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A numerical experiment on wind effects in the Adriatic sea

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Geofisica. — *A numerical experiment on wind effects in the Adriatic sea.* Nota di FRANCO STRAVISI (*), presentata (**) dal Socio A. MARUSSI.

RIASSUNTO. — Si studiano gli effetti di una forza tangenziale costante sul livello e sulla corrente orizzontale media del mare Adriatico, per mezzo delle equazioni di storm surge linearizzate. Tali equazioni sono risolte numericamente con uno schema esplicito alle differenze finite, la cui coerenza è assicurata dalla validità delle leggi di conservazione per l'energia meccanica e per la massa totale.

INTRODUCTION

This paper studies the effects induced by a constant wind force in the Adriatic sea, by means of the standard storm surge equations, numerically solved with a difference scheme. The Author has previously [8, 9, 10] applied this scheme to rectangular, circular, and elliptical basins, with particular emphasis on boundary conditions. Mechanical energy and mass are conserved, as a necessary (but not sufficient) condition for the reliability of the employed numerical procedure.

BASIC EQUATIONS

The storm surge equations derive from the Navier–Stokes and continuity equations for a fluid of constant density ρ, linearized and vertically averaged in the long wave approximation [2, 5, 9, 13]. Dealing with a small adjacent sea, its equipotential free surface at rest is approximated by an (x, y) plane, normal to a uniform earth field $\boldsymbol{g}$; the vertical component $1/2 f$ of earth's rotation vector is considered as a constant.

The continuity and momentum equations are:

$$(1) \qquad \begin{cases} \dfrac{\partial \eta}{\partial t} = -\nabla \cdot (\mathrm{H}\mathbf{U}) \\ \dfrac{\partial \mathbf{U}}{\partial t} = -g\nabla\eta - \mathrm{K}\mathbf{U} + \mathbf{C} + \mathbf{F} \end{cases}$$

with:

η	sea level, referred to the surface at rest;
$\mathbf{U} \equiv (\mathrm{U}, \mathrm{V})$	vertically averaged horizontal velocity;
$\mathbf{C} \equiv f(\mathrm{V}, -\mathrm{U})$	Coriolis acceleration;
$\mathbf{F} \equiv (\mathrm{X}, \mathrm{Y})$	wind force per unit mass on the sea surface;
H	depth of the basin;
K	constant bottom friction coefficient;
$\nabla \equiv (\partial_x, \partial_y)$	gradient operator.

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The system (1) is linear, and invariant for a rotation of the frame of reference in the (x, y) plane. The boundary condition at the coast is that the normal component of the velocity must vanish; on the open end of the basin a node for the level is imposed ($\eta = 0$), with the assumption that the external sea has an infinite capacity.

Conservation laws can be easily obtained [9] from equations (1):

$$E(t) = W(t) + E(0) \tag{2}$$

$$\mathcal{V}(t) = \mathcal{V}_e(t) + \mathcal{V}(0) \tag{3}$$

where:

$$E(t) = \frac{1}{2}\rho \int_S dS\,[g\eta^2 + H\mathbf{U}^2] \tag{4}$$

$$W(t) = \rho \int_0^t dt \int_S dS\, H\,[\mathbf{F}\cdot\mathbf{U} - K\mathbf{U}^2] \tag{5}$$

$$\mathcal{V}(t) = \int_S dS\,\eta \tag{6}$$

$$\mathcal{V}_e(t) = -\int_0^t dt \int_{\mathcal{A}} d\mathcal{A}\, H\mathbf{U}\cdot\boldsymbol{n} \tag{7}$$

are: (4) the total potential and kinetic energy, (5) the work done by the external force **F** and the resistence $-K\mathbf{U}$, (6) the total volume, referred to the rest one, (7) the volume exchange through the open boundary $\mathcal{A}$, with outward directed normal $\boldsymbol{n}$; S is the surface of the basin. No energy exchange appears in (2) because of the imposed boundary condition on $\mathcal{A}$.

