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Spatial Mechanisms with Guidance Analysis using Curvilinear Co-ordinates

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Meccanica razionale. — *Spatial Mechanisms with Guidance Analysis using Curvilinear Co-ordinates.* Nota di DEMETRIO MANGERON [1],[2], MEHMET N. OĞUZTÖRELI [3] e NICOLAE PLITEA [4] presentata [*] dal Socio B. FINZI.

RIASSUNTO. — Gli Autori, prendono le mosse dai risultati di una serie di loro studi concernenti l'elaborazione di vari nuovi metodi d'indagine dei problemi di analisi e di sintesi dei meccanismi e delle macchine, pubblicati soprattutto nella « Revue Roumaine des Sciences Techniques, Série de Mécanique appliquée » [3]–[6], e facendo uso sistematico delle coordinate curvilinee, espongono un metodo di analisi cinematica dei meccanismi spaziali con uno, due e tre gradi di mobilità e con elemento centrale guidato in cinque, quattro oppure tre punti.

1. In a set of their previous papers the authors introduced the *any order reduced accelerations method*, the *tangential method*, the *various numerical corps method*, the *matrix-tensor method*, the *new Analytical Dynamics Equations method*, the *integral-equations method* [1]–[7], useful in the theory of mechanisms and machines, and constructed different *mechanical-electronic devices* concerning mechanisms design. A part of their results was published in the "Rendiconti dell'Accademia Nazionale dei Lincei" [8]–[9] or in the "Romanian Journal of Applied Mechanics" [5], and was subsequently enlarged and successfully applied by numerous scientists, for instance K. Ogawa and K. Yamauchi (Japan), P. A. Lebedev, F. L. Litvin, J. S. Zilberman, E. G. Berger (USSR), John J. Uicker, Jr., M. A. Chace (USA), S. Niang (Rep. of Senegal), Tchang Tsu-siang (People's Republic of China) a.o.a.o.

The paper under consideration, having the above mentioned works as a starting point, proposes to develop a general method of kinematic analysis of spatial mechanisms with one, two and three degrees of mobility with a central element guided in five, four and three points. It is also related with our very recent Note published in the same "Rendiconti Lincei" [13] and concerned with the kinematic study of the rigid body subjected to a constrained

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(2) The first of the authors wishes to express his sincere thanks to the University of Alberta for the invaluable conditions offered to him to develop his work in the Department of Mathematics of this University.

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(4) Polytechnic Institute of Cluj, Socialist Republic of Romania.

(5) The authors are pleased to underline that an invaluable number of useful ideas and references pertaining both to rational and applied mechanics was derived from the well known "Meccanica razionale" by Bruno Finzi.

motion. Using the curvilinear co-ordinates of the guided points the position equations of the mechanisms of interest in their kinematic analysis are established.

2. THE MOVEMENT OF A RIGID BODY WITH ONE DEGREE OF FREEDOM

2 a. *The case of guidance in five points*.—Let us consider a rigid body subjected to contraints (fig. 1) in such a manner that its points A_i $(i = 1, 2, \cdots, 5)$ move respectively on fixed surfaces (S_i) $(i = 1, 2, \cdots, 5)$.

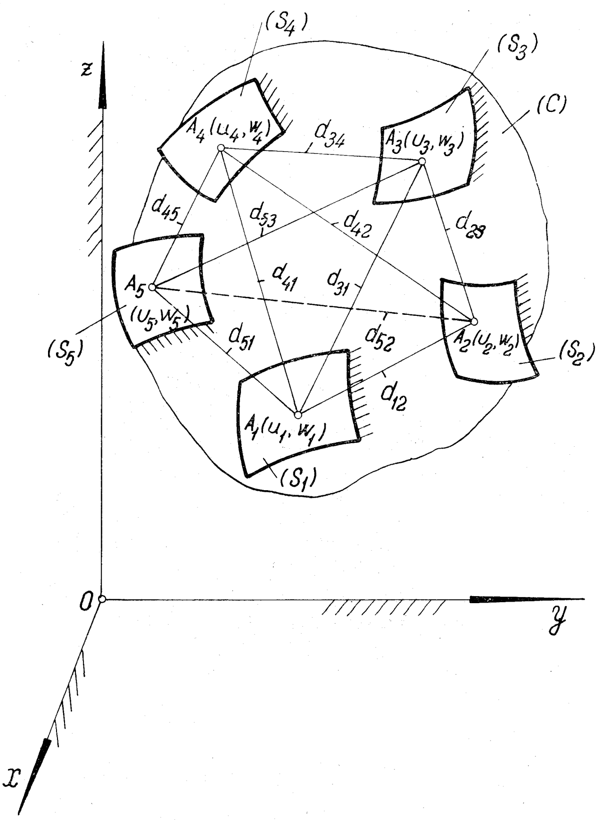


Fig. 1.

