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GLEB WATAGHIN

On the "cutoff method" and on the multiple production in high energy collisions

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SEZIONE II

(Fisica, chimica, geologia, paleontologia e mineralogia)

Fisica teorica. — *On the "cutoff method" and on the multiple production in high energy collisions.* Nota [*] del Socio GLEB WATAGHIN.

RIASSUNTO. — Nella prima parte si dimostra, applicando il metodo del «taglio relativistico» agli urti di particelle qualsiasi di elevata energia baricentrica, che questi urti devono dar luogo necessariamente al fenomeno della produzione multipla, d'accordo coll'esperienza. Nella seconda parte si citano alcune parti del rapporto presentato dall'A. al simposio sui raggi cosmici di Rio de Janeiro nel 1941, in cui tale fenomeno è predetto e descritto.

The purpose of this Note is to point out some aspects of an attempt to eliminate the ultraviolet divergencies in a quantized field theory, proposed in earlier papers [1] and sometimes called "the cutoff method". Let us consider e.g. the energy losses in successive collisions of a high energy cosmic ray proton which are accompanied by a multiple production. The elastic collisions of the incident proton with a nucleon can be considered in the c.m. frame and the cutoff reduces the probability of such collisions, for $s^{1/2}$ higher than a critical value of the cutoff ($\sim 2 m_p c^2$). Then the unitarity requires that new channels must arise. Observation shows, in accord with the relativistic cutoff method, that the mass spectrum of stable particles has an upper limit (the proton mass). Therefore in the new channels particles must have low momenta and low masses. Due to the conservation of the energy the outgoing particles must be many. This conclusion does not depend on the nature of the colliding particles, if the cutoff is universal. The present day experimental knowledge confirms the dominance of the multiple production at high values of s.

In cases of multiple production the observation shows some time the existence of groups of associated outgoing particles (e.g. pions of a "fireball" or "muon bundles") correlated by a common small domain in which they are created. In such cases a new reference system (a c.m. system of the "fireball") is needed to describe, in a simple way, the symmetry and the dynamics of the process.

The appearance of these groups can be due to the formation of excited unstable states (resonances) or of an intermediate boson or, in general case, of a cascade of clusters of particles which decay.

Let us recall the definition of the above mentioned cutoffs. Let P_μ be the total 4-momentum of the incident particles and P'_μ the total 4-momentum

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(1) «Zs. cf.Phys.», *88*, 92 (1934), «La Ricerca Scientifica», S. II, anno 1936, v. 11, n. 9, pag. 517 «Nuovo Cimento», VII, p. 166, 1950.

of the group of associated particles. One can choose in a Lorentz reference frame 4 unit vectors $u_\mu^{(1)}\, u_\mu^{(2)}\, u_\mu^{(3)}\, u_\mu^{(4)}$ belonging to the momentum space in the following way: $u_\mu^{(4)}$ is a time-like unit vector parallel to the total 4–momentum P_μ $\left(u_\mu^{(4)} = \frac{P_\mu}{(P_\mu p^\mu)^{1/2}}\right)$ $u^{(2)}\, u^{(3)}\, u^{(4)}$ are space-like unit vectors forming with $u^{(4)}$ an orthogonal set. In the case of two colliding particles one can choose $u_\mu^{(1)}$ parallel to their relative velocity in the c.m. frame. A reference frame in space-time with axis parallel to the $u^{(a)}$ can be defined with the origin in a point of the interaction domain. Indicating by I_a $(a = 1, 2, 3, 4)$ the projections of an arbitrary vector P_μ on $u^{(a)}$, multiplied by a universal constant l: $I_a = l\,(p^\mu u_\mu^{(a)})$ we define in the momentum space the cutoff operators G^+ associated with a created particle e.g. as follows:

$$G^+ = G^+(u', p) = \frac{1}{(1 + I_1^2 + I_2^2 + I_3^2)^2}\,\frac{1}{1 + iI_4}\,. \tag{1}$$

and the operators associated with the annihilation operator:

$$G^- = G^-(u, p) = \frac{1}{(I_1^2 + I_2^2 + I_3^2)^2}\,\frac{1}{1 - iI_4}\,. \tag{2}$$

Here: $l = \frac{\hbar}{2 m_p c}$ where m_p is the proton mass.