DIFFERENCE SCHEME

A forward time differences explicit scheme [7, 8, 9, 10] has been adopted:

$$\eta^{i+1} = \eta^i - \Delta t\,\{D_x(HU)^i + D_y(HV)^i\} \tag{8}$$

$$U^{i+1} = (1 - K\,\Delta t)\,U^i + \Delta t\,\{fV^i - g\,D_x(\eta)^{i+1} + X^i\} \tag{9}$$

$$V^{i+1} = (1 - K\,\Delta t)\,V^i - \Delta t\,\{fU^{i+1} + gD_y(\eta)^{i+1} - Y^i\}\,. \tag{10}$$

By means of (8, 9, 10) the η, U, V fields are evaluated, separately and in the indicated order, in all their own grid points, stepping from $i\,\Delta t$ to $(i+1)\,\Delta t$, the initial conditions η^0, U^0, V^0 being assigned.

D_x, D_y are central difference operators in the interior, and forward or backward operators at the boundary points. The grid arrangement is shown in fig. 1: at each velocity point, the missing component of **U** is evaluated with

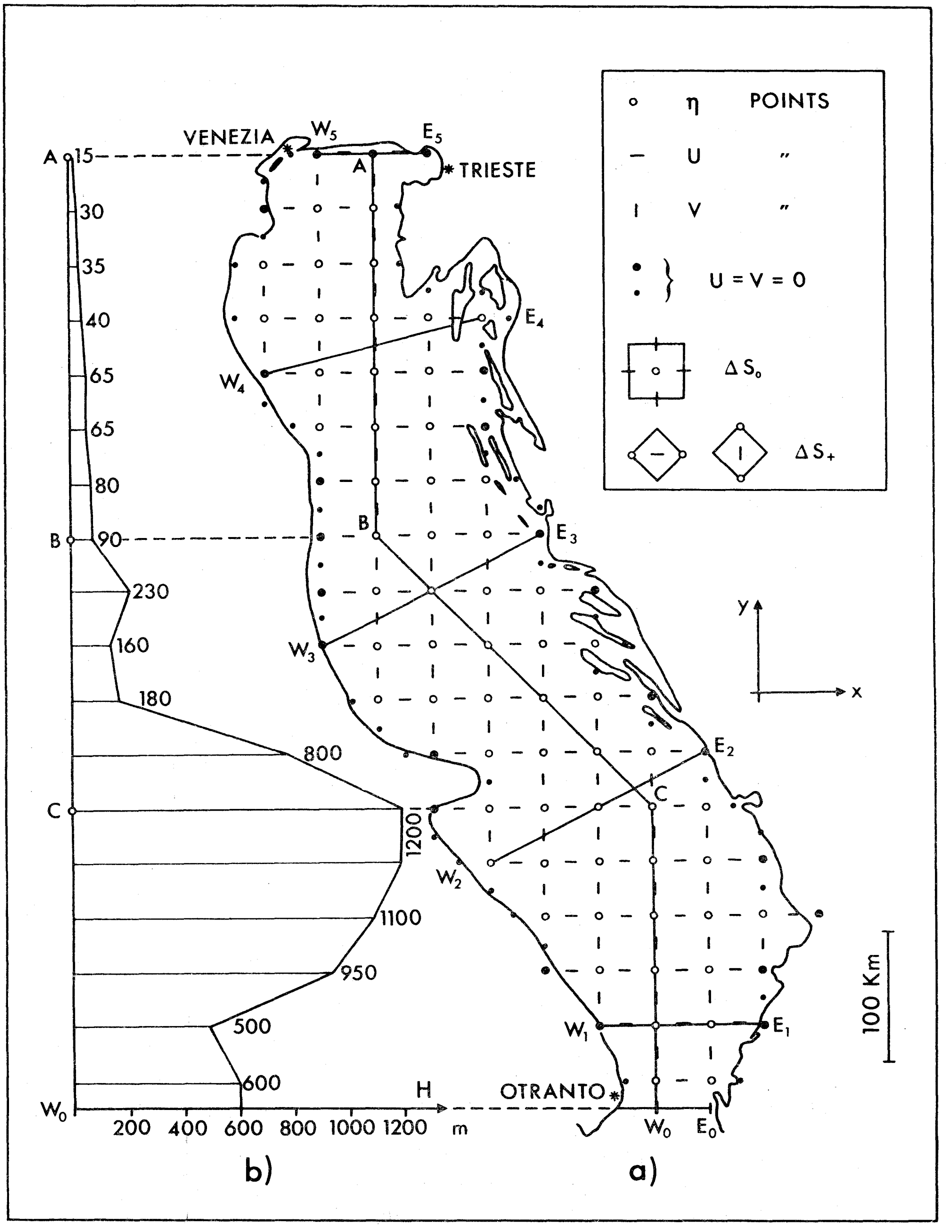

Fig. 1. – Adriatic sea: (*a*) grid, reference points and corresponding transverse sections. (*b*) Depth profile along the A–B–C–W_0 section.

an arithmetical mean. At the points by the coast of the basin, both of the velocity components are taken to be zero [9, 10]. The open contour $\mathcal{C}$ is a straight segment in the x direction, at $y = 0$. The finite surface elements are squares centered at the $\eta^{[o]}$, or U ,V$^{[+]}$ points (fig. 1):

$$\Delta S_o = 4\,\Delta x\,\Delta y \qquad \Delta S_+ = 2\,\Delta x\,\Delta y\,.$$

Energy, work and volume integrals (4, 5, 6, 7) reduce to summations:

$$E^i = \frac{1}{2} \Delta S_+ \sum_+ H_+ [U^2 + V^2]^i_+ + \frac{1}{2} g \Delta S_o \sum_o [\eta^i_o]^2 \tag{11}$$

$$W^i = \Delta t \, \Delta S_+ \sum_{n=0}^{i} \sum_+ H_+ \{[XU + YV]^n_+ - K [U^2 + V^2]^n_+ \} \tag{12}$$