The cartesian co-ordinates of points A_i may be expressed in relation to the curvilinear co-ordinates on the (S_i) surfaces as follows:

$$x_i = x_i(u_i, w_i) \quad , \quad y_i = y_i(u_i, w_i) \quad , \quad z_i = z_i(u_i, w_i) . \tag{2.2a.1}$$

The unique generalized co-ordinate taken into consideration is the curvilinear co-ordinate u_1 of point A_1 whose variation with time is considered to be known

$$u_1 \equiv q_1 = q_1(t) . \tag{2.2a.2}$$

Assuming that the distances between the five points are constant, ten position equations are obtained of which only nine are independent. The position equations are

$$(2.2a.3)\qquad [x_k(u_k, w_k) - x_j(u_j, w_j)]^2 + [y_k(u_k, w_k) - y_j(u_j, w_j)]^2 + \\ + [z_k(u_k, w_k) - z_j(u_j, w_j)]^2 - d_{jk}^2 = 0 \qquad (k, j = 1, 2, \cdots, 5),$$

where we marked with d_{jk} the known distance between points A_k and A_j.

The system of nine equations (2.2a.3) with unknown elements w_1 and u_i, w_i $(i = 2, 3, 4, 5)$ as well as the position equations we established in what follows are solved by successive approximations. The algorithmic details and the convergence conditions required for computer programming will be exposed in the "Bulletin of the Polytechnic Institute of Jassy". After solving the system (2.2a.3), the cartesian co-ordinates of points A_i $(i = 1, 2, \cdots, 5)$ are calculated with relations (2.2a.1).

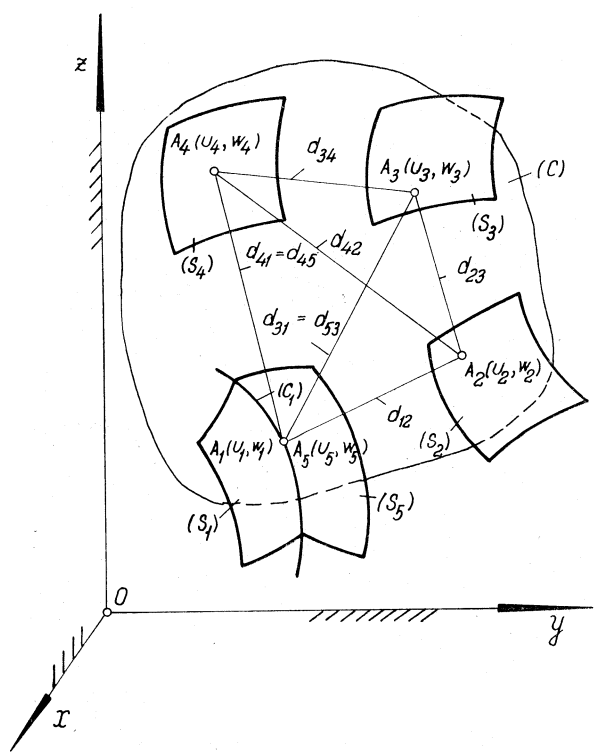


Fig. 2.