Each form-factor in the internal lines of the scattering matrix T_{fi} written in the interaction representation is associated with a cutoff $G_{\text{out}}^+(q, u')\, G_{\text{in}}^-(q, u)$ which is spherically symmetric with respect to the space components, but acts separately on the time- or energy-component and depends on the vacuum states defined resp. by the 4–vectors u and u' where u' refers to P'_μ.

Recent experiments on fireballs, or groups of associated outgoing particles, show an agreement with the following rule: in the c.m. frame of the fireball one must apply the spherically symmetric cutoff (1) to each of the outgoing clusters of particles. The observation shows that these outgoing particles have, on average, momenta which are $\lesssim m_p c$. The observed multiplicity of particles seems to be proportional to the $E_{\text{cm}}^{1/2} = s^{1/2}$. This multiplicity is in agreement with the assumption that the collision is a cascade process: first some unstable particles are formed and then they decay. At the end of this Note a quotation of a part of an early Note (1941) is given, in which a first calculation of the multiplicity, based on the introduction of relativistic cutoffs and on spherical symmetry in the c.m. frame, is made.

The association of the operators G^+ and G^- to the operators of creation and annihilation of particles gives rise to an important modification of the commutation and anti-commutation relations of the second quantization, in which the factor G^+G^- appears [1] (1936). Similar modification must be introduced in the commutation relations of the "first quantization".

In order to consider the Dirac spinors and the corresponding equations of an electron, one needs to introduce (in the theory of general relativity) a "four legs" or "tetradic" (comoving) reference frame. The above formalism can be adopted also in this case. The fundamental interaction

Lagrangian or Hamitonian can be refered to the interactions of " quarks " or of some more general " infra-structure " particles. The free particle propagation (of ingoing or outgoing particles) remains uninfluenced by these interactions, and there is no possibility of describing the propagation inside the interaction domains, where laws of probability must be applied.

The problems of the selfenergy can be solved with a similar method, applying relativistic cutoff operators. Also in this case the interaction is confined to a small domain of the c.m. reference system of the particle and the vacuum state is defined by means of 4–vectors $u_{\mu}^{(a)}$ to the same frame.

QUOTATION OF SOME PARTS OF THE NOTE: "ON THE PRODUCTION OF GROUPS OF MESOTRONS BY HIGH ENERGY COLLISIONS" (2)

The purpose of the following remarks is to show some very simple features of the distribution of energy and momenta in a group of particles created in a high energy collision. The expected distribution of probabilities can be derived from a straightforward application of the Lorentz transformation to the collision problem.

Starting from the assumption of relativistic invariance and of the validity of conservation laws we shall confine ourselves to the case of collision of a primary cosmic ray particle (a proton) with a nucleus at rest in the high atmosphere, and we shall admit the correctness of some ideas about these collisions made plausible by experimental data and by some theoretical considerations of Bohr, Heisenberg and our own [1].

We assume that the energy E of the primary proton is absorbed in one or two collision processes, so that the energy loss ΔE during one collision is of the order of magnitude of E. The experimental evidence supports the idea that the primary mesotron producing particles are rapidly absorbed in a layer of atmosphere equivalent in mass to few cm. Hg. This rapid absorption can be understood by admitting a cross-section $\sim 10^{-26}\,\text{cm}^2$ for a collision with an energy loss $(\Delta E/E) \sim 1$.

Another explanation of the rapid absorption can be derived from the assumption that the primary particle suffers a great number of collisions with small relative energy losses. This assumption requires a much larger cross-section and thus seems to be less plausible.

Let us consider for definiteness the collision of a primary high energy proton with a proton or a neutron at rest. We assume that in the reference frame Σ_0 of the center of masses of two colliding particles almost all the relative energy of colliding particles is transformed into the energy of the created particles: mesotrons, electrons and photons.

(2) Symposium on cosmic rays, Brazilian Academy of Sciences, Rio de Janeiro 1941, published in 1943.

Obviously one must expect that the number and nature of particles produced in these collisions can present many different cases. In this paper we limit ourselves to study the case in which only mesotrons are produced. A theoretical examination of the probability of various possible distributions of momenta and energies between n created particles (where n is also variable) leads us to establish the following expected features of the high energy collision. We find that in the most probable distribution the created mesotrons have, in reference frame Σ_0 an energy $\sim 3\,\mu c^2$ (where μ is the rest mass of a mesotron) and the two "nucleons" (protons or neutrons) have after the collision an energy $\sim 1{,}1\,\mathrm{M}c^2 \sim 13\,\mu c^2 \cdot [\mathrm{M}c^2 = 9{,}4 \times 10^9$ e.v.].