$$\mathcal{V}^i = \Delta S_o \sum_o \eta^i_o \tag{13}$$

$$\mathcal{V}^i_e = \Delta t \, \Delta x \sum_{n=0}^{i} \sum_k \alpha_k H_{k,0} V^n_{k,0} \, . \tag{14}$$

Density $\rho = 1$ c.g.s., and does not appear in (11, 12); the open end configuration is taken into account by suitable α_k weights in (14), depending from the considered $(k, 0)$ point on $\mathcal{A}$.

A working criterion for the numerical stability of (8, 9, 10) is the Courant–Friedrich–Lewy one:

$$\Delta t \leq \sqrt{2} \, \Delta s / \sqrt{gH}$$

where $\Delta s = \min(\Delta x, \Delta y)$ and $H = \max(H(x, y))$.

NUMERICAL EXPERIMENTS

The solution of the storm surge system for a rectangular basin is of the kind:

$$\begin{cases} \eta = \eta_0 + \eta_F \\ \mathbf{U} = \mathbf{U}_0 + \mathbf{U}_F \end{cases}$$

where η_0, $\mathbf{U}_0$ are a superposition of $e^{-(K/2)t}$ damped free modes, η_F, $\mathbf{U}_F$ are characteristic of the wind force $\mathbf{F}$ [9, 10]. In the case of $\mathbf{F}$ constant, directed along the y axis of the basin, η_F represents a plane surface, with a slope F/g in the y direction, and $\mathbf{U}_F$ is equal to zero; so the water oscillates around this configuration, reaching it asymptotically in time. The total mechanical energy $E(t)$ and volume $\mathcal{V}(t)$, whose mean and asymptotic values are those pertaining to the described η_F, $\mathbf{U}_F$ configuration, have a similar behaviour. These general features can be applied to a real basin, with irregular boundary.

The Adriatic basin and grid are represented in fig. 1. The frame of reference is so oriented that the y axis is parallel to the opening at Otranto ($y = 0$), so the boundary conditions there are: $\eta = 0$, $U = 0$.