2 b. *The case of guidance in four points.*—In this case the point A_1 moves (fig. 2) on the fixed curve (C_1) and the points A_i $(i = 2, 3, 4)$ move respectively on the surfaces (S_i) $(i = 2, 3, 4)$. This case is reduced to the previous one by considering that point A_5 coincides with point A_1 and is solved with the same nine position equations (2.2a.3) in which $d_{51} = 0$,

$d_{53} = d_{31}$, $d_{54} = d_{41}$. If the cartesian co-ordinates of point A_1 are expressed in terms of curvilinear co-ordinate u_1 on curve (C_1), or, in other words, one has

$$x_1 = x_1(u_1) \quad , \quad y_1 = y_1(u_1) \quad , \quad z_1 = z_1(x_1) \, , \tag{2.2b.1}$$

then the number of equations decreases to six

$$(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2 - d_{jk}^2 = 0 \quad (j \, , k = 1 \, , 2 \, , 3 \, , 4) \, , \tag{2.2b.2}$$

where x_1 , y_1 , z_1 are to be replaced by the given values of (2.2b.1), and x_i , y_i , z_i are replaced by the values given by the relation (2.2a.1) for $i = 2 , 3 , 4$. Choosing, as generalized co-ordinate, the curvilinear co-ordinate on curve (C_1) of point A_1 whose variation with time is given by (2.2a.2), the system of six position equations (2.2b.2) has the unknowns $u_i , w_i \, (i = 2 , 3 , 4)$.

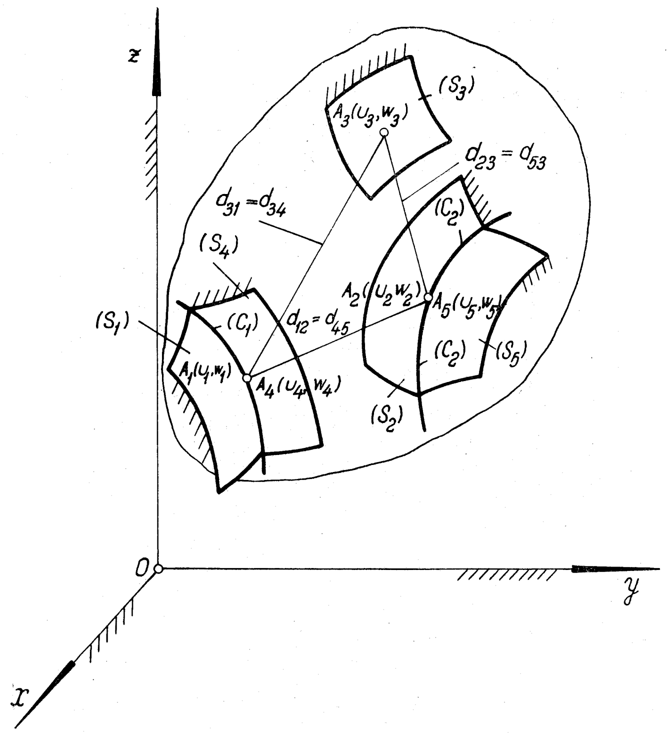


Fig. 3.

2 c. *The case of guidance in three points.*—In this case (fig. 3) the points $A_i \, (i = 1 , 2)$ move respectively on two fixed curves $(C_i) \, (i = 1 , 2)$ and point A_3 moves on the fixed surface (S_3). This case, too, is reduced to the first one, provided one considers that the point A_4 coincides with the point A_1 and the point A_5 coincides with the point A_2. The equations (2.2a.3) will be position equations for this case, where $d_{41} = 0$, $d_{52} = 0$, $d_{12} = d_{45}$, $d_{31} = d_{34}$, $d_{23} = d_{53}$.

If the cartesian co-ordinates of points A_i $(i = 1, 2)$ are expressed in terms of curvilinear co-ordinates on curves (C_i) $(i = 1, 2)$ with relations

$$x_i = x_i(u_i) \quad , \quad y_i = y_i(u_i) \quad , \quad z_i = z_i(u_i) \qquad (i = 1, 2), \tag{2.2c.1}$$

the number of position equations is reduced to three

$$(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2 - d_{jk}^2 = 0 \qquad (j, k = 1, 2, 3), \tag{2.2c.2}$$

where x_i, y_i and z_i $(i = 1, 2)$ are replaced by the values given by (2.2c.1) and x_3, y_3 and z_3 are replaced with the values given by (2.2a.1) for $i = 3$. Choosing as generalized co-ordinate the curvilinear co-ordinate u_1 whose variation with time is considered to be known and is given by (2.2a.2), the system (2.2c.2) has three unknowns u_2, u_3 and w_3.