In the reference frame of the center of masses Σ_0 the energy $\Delta \mathrm{E}_0$ lost in the collision is equally divided between n mesotrons so that:

$$\Delta \mathrm{E}_0 = \mathrm{E}_{01} + \mathrm{E}_{02} - 2{,}2\,\mathrm{M}c^2 \sim n\,(3\,\mu c^2) \tag{1}$$

where $\mathrm{E}_{01} = \mathrm{E}_{02}$ are the energies of the nucleons before collision. From the symmetry considerations, the distribution of probabilities in the momentum space of the created particles can be represented as a sum of terms corresponding to spherical harmonics of different orders. Whatever will be the group of terms of this representation involved in the description of the various possible types of collision, the principal features of the high energy collision and data reproduced in Table I are independent of it.

Transforming the distribution given in Σ_0 to the terrestrial reference frame Σ in which one of the nucleons is initially at rest, ($p_2 = 0$) one has:

$$\begin{aligned} p_2 &= 0 = \gamma\left(p_{02} - \beta\,\frac{\mathrm{E}_{02}}{c}\right) \\ \mathrm{E}_2 &= \mathrm{M}c^2 = \gamma\,(\mathrm{E}_{02} - \beta\,c\,\mathrm{P}_{02}) \\ \mathrm{E}_1 &= \gamma\,(\mathrm{E}_{01} + \beta\,c\,\mathrm{P}_{01}) \sim 2\,\gamma\,\mathrm{E}_{01} \end{aligned} \tag{2}$$

where $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ and $c\beta$ is the velocity of Σ_0 relative to E and where the other symbols have an obvious significance.

It follows:

$$\mathrm{E}_{01} = \mathrm{E}_{02} = \gamma\,\mathrm{M}c^2 \qquad \mathrm{E}_1 \sim 2\,\gamma^2\,\mathrm{M}c^2 \tag{3}$$

and from (I):

$$\begin{aligned} &\gamma - 1{,}1 \sim \frac{3}{2}\,n\,\frac{\mu}{\mathrm{M}} \sim \frac{n}{8} \\ &\mathrm{E}_1 \sim 2\,\mathrm{M}c^2\left(1{,}2 + \frac{n}{4} + \frac{n^2}{64}\right) \\ &\bar{\mathrm{E}}_\mu \sim \frac{1}{4}\,\gamma\,\mathrm{M}c^2 \sim \frac{1}{8\gamma}\cdot \mathrm{E}_1 \end{aligned} \tag{3'}$$

where E_1 is the energy of of the incoming primary particle and E_μ is the average energy of the created mesotrons. In the case $\gamma \gg 1$ they are emitted within a solid angle of the order of $1/\gamma^2$.

TABLE I

n	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$	Primary energy E_1	Average energy of the mesotrons $\overline{E}_\mu$	Max. of $\overline{E}_\mu$	Min. of $\overline{E}_\mu$
4 . . .	1.6	4.8×10^9	4×10^8	8×10^8	2×10^7
8 . . .	2.1	8.3×10^9	5×10^8	10^9	2.5×10^7
16 . . .	3.1	1.8×10^{10}	7×10^8	1.4×10^9	3.5×10^7
24 . . .	4.1	3.2×10^{10}	10^9	2×10^9	5×10^7
71 . . .	10	1.9×10^{11}	2.4×10^9	4.8×10^9	1.2×10^8
8×10^2 . . .	10^2	1.9×10^{13}	2.4×10^{10}	4.8×10^{10}	1.2×10^9
8×10^3 . . .	10^3	1.9×10^{15}	2.4×10^{11}	4.8×10^{11}	1.2×10^{10}
8×10^4 . . .	10^4	1.9×10^{17}	2.4×10^{12}	4.8×10^{12}	1.2×10^{11}

In Table I are indicated the numerical data for the resulting distribution, for energy range of primaries from $5\cdot10^9$ until $2\cdot10^{17}$ e.v. The fluctuations of n and of the angular distribution of mesotrons in Σ_0 can greatly affect the values of $E\mu$ for low energies E_1, but should be less important for high values of n.