The basin is initially at rest; for $t \geq 0$ a constant wind force is directed along the longitudinal axis towards the closed end of the Adriatic. Computations are performed for 120 hours (real time) for two models: one of constant depth, the other for "real" depth, both for $F = 6.4 \times 10^{-4}$ cm sec^{-2}. Using $\mathbf{F} = \gamma H^{-1} w \boldsymbol{w}$ and $\gamma = 3.2 \times 10^{-6}$ and $H = 200$ m, this corresponds to a wind velocity $w = 20$ m/sec. The bottom friction coefficient is taken $K = 2 \times 10^{-5}$ sec^{-1}.

A constant mean depth H = 200 m is assumed. Fig. 2 shows computed sea level $\eta\,(W_i\,, t)$ in function of time for five points W_i ($i = 1, \cdots 5$) on the west coast. The first longitudinal seiche with $T = 23^h$ is evident; it is the dominant mode because of the unidirectional wind force assumed; from the nodal line at the open end the amplitude increases northward, reaching a rather large value of 28 cm at W_5 due to the discontinuous impact of **F** at

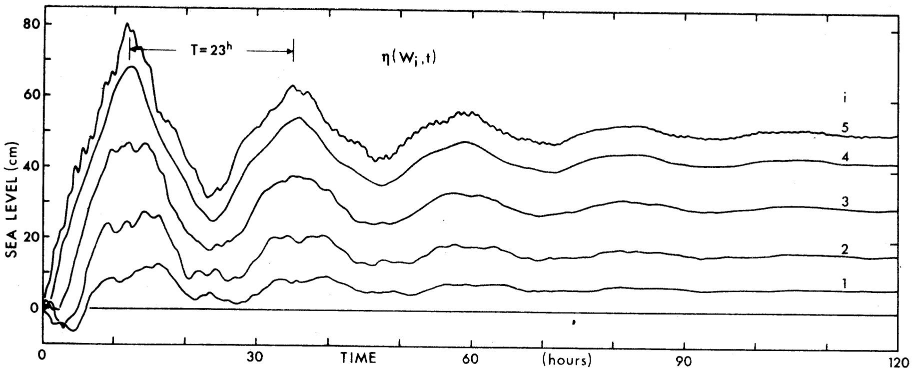

Fig. 2. – Sea level η in function of time at $W_1, \cdots, W_5$ on the west coast of the Adriatic (H = 200 m), with constant wind force.

$t = 0$. The minor short period damped oscillations are transverse modes, as seen from the elevation profiles on the corresponding $W_i - E_i$ sections. The asymptotic values correspond to a plane surface η_F.

Fig. 4 shows the equal level lines after 10^h, that is toward maximum elevations (there is a piling-up of water at the east coast), and after 110^h, when the energy passes through its equilibrium value and the asymptotic

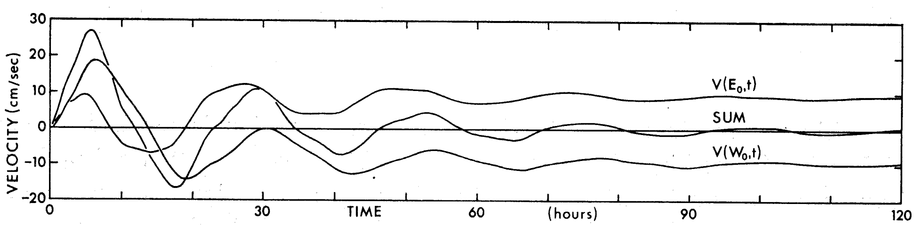

Fig. 3. – Velocity V in function of time at W_0, E_0, and sum, on the open end of the Adriatic (H = 200 m), with constant wind force.

plane surface is approximated. Fig. 3 shows the velocity $V(W_0\,, t)\,, V(E_0\,, t)$ at the open end, and their sum; while the total flux tends to zero with time, a counterclockwise steady circulation appears in the mouth region after the first period. This circulation does not occur in a rectangular open basin, and is probably due to the asymmetry of the opening in the Adriatic. In the remaining basin, the current tends to zero with time.

