + 2\,\varphi_{\lambda\mu}\,S_{\nu}{}^{k}\} - \\ & - \frac{R}{(n+2)(n+4)}\{g_{\lambda\nu}\,\delta_{\mu}^{k} - g_{\mu\nu}\,\delta_{\lambda}^{k} + \varphi_{\lambda\nu}\,\varphi_{\mu}{}^{k} - \varphi_{\mu\nu}\,\varphi_{\lambda}{}^{k} + 2\,\varphi_{\lambda\mu}\,\varphi_{\nu}{}^{k}\} \end{aligned}$$

coincides with $P_{\lambda\mu\nu}{}^{k}$ given by (2.5), we therefore have the following theorem:

THEOREM 2.2. *A Kählerian Einstein space with recurrent Bochner curvature tensor reduces to a space of constant curvature or the recurrence vector* a_ε *of the space is null.*

THEOREM 2.3. *In a Kählerian projective recurrent space the recurrence vector* K_ε *is a gradient vector.*

Proof. From (1.4) we have the following equation

$$
\begin{aligned}
(2.10)\qquad & P_{\lambda\mu\nu\omega,\varepsilon,\eta} - P_{\lambda\mu\nu\omega,\eta,\varepsilon} + P_{\nu\omega\varepsilon\eta,\lambda,\mu} - P_{\nu\omega\varepsilon\eta,\mu,\lambda} + P_{\varepsilon\eta\lambda\mu,\nu,\omega} - \\
& - P_{\varepsilon\eta\lambda\mu,\omega,\nu} = [R_{\lambda\mu\nu\omega,\varepsilon,\eta} - R_{\lambda\mu\nu\omega,\eta,\varepsilon} + R_{\nu\omega\varepsilon\eta,\lambda,\mu} - R_{\nu\omega\varepsilon\eta,\mu,\lambda} + \\
& + R_{\varepsilon\eta\lambda\mu,\nu,\omega} - R_{\varepsilon\eta\lambda\mu,\omega,\nu}] + [\{g_{\mu\omega}(R_{\lambda\nu,\varepsilon,\eta} - R_{\lambda\nu,\eta,\varepsilon}) - \\
& - g_{\lambda\omega}(R_{\mu\nu,\varepsilon,\eta} - R_{\mu\nu,\eta,\varepsilon}) + \varphi_{\mu\omega}(S_{\lambda\nu,\varepsilon,\eta} - S_{\lambda\nu,\eta,\varepsilon}) - \\
& - \varphi_{\lambda\omega}(S_{\mu\nu,\varepsilon,\eta} - S_{\mu\nu,\eta,\varepsilon}) + 2\,\varphi_{\nu\omega}(S_{\lambda\mu,\varepsilon,\eta} - S_{\lambda\mu,\eta,\varepsilon})\} + \\
& + \{g_{\omega\eta}(R_{\nu\varepsilon,\lambda,\mu} - R_{\nu\varepsilon,\mu,\lambda}) - g_{\nu\eta}(R_{\omega\varepsilon,\lambda,\mu} - S_{\omega\varepsilon,\mu,\lambda}) + \\
& + \varphi_{\omega\eta}(S_{\nu\varepsilon,\lambda,\mu} - S_{\nu\varepsilon,\mu,\lambda}) - \varphi_{\nu\eta}(S_{\omega\varepsilon,\lambda,\mu} - S_{\omega\varepsilon,\mu,\lambda}) + \\
& + 2\,\varphi_{\varepsilon\eta}(S_{\nu\omega,\lambda,\mu} - S_{\nu\omega,\mu,\lambda})\} + \{g_{\eta\mu}(R_{\varepsilon\lambda,\nu,\omega} - R_{\varepsilon\lambda,\omega,\nu}) - \\
& - g_{\varepsilon\mu}(R_{\eta\lambda,\nu,\omega} - R_{\eta\lambda,\omega,\nu}) + \varphi_{\eta\mu}(S_{\varepsilon\lambda,\nu,\omega} - S_{\varepsilon\lambda,\omega,\nu}) - \\
& - \varphi_{\varepsilon\mu}(S_{\eta\lambda,\nu,\omega} - S_{\eta\lambda,\omega,\nu}) + 2\,\varphi_{\lambda\mu}(S_{\varepsilon\eta,\nu,\omega} - S_{\varepsilon\eta,\omega,\nu})\}]/(n+2)\,.
\end{aligned}
$$