The general conclusions are: for high energies ($\gamma \gg 1$) the number n and the average energy $\overline{E}_\mu$ of mesotrons are proportional to the square root of the primary energy E_1. In each collision the expected spectral distribution of created mesotrons covers a range $E_{(max} - E_{(min.}$ proportional to E_μ.

In the cases $\gamma \gg 1$ one can put $\beta = 1$ and the representation of the transformation law of impulses can be obtained in a two dimensional section as a product of a longitudinal stress with the ratio γ and a subsequent translation equal to: $\gamma \cdot 3\mu c^2$, both applied to a circle of the radius $\sqrt{8}\mu c^2$.

The wide angular distribution is of the greatest importance for low energy values of E which correspond to the greatest intensity in the primary spectrum.

Neglecting the effect of earth magnetic field and admitting an isotropic distribution of intensity outside of the atmosphere, one can say that the primaries entering within a great zenith angle give a greater contribution to the mesotron production and give rise to penetrating particles of energy $\sim 10^8$ e.v. in the vertical direction.

For values of $\gamma \gg 1$ the angular distribution is confined to a solid angle of the order of $1/\gamma^2$ and the angular distance varies as $1/\gamma$ or as $\sqrt{2\mu c^2}/\sqrt{E_1}$ (because of: $E_1 \sim 2\gamma^2\mu c^2$).

There are two main problems to solve: 1) To deduce the primary energy spectrum from the observed energy distribution of mesotrons. 2) To calculate the spectral energy distribution of mesotrons incident within a small solid angle and due to primaries of a given spectral distribution coming from all directions. Both will be object of a further publication.

As Prof. A. H. Compton observed in the discussion, another relation between the present calculation and the work of Dr. V. Wilson can be established. The value of E $2 \cdot 10^{11}$ e.v. of mesotron energy as compared with 10^{15} e.v. of a primary particle represents approximately the maximum values respectively of the energies that Volney Wilson gets for the particles that go to great depths and which N. Hilberry gets for the particles that enter the atmosphere. The depths to which these particles penetrate are of the order of 1,000 meters of water equivalent, corresponding roughly to energy losses through the ionization alone of the order of 10^{11} e.v. which is approximately the value of $\overline{E}_{\mu}$. When one goes much farther than that, almost nothing can be detected at all. At 2,000 meters of water equivalent, if one is to find a cosmic ray, one must wait for days before one particle comes. That would correspond closely to the limit of high energy of the incoming particle which is of the order of 10^{16} e.v.

In the "Appendix" to the above Note of the Brazilian Symposium of 1941, the cutoff method was applied to deduce the properties of the multiple production mentioned above.

References

[1] N. Bohr, Congress in Rome, 1931, p. 121; Solvay Congress 1933, p. 216. W. Heisenberg, «Zs. f. Phys.», *101*, 533 (1936); *110*, 251 (1938); *113*, 61 (1939); «Ann. d. Phys.», *32*, 20 (1938). G. Wataghin, «Zs. f. Phys.», *66*, 650 (1930); *88*, 92 (1934); «Nature», *142*, 393 (1938); «Comptes Rendues», *207*, 358–421 (1938); «Phys. Rev.», *56*, 1245 (1939).

[2] P. A. Pompéia, M. D. de Souza Santos and G. Wataghin, «Phys. Rev.», *57*, 61, January 1th (1940); *57*, 339 (1940); *59*, 902 (1941); «Ann. Ac. Bras. Ciências», T. XII, Setembro 1940. Other experiments on associated penetrating particles were made independently by D. K. Fromen, V. Josephson and J. C. Stearns, «Phys. Rev.», *57*, 335 (1940); L. Jánossy and P. Ingleby, «Nature», *145*, 511 (1940).

Note added during the revision of proofs.: Later L. Jánossy has studied these showers with a different technique («Proc. Roy. Soc. A.» *179*, 361 (1942); L. Jánossy and G. D. Rochester, «Nature», *150*, 633 (1942)). Also P. Auger and J. Daudin have observed associated penetrating particles in the extensive showers, «C.R.», *212*, 24 (1941); «Phys. Rev.», *61*, 549 (1942).

[3] «Phys. Rev.», *57*, 61 (1940); and P. Auger and J. Daudin, «Phys. Rev.», *61*, 549 (1942).