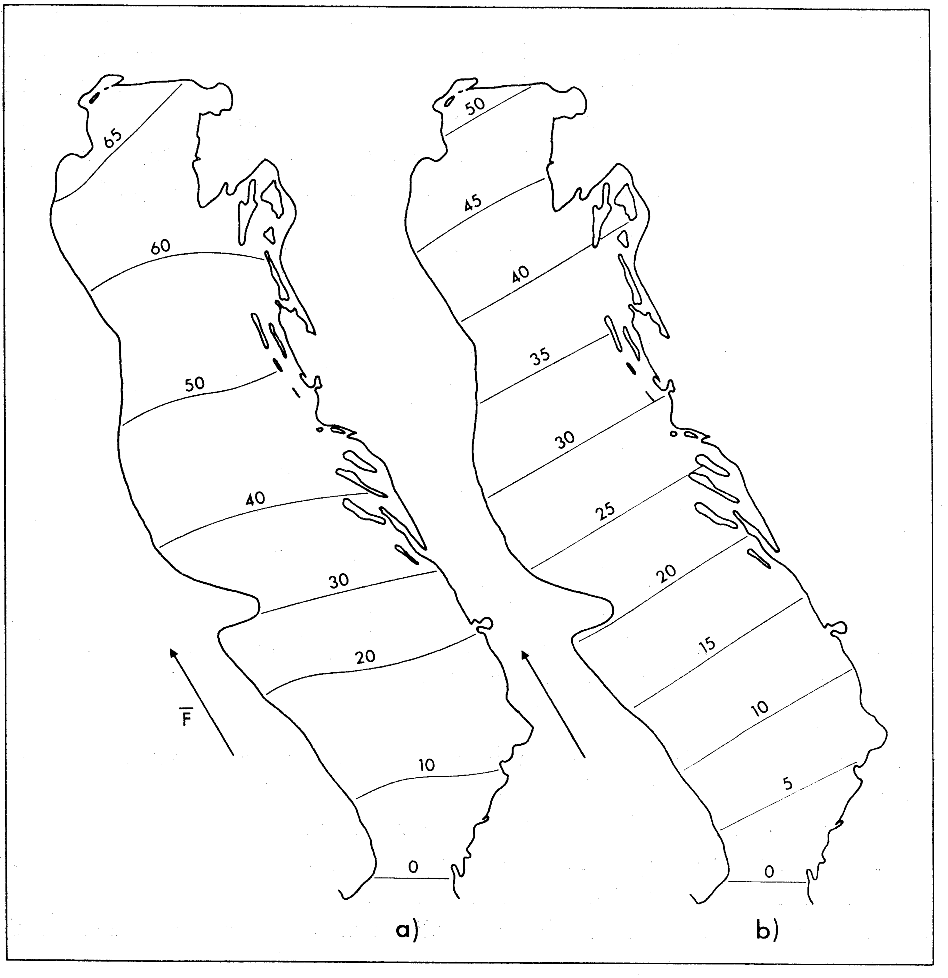

Fig. 4. – Equal level lines (in cm) after (*a*) 10^h, (*b*) 110^h of constant **F** wind force on the Adriatic (H = 200 m).

Looking at the instant when the maximum elevation is reached at each E_i, W_i point (smoothing the corresponding $\eta(t)$ curve), a wave is found travelling counterclockwise with a velocity about 42 m/sec along the east coast, and about 45 m/sec down the west coast, with a period of about 9^h. This corresponds in direction and speed to a Kelvin wave, whose velocity for H = 200 m is 44 m/sec.

In general it can be observed that the first maximum incoming flux from the external sea appears about T/4 after the wind started (fig. 3), while the first maximum elevation on the north coast of the basin (W_5 — E_5) is reached after T/2 (fig. 2).

Computed total mechanical energy, work, volume, volume exchange (11 ,··· 14) do satisfy the conservation laws (2, 3) (fig. 5, 6): their general behaviour resembles that for a rectangular basin.

