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## The Inner core of the Earth

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**Geofisica.** — *The Inner core of the Earth.* Nota (*) del Socio ALAN HUGH COOK.

RIASSUNTO. — È dimostrato che nuovi valori della velocità delle onde elastiche traverse nel nucleo interiore della Terra permettono di stimare i valori dell'incompressibilità e della densità al centro della Terra.

1. The idea that the liquid core of the Earth contains an inner core in which, $\alpha$, the velocity of compressional (P) elastic waves is greater than in the surrounding liquid was suggested by Miss I. Lehmann (1936) as an explanation of the fact that the amplitudes of P–waves arriving within the core shadow zone between 104° and 140° from a source are greater than that can be explained by diffraction round the boundary between the core and the mantle. If the boundary between the core and inner core is sharp, then it should be possible to detect seismic pulses that have been reflected at that boundary; the difficulty is that such pulses will generally arrive at the surface after pulses which have travelled by other paths and so will be lost in noise generated by earlier pulses. Caloi (1961) claimed first to observe pulses of compressional waves reflected at nearly normal incidence from the boundary of the inner core (*PKiKP*), so demonstrating that the boundary is indeed sharp, and now, as a result of more recent observations it is possible to construct a fairly well defined model of the inner core.

Our knowledge of the elastic properties of the interior of the Earth is derived from three sources—the times of travel of pulses travelling as body waves, the dispersion of surface waves and the periods of free oscillations of the Earth as a whole. Surface waves do not penetrate to the inner core and carry no information about it. Travel times of bodily waves that penetrate to the inner core are difficult to determine because the pulses arrive at the surface in the noise generated by earlier pulses, and the amplitudes of fundamental modes of free oscillation are very small in the inner core so that the properties of the inner core have little effect upon the periods of the fundamental modes. The effectiveness of recent observations in the study of the inner core arises from the use of arrays of seismometers to detect pulses from the inner core in the presence of noise and from the identification of overtones (with large amplitudes in the inner core) in the spectra of free oscillations.

2. In 1970, Engdahl, Flinn and Romney announced at the annual meeting of the Seismological Society of America that they had detected pulses reflected at near normal incidence from the boundary of the inner core in the records of the Large Aperture Seismic Array (LASA) in Montana. Bolt and

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Qamar (1970) compared the amplitude of the pulse reflected from the inner core (*PKIKP*) with that reflected from the core (*PcP*); they estimated the reflexion coefficient at the core-mantle boundary from the known elastic properties of the core and mantle and so derived an estimate of the reflexion coefficient at the boundary of the inner core. The reflexion coefficient for *PcP* is proportional to $|\rho_{\text{core}}\,\alpha_{\text{core}} - \rho_{\text{mantle}}\,\alpha_{\text{mantle}}|$. It is quite sensitive to the values of density ($\rho$) and $\alpha$ adopted for the core and the mantle, and so Bolt and Qamar express their result as a minimum value for the ratio $\rho_c/\rho_i$ where $\rho_i$ is the density of the inner core and $\rho_c$ that of the core just outside. Their result is insensitive to the value of the shear wave velocity in the inner core and to the attenuation in the core. Bolt and Qamar concluded that $\rho_c/\rho_i$ is not less than 0.875. They also find that the radius of the inner core is 1216 km. Because $\rho_c/\rho_i$ is insensitive to the value of $\beta$, the velocity of shear waves in the inner core, the observation of the reflected pulse says nothing about whether the inner core is solid.

The first direct evidence that the modulus of rigidity of the inner core is not negligibly small came from the periods of certain overtones of free oscillations of the Earth. Because the displacement of an overtone in the inner core is relatively greater than that of a fundamental mode, the periods of certain overtones are more sensitive to the properties of the inner core than are those of fundamentals, and because some of those overtones decay much more slowly than fundamentals they can be distinguished. Dziewonski and Gilbert (1971) identified such overtones and compared their calculated periods with observation, finding that if they assumed the inner core to be solid the r.m.s. difference between observed and calculated periods was 0.12 % while it was 1 % if they assumed the inner core to be liquid. They concluded that $\beta$, the velocity of shear waves in the inner core, would lie between 3.0 and 3.5 km/s.