3. The movement of a rigid body with two degrees of mobility

3 a. *The case of guidance in four points*.—In this case (fig. 4) the points A_i $(i = 1, 2, 3, 4)$ move respectively on the fixed surfaces (S_i) $(i = 1, 2, 3, 4)$. The generalized co-ordinates are to be, for instance, the

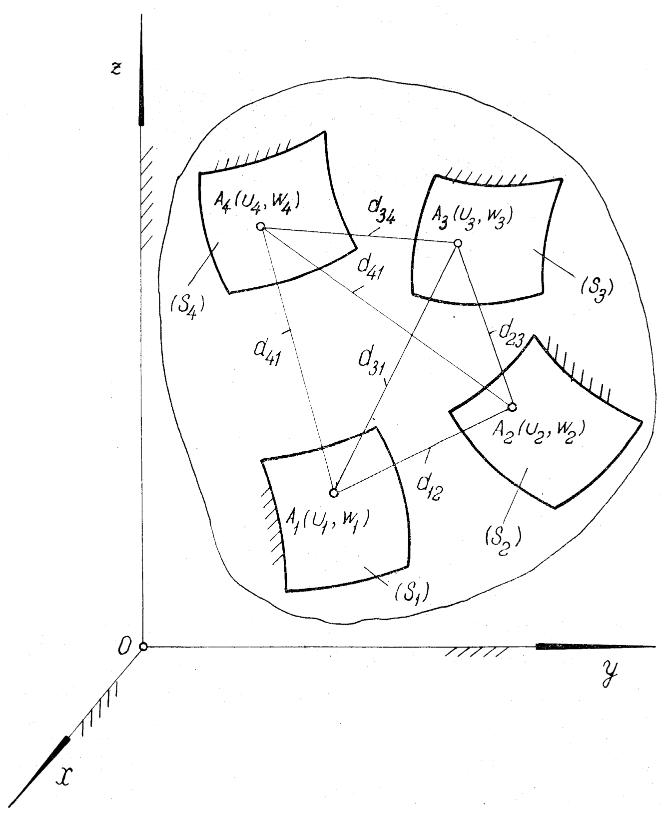


Fig. 4.

curvilinear co-ordinates u_1 and u_2 of points A_i $(i = 1, 2)$, whose variation laws with time are considered known

$$u_i \equiv q_i = q_i(t) \qquad (i = 1, 2). \tag{3.3a.1}$$

Six position equations are obtained in this case

$$[x_k(u_k, w_k) - x_j(u_j, w_j)]^2 + [y_k(u_k, w_k) - y_j(u_j, w_j)]^2 + \\ + [z_k(u_k, w_k) - z_j(u_j, w_j)]^2 - d_{jk}^2 = 0 \qquad (j, k = 1, 2, 3, 4), \tag{3.3a.2}$$

where the unknowns are w_1, w_2, u_3, w_3, u_4 and w_4.

3 b. *The case of guidance in three points.*—In this case (fig. 5) the point A_1 moves on the fixed curve (C_1) and points A_i $(i = 2, 3)$ move respectively on the surfaces (S_i) $(i = 2, 3)$. This case is reduced to the previous one provided that the point A_4 coincides with the point A_1. The position equations will be (3.3a.2) in which $d_{41} = 0$, $d_{42} = d_{12}$, $d_{34} = d_{31}$.