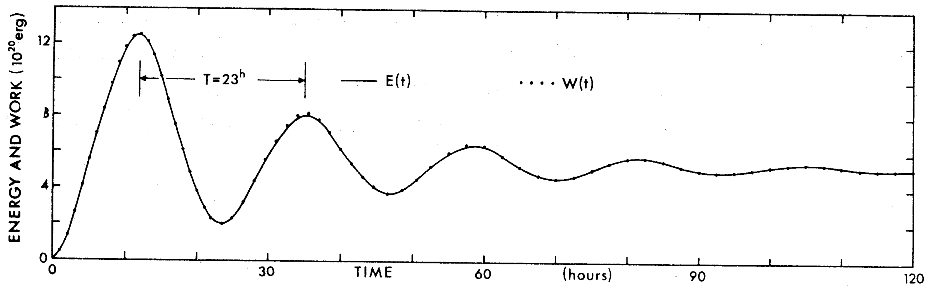

Fig. 5. – Mechanical energy balance for the Adriatic (H = 200 m), constant wind force.

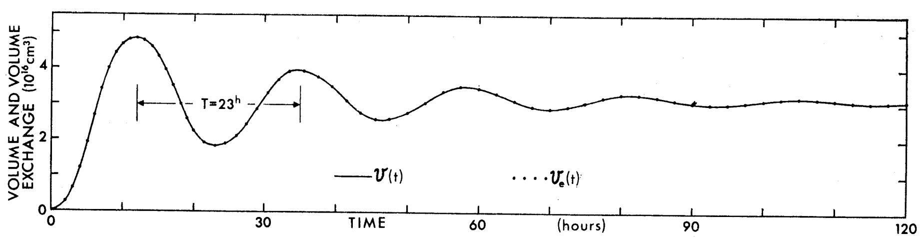

Fig. 6. – Volume (mass) balance for the Adriatic (H = 200 m), constant wind force.

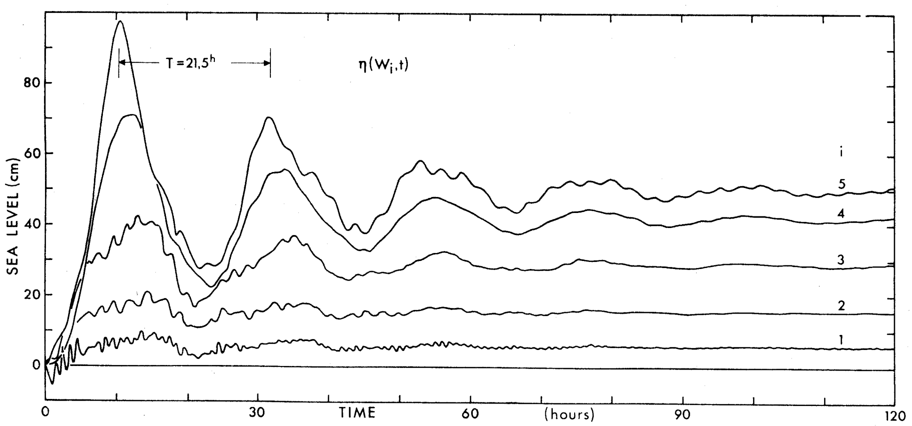

Fig. 7. – Sea level η in function of time at $W_1, \cdots, W_5$ on the west coast of the Adriatic (H real), with constant wind force.

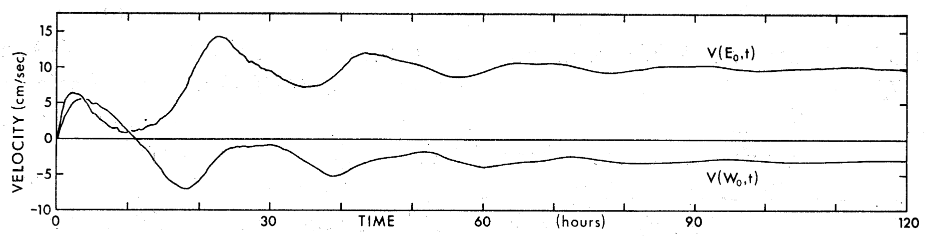

Fig. 8. – Velocity V in function of time at W_0, E_0 on the open end of the Adriatic (H real), with constant wind force.