In a Kählerian projective recurrent space the relation

$$R_{\lambda\omega,\varepsilon} - K_\varepsilon R_{\lambda\omega} = \frac{1}{n}(R_{,\varepsilon} - K_\varepsilon R)\, g_{\lambda\omega}\,,$$

gives

$$(2.11)\qquad R_{\lambda\nu,\varepsilon,\eta} - R_{\lambda\nu,\eta,\varepsilon} = (n+2)\, K_{\varepsilon\eta}\, P_{\lambda\nu}/n\,,$$

where we have put

$$K_{\varepsilon\eta} = K_{\varepsilon,\eta} - K_{\eta,\varepsilon}\,.$$

Applying Ricci's identity to the first term on the right hand side of (2.10) and using

$$P_{\lambda\mu\nu\omega,\varepsilon,\eta} - P_{\lambda\mu\nu\omega,\eta,\varepsilon} = K_{\varepsilon\eta}\, P_{\lambda\mu\nu\omega}$$

we have

$$
\begin{aligned}
& K_{\varepsilon\eta}\,[\{P_{\lambda\mu\nu\omega} - g_{\mu\omega} P_{\lambda\nu} - g_{\lambda\omega} P_{\mu\nu})/n - (\varphi_{\mu\omega}\,\varphi_\lambda{}^\alpha P_{\alpha\nu} - \varphi_{\lambda\omega}\,\varphi_\mu{}^\alpha P_{\alpha\nu})/n - \\
& - 2\,(\varphi_{\nu\omega}\,\varphi_\lambda{}^\alpha P_{\alpha\mu})/n\} + K_{\lambda\mu}\,\{P_{\nu\omega\varepsilon\eta} - (g_{\omega\eta} P_{\nu\varepsilon} - g_{\nu\eta} P_{\omega\varepsilon})/n - \\
& - (\varphi_{\omega\eta}\,\varphi_\nu{}^\alpha P_{\alpha\varepsilon} - \varphi_{\nu\eta}\,\varphi_\omega{}^\alpha P_{\alpha\varepsilon})/n - 2\,\varphi_{\varepsilon\eta}\,\varphi_\nu{}^\alpha P_{\alpha\omega}/n\} + \\
& + K_{\nu\omega}\,\{P_{\varepsilon\eta\lambda\mu} - (g_{\eta\mu} P_{\varepsilon\lambda} - g_{\varepsilon\mu} P_{\eta\lambda})/n - (\varphi_{\eta\mu}\,\varphi_\varepsilon{}^\alpha P_{\alpha\lambda} - \varphi_{\varepsilon\mu}\,\varphi_\eta{}^\alpha P_{\alpha\lambda})/n - \\
& - 2\,\varphi_{\lambda\mu}\,\varphi_\varepsilon{}^\alpha P_{\alpha\eta}/n\}] = 0\,.
\end{aligned}
$$

Using the relation

$$P_{\lambda\omega} = P_{\lambda\mu\nu\omega}\, g^{\mu\nu} = n\,[R_{\lambda\omega} - \frac{R}{n} g_{\lambda\omega}]/(n+2)$$

and putting

$$U_{\lambda\mu\nu\omega} = R_{\lambda\mu\nu\omega} + \frac{R}{n\,(n+2)}\,(g_{\lambda\nu}\, g_{\mu\omega} - g_{\mu\nu}\, g_{\lambda\omega} +$$

$$+ \varphi_{\lambda\nu}\, \varphi_{\mu\omega} - \varphi_{\mu\nu}\, \varphi_{\lambda\omega} + 2\, \varphi_{\lambda\mu}\, \varphi_{\nu\omega})\,,$$

the above equation reduces to

$$\text{(2.12)} \qquad K_{\varepsilon\eta}\, U_{\lambda\mu\nu\omega} + K_{\lambda\mu}\, U_{\nu\omega\varepsilon\eta} + K_{\nu\omega}\, U_{\varepsilon\eta\lambda\mu} = 0\,.$$

Evidently the following equation holds:

$$\text{(2.13)} \qquad U_{\lambda\mu\nu\omega} = U_{\nu\omega\lambda\mu}\,.$$

Now, we shall introduce Walker's Lemma [3].