Pulses which have passed through the inner core as shear waves (*PKJKP*) have now been identified by Julian, Davies and Sheppard (1972). They looked in the records of the LASA array for signals that had the times to the array and velocities across the array calculated supposing $\beta$ to be 3 km/s in the inner core. They found 5 earthquakes that gave such signals at the most favourable distance and showed that data were best fitted by a value of $\beta$ between 2.9 and 3 km/s. They found no evidence that $\beta$ varied with radius.

3. The information on the change of density at the boundary of the inner core and on the value of $\beta$ enable limits to be placed on the bulk modulus, $K$, and density, $\rho$, within the inner core.

$K/\rho$ is commonly denoted by $\Phi$ and is equal to $\alpha^2 - \frac{4}{3}\beta^2$, where $\alpha$ is the velocity of longitudinal elastic waves.

Let $\delta\Phi$ be the change of $\Phi$, $\delta K$ the change of $K$ and $\delta\rho$ the change of $\rho$ in crossing the boundary from the outer to the inner core. Then

$$\frac{\delta\Phi}{\Phi} = \frac{\delta K}{K} - \frac{\delta\rho}{\rho},$$

or, since according to Bolt and Qamar, $\delta\rho/\rho$ is less than 0.125,

$$\frac{\delta K}{K} \not> \frac{\delta\Phi}{\Phi} + 0.125 .$$

Further, for stability, $\delta\rho/\rho$ cannot be negative and so

$$\frac{\delta K}{K} \not< \frac{\delta\Phi}{\Phi} .$$

Now, according to Buchbinder (1971), $\alpha$ is 10,26 km/s at the base of the outer core and is 10.84 km/s just within the inner core. Since $\beta$ is 0 just outside and 3 km/s just within the inner core, $\Phi$ is 105.30 $km^2/s^2$ just outside and 106.92 $km^2/s^2$ just within the inner core. Then $\delta\Phi/\Phi = 0.0144$ and $0.014 \leq \delta K/K \leq 0.139$.

Other estimates of the values of $\alpha$ in the core (Haddon and Bullen, 1969) would imply larger changes in $K$ at the boundary of the inner core, but in view of the evidence that $K$ depends primarily on pressure within the Earth and only in a minor way upon composition (see Cook, 1972 for a recent summary of the evidence) the estimate entailed by Buchbinder's velocities seems the more probable.

According to the models constructed by Derr (1969), with which those of Press (1971) are consistent, the value of $K$ at the base of the inner core is $13.05 \times 10^{11}$ $N/m^2$; just within the inner core $K$ may be expected to lie between 13.2 and $14.9 \times 10^{11}$ $N/m^2$.

4. If it is assumed that the inner core is chemically homogenous, it now becomes possible to estimate the density at the centre of the Earth. Let $\delta\Phi$, $\delta K$, $\delta\rho$, be the changes of $\Phi$, $K$ and $\rho$ from the boundary of the inner core to the centre. According to Buchbinder (1971) $\alpha$ is 11.28 km/s at the centre and so $\Phi$ is 115.28 $km^2/s^2$. Thus

$$\frac{\delta\Phi}{\Phi} = 0.076 ,$$

taking $\Phi$ to be the mean value in the inner core.

Now if the inner core is homogenous, there is no change of density on account of changes of chemical composition or crystal structure and so

$$\frac{\delta\rho}{\rho} = \frac{\delta p}{K} ,$$

where $\delta p$ is the increase of pressure within the inner core. Hence

$$\frac{\delta\Phi}{\Phi} = \frac{\delta K - \delta p}{K}$$

or

$$\delta K = \delta p + K\, \delta\Phi/\Phi .$$

Derr (1969) finds that $\delta p = 0.42 \times 10^{11}\,\text{N/m}^2$ and so

$$\delta K = 0.042 + 0.076\,K$$
$$= 1.10 \times 10^{11}\,\text{N/m}^2$$

if $K$ is taken to be $14.0 \times 10^{11}\,\text{N/m}^2$.