The real depth is now considered for the Adriatic basin: its longitudinal section along the A—B—C—W_0 line is shown in fig. 1. Computations give the following differences between the constant and variable depth cases: (*a*) the first longitudinal seiche period is $T = 21.5^h$, very close to the observed

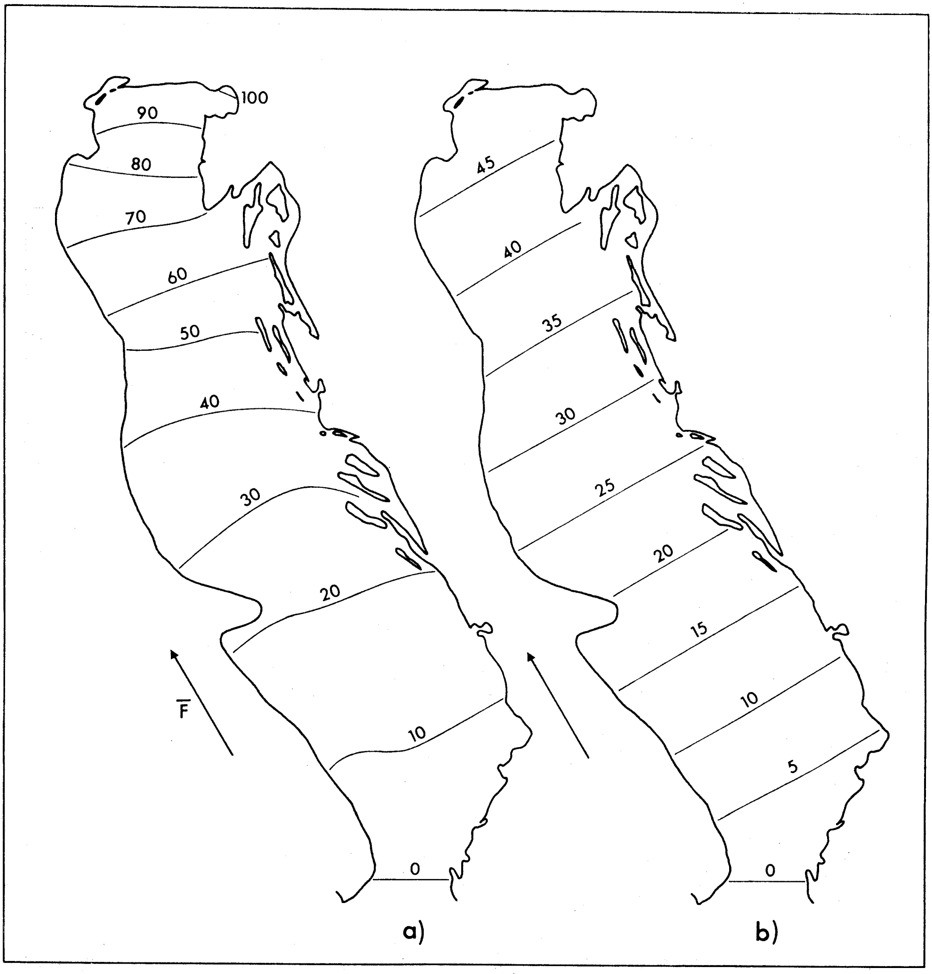

Fig. 9. – Equal level lines (in cm) after (*a*) 10^h, (*b*) 110^h of constant **F** wind force on the Adriatic (H real).