LEMMA. *If $a_{\alpha\beta}$, b_{γ} are numbers satisfying*

$$\text{(2.14)} \qquad a_{\alpha\beta} = a_{\beta\alpha} \quad , \quad a_{\beta\gamma}\, b_{\alpha} + a_{\gamma\alpha}\, b_{\beta} + a_{\alpha\beta}\, b_{\gamma} = 0\,,$$

then either all the $a_{\alpha\beta}$ are zero or all the b_{γ} are zero.

Equations (2.12) and (2.13) are of the form (2.14) as suffixes α, β, γ being replaced by the pairs $\lambda\mu$, $\nu\omega$, $\varepsilon\eta$. Since the space is not of constant curvature, $U_{\lambda\mu\nu\omega}$ is not all zero. Therefore it follows that $K_{\varepsilon\eta} = 0$. This implies that $K_{\varepsilon,\eta} = K_{\eta,\varepsilon}$, which is the condition for K_{ε} to be a gradient vector. This completes the proof.

THEOREM 2.4. *A Kählerian projective recurrent space is Kählerian recurrent or an Einstein space whose recurrence vector is null.*

Proof. Putting

$$\text{(2.15)} \qquad P_{\lambda\omega} = P_{\lambda\mu\nu\omega}\, g^{\mu\nu}$$

we get

$$\text{(2.16)} \qquad P_{\lambda\omega} = n\left(R_{\lambda\omega} - \frac{R}{n} g_{\lambda\omega}\right)/(n+2)\,,$$

which gives $P_{\lambda\omega,\varepsilon} = K_{\varepsilon}\, P_{\lambda\omega}$.

Multiplication by $\varphi_{\varepsilon}^{\ \lambda}$ reduces (2.16) to

$$\text{(2.17)} \qquad P_{\lambda\omega}\, \varphi_{\varepsilon}^{\ \lambda} = n\left[S_{\varepsilon\omega} - \frac{R}{n} \varphi_{\varepsilon\omega}\right]/(n+2)\,.$$

Taking into consideration that K_{ε} is a gradient vector we get

$$P_{\lambda\omega,\varepsilon,\eta} - P_{\lambda\omega,\eta,\varepsilon} = 0\,.$$

Applying Ricci's identity to the last equation, we have

$$P_{a\omega}\, R_{\eta\varepsilon\lambda}^{\ \ \ \ a} + P_{\lambda a}\, R_{\eta\varepsilon\omega}^{\ \ \ \ a} = 0\,.$$

The above equation implies

$$P_{a\omega}\left[P_{\eta\varepsilon\lambda}{}^{a}-\frac{1}{n+2}(R_{\eta\lambda}\,\delta_{\varepsilon}{}^{a}-R_{\varepsilon\lambda}\,\delta_{\eta}{}^{a}+S_{\eta\lambda}\,\varphi_{\varepsilon}{}^{a}-S_{\varepsilon\lambda}\,\varphi_{\eta}{}^{a}+2\,S_{\eta\varepsilon}\,\varphi_{\lambda}{}^{a})\right]+$$

$$+P_{\lambda a}\left[P_{\eta\varepsilon\omega}{}^{a}-\frac{1}{n+2}(R_{\eta\omega}\,\delta_{\varepsilon}{}^{a}-R_{\varepsilon\omega}\,\delta_{\eta}{}^{a}+S_{\eta\omega}\,\varphi_{\varepsilon}{}^{a}-S_{\varepsilon\omega}\,\varphi_{\eta}{}^{a}+2\,S_{\eta\varepsilon}\,\varphi_{\omega}{}^{a})\right]=0\,.$$