Thus the value of $K$ at the centre of the Earth would lie between 14.3 and $16.0 \times 10^{11}\,\text{N/m}^2$ if Buchbinder's (1971) values of $\alpha$ are accepted.

Taking a mean value of $15 \times 10^{11}\,\text{N/m}^2$ for $K$, $\delta\rho/\rho$, which is $\delta p/K$, is found to be 0.03. Derr finds that $\rho$ is 12.350 kg/m³ at the base of the inner core and so according to Bolt and Qamar (1970) the density just within the inner core must lie between 12350 and 14100 kg/m³; the density at the centre of the Earth must then lie between 12730 and 14480 kg/m³.

With the foregoing estimates of the density, the shear modulus of the inner core has a value lying between 1.11 and $1.19 \times 10^{11}\,\text{N/m}^2$ at the boundary of the inner core and between 1.15 and $1.23 \times 10^{11}\,\text{N/m}^2$ at the centre, that is to say, approximately 0.08 of the bulk modulus; Poisson's ratio is very close to 1/2. No ordinary solid is known with such a small value of the ratio $\mu/K$.

5. A principal aim in studying the density and elastic moduli in the inner core is to attempt to decide whether the inner core is a solid form of the outer core material or whether it has a different chemical composition. One approach would be to examine the dependence of $K$ on pressure. Within most of the Earth, $K$ follows the rule

$$K = K_0 + bp$$

where $b$ is close to 3.3 (Cook, 1972).

With this rule

$$\frac{\delta\Phi}{\Phi} = (b - 1)\,\frac{\delta p}{K}$$

and it will be found that $b$ is 3.6 for the inner core.

Bearing in mind the small range of pressure in the inner core, that value is not significantly different from the value in the main body of the Earth. However, whereas there is no evidence of any significant discontinuity of $K$ at the boundary of the mantle and core, where there is a large change of density and most probably a change of composition, this note shows that $K$ certainly suffers a jump at the boundary of the inner core. It is tempting to speculate that $K$ would increase on solidification, but there are no experimental data that bear on the question. On the other hand, such indication as there is suggests that the slope $b$ is greater in the inner core than in the core, yet experiment and theory show that $b$ normally decreases as the pressure increases. So far as it goes, this consideration implies that the inner core differs in chemical composition from the outer core.

6. The main point of this Note is that values of $K$ and $\rho$ can be estimated for the inner core as a consequence of the observation of Bolt and Qamar (1970) and of the determination of $\beta$ within the inner core. By comparing the values of $K$ in the inner core and outer core there may be some indication that the material of the inner core differs in composition from that of the core.

## References

BOLT B. A. and QAMAR A., « Nature (London) », *228*, 148–150 (1970).
BUCHBINDER G. G. R., « Bull. Seismol. Soc. Amer. », *61*, 429–456 (1971).
CALOI P., « Geophys. J. R. Astrom. Soc. », *4*, 137–150 (1961).
COOK A. H., « Proc. Roy. Soc. A », *328*, 301–336 (1972).
DERR J. S., « J. Geophys. Res. », *74*, 5202–5220 (1969).
DZIEWONSKI A. M. and GILBERT F. « Nature (London) », *234*, 465 (1971).
HADDON R. A. W. and BULLEN K. E., « Phys. Earth Planet. Interiors », *2*, 35 (1969).
JULIAN B. R., DAVIES D. and SHEPPARD R. M. « Nature (London) », *235*, 317–8 (1972).
LEHMANN I., « Pub. Bureau Seismol. Int., Trav. Sci. Sér. A » *14*, 87–115 (1936).
PRESS F., « Q. Jnl. R. Astron. Soc. », *12*, 232–243 (1971).