periods of about 21.3^h [6]; its amplitude increases rapidly in the north (48 cm at W_5), because of shallow water effects (fig. 7, 9 a). (*b*) The transverse seiches, related to the transverse profiles of the basin by Merian's formula, are: the 1^h seiche through the Otranto channel (section 1), those of about 2^h through the middle part of the basin (sections 2, 3), and the 3^h seiche in the gulf of Venice (section 5) (fig. 7); the last one has been observed [1]. (*c*) A counter-clockwise travelling Kelvin type wave is found with a wave velocity of about 34 m/sec along the east coast, 26 m/sec around the northern part of the basin, 79 m/sec down the west coast. These values correspond to a depth of 116, 67, 630 m respectively; the travel time is about 12.5 hours. (*d*) The first maximum incoming flux takes place about 3^h after the wind started, instead of $T/4 = 5.4^h$ (fig. 8), while the maximum elevation on the closed end appears after $T/2$ (fig. 7), as before.

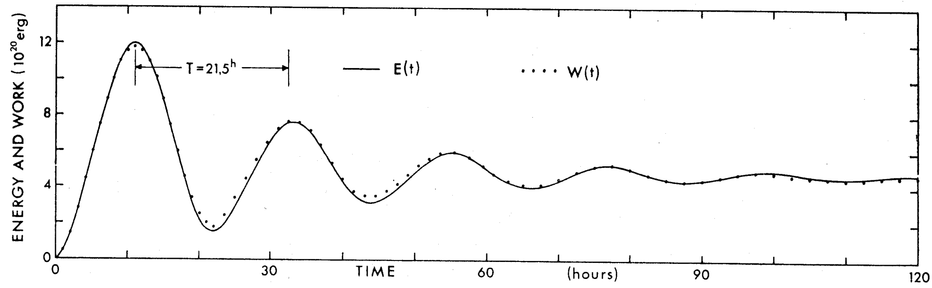

Fig. 10. – Mechanical energy balance for the Adriatic (H real), constant wind force.

The same counterclockwise circulation is found in the mouth region; in fig. 8 V (W_0, *t*) and V (E_0, *t*) have different asymptotic values because of the different depth at the two points, such that the total flux tends to zero. The asymptotic equilibrium surface is still a plane one (fig. 9 b). Conservation laws are satisfied by the numerical scheme (fig. 10, 11).

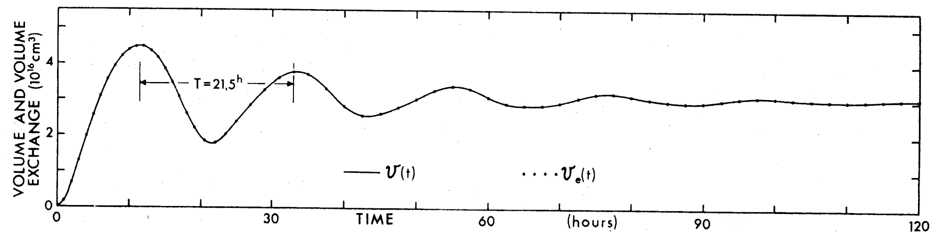

Fig. 11. – Volume (mass) balance for the Adriatic (H real), constant wind force.

Characteristic data for the numerical computation are:

$F = 6.4 \times 10^{-4}$ cm sec^{-2};
$X = -3.2 \times 10^{-4}$ cm sec^{-2};
$Y = 5.5 \times 10^{-4}$ cm sec^{-2};
$K = 2 \times 10^{-5}$ sec^{-1};
$g = 980$ cm sec^{-2};
$f = 9.9 \times 10^{-5}$ sec^{-1};
$\Delta x = \Delta y = 20.6$ km;
H = 200 m : $\Delta t = 6^m$ time length: $120^h = 1200\,\Delta t$;
H real : $\Delta t = 3^m$ time length: $120^h = 2400\,\Delta t$.

Computing time is about 2.5 minutes for 100 grid points and 1000 time steps, on the IBM 7044 machine.

These numerical experiments demonstrate that the employed difference scheme adequately conserves total energy and mass in a very general case; this property, together with the preceding [8, 9, 10] tests, assures that the scheme can be applied with a high degree of reliability to hindcasting and forecasting sea levels.

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