Substituting $R_{\lambda\omega}=\{(n+2)\,P_{\lambda\omega}+Rg_{\lambda\omega}\}/n$ into the above equation we have

$$P_{a\omega}\,P_{\eta\varepsilon\lambda}{}^{a}+P_{\lambda a}\,P_{\eta\varepsilon\omega}{}^{a}-\frac{R}{n(n+2)}\,\cdot \tag{2.18}$$

$$\cdot\,[(g_{\eta\lambda}\,P_{\varepsilon\omega}-g_{\varepsilon\lambda}\,P_{\eta\omega}+\varphi_{\eta\lambda}\,\varphi_{\varepsilon}{}^{a}\,P_{a\omega}-\varphi_{\varepsilon\lambda}\,\varphi_{\eta}{}^{a}\,P_{a\omega}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\lambda}{}^{a}\,P_{a\omega})+$$

$$+(g_{\eta\omega}\,P_{\lambda\varepsilon}-g_{\varepsilon\omega}\,P_{\lambda\eta}+\varphi_{\eta\omega}\,\varphi_{\varepsilon}{}^{a}\,P_{\lambda a}-\varphi_{\varepsilon\omega}\,\varphi_{\eta}{}^{a}\,P_{\lambda a}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\omega}{}^{a}\,P_{\lambda a})]=0\,.$$

Differentiating (2.18) covariantly, we obtain

$$2\,K_{\sigma}\,P_{a\omega}\,P_{\eta\varepsilon\lambda}{}^{a}+2\,K_{\sigma}\,P_{\lambda a}\,P_{\eta\varepsilon\omega}{}^{a}-\frac{(R_{,\sigma}+RK_{,\sigma})}{n(n+2)}\,\cdot \tag{2.19}$$

$$\cdot\,[(g_{\eta\lambda}\,P_{\varepsilon\omega}-g_{\varepsilon\lambda}\,P_{\eta\omega}+\varphi_{\eta\lambda}\,\varphi_{\varepsilon}^{a}\,P_{a\omega}-\varphi_{\varepsilon\lambda}\,\varphi_{\eta}^{a}\,P_{a\omega}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\lambda}^{a}\,P_{a\omega})+$$

$$+(g_{\eta\omega}\,P_{\lambda\varepsilon}-g_{\varepsilon\omega}\,P_{\lambda\eta}+\varphi_{\eta\omega}\,\varphi_{\varepsilon}^{a}\,P_{\lambda a}-\varphi_{\varepsilon\omega}\,\varphi_{\eta}^{a}\,P_{\lambda a}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\omega}{}^{a}\,P_{\lambda a})]=0\,.$$

Multiplying (2.18) by $2\,K_{\sigma}$ and subtracting from (2.19), we have

$$\frac{(R_{,\sigma}-RK_{\sigma})}{n(n+2)}[(g_{\eta\lambda}\,P_{\varepsilon\omega}-g_{\varepsilon\lambda}\,P_{\eta\omega}+\varphi_{\eta\lambda}\,\varphi_{\varepsilon}^{a}\,P_{a\omega}- \tag{2.20}$$

$$-\varphi_{\varepsilon\lambda}\,\varphi_{\eta}^{a}\,P_{a\omega}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\lambda}^{a}\,P_{a\omega})+(g_{\eta\omega}\,P_{\lambda\varepsilon}-g_{\varepsilon\omega}\,P_{\lambda\eta}+$$

$$+\varphi_{\eta\omega}\,\varphi_{\varepsilon}^{a}\,P_{\lambda a}-\varphi_{\varepsilon\omega}\,\varphi_{\eta}^{a}\,P_{\lambda a}+2\,\varphi_{\eta\varepsilon}\,\varphi_{\omega}{}^{a}\,P_{\lambda a})]=0\,.$$

Transvecting (2.20) with $g^{\eta\lambda}$ and after some simplification we obtain

$$\frac{(R_{,\sigma}-RK_{\sigma})}{n(n+2)}\,P_{\varepsilon\omega}=0$$

where we have used $P_{\lambda\eta}\,g^{\eta\lambda}=0$. It follows that $R_{,\sigma}=RK_{\sigma}$ or $P_{\varepsilon\omega}=0$. If the former equation holds, then the Kählerian projective recurrent space is Kählerian recurrent (Theorem 3.3, [2]). If the latter equation holds, then the Kählerian projective recurrent space is an Einstein one. From Theorem 2.1 the recurrence vector is null. This completes the proof.

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