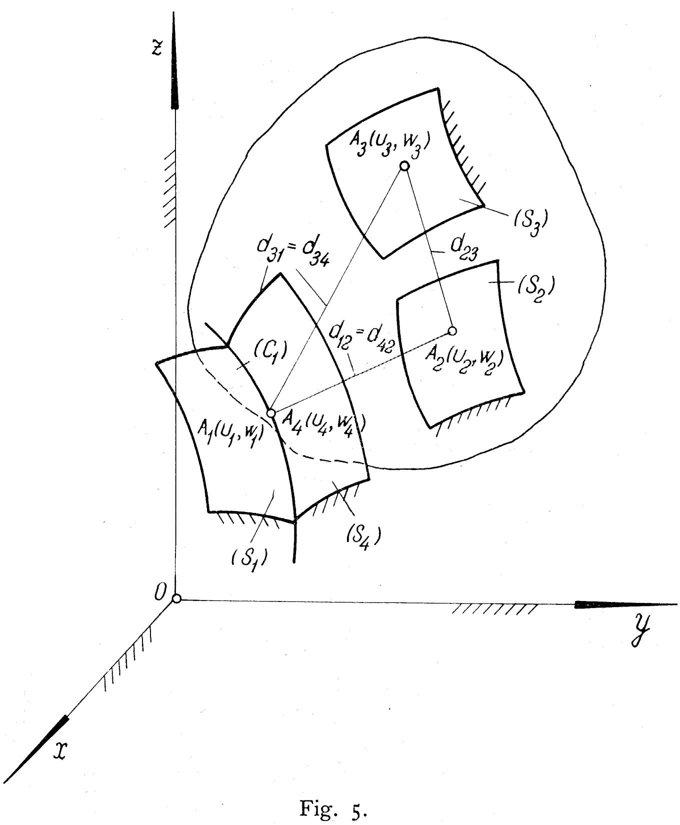


Fig. 5.

If the cartesian co-ordinate of points A_1 can be expressed in terms of curvilinear co-ordinate u_1 on the curve (C_1) with the relations (2.2b.1) and supposing we choose as generalized co-ordinates the curvilinear co-ordinates u_1

and u_2 whose variation with time is given by the relations (3.3a.1), then the unknowns w_1, u_3 and w_3 follow from solving the system of three equations of position

$$(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2 - d_{jk}^2 = 0 \qquad (j, k = 1, 2, 3), \tag{3.3b.1}$$

in which x_1, y_1 and z_1 are replaced by the values given by (2.2b.1) and x_i, y_i and z_i $(i = 2, 3)$ are replaced by the values given by (2.2a.1) for $i = 2, 3$.

4. The movement of a rigid body with three degrees of freedom

4 a. *The case of guidance in three points.*—In this case the points A_i $(i = 1, 2, 3)$ move respectively on the fixed surfaces (S_i) $(i = 1, 2, 3)$. The rigid body having three degrees of freedom (fig. 6), we choose as gene-

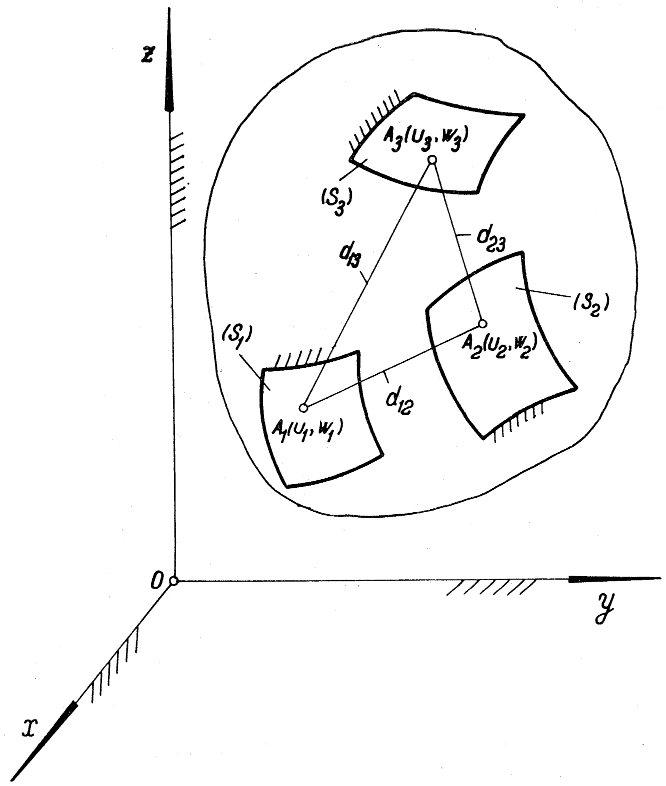


Fig. 6.

ralized co-ordinates one of the curvilinear co-ordinate of points A_i on the surfaces (S_i), for instance u_1, u_2, and u_3 whose variation with time is considered to be known

$$u_i \equiv q_i = q_i(t) \qquad (i = 1, 2, 3). \tag{4.4a.1}$$

Three position equations are obtained in this case

$$(4.4a.2) \qquad [x_k(q_k, w_k) - x_j(q_j, w_j)]^2 + [y_k(q_k, w_k) - y_j(q_j, w_j)]^2 +$$
$$+ [z_k(q_k, w_k) - z_j(q_j, w_j]^2 - d_{jk}^2 = 0 \qquad (j, k = 1, 2, 3)$$

and the unknowns of this system are w_i $(i = 1, 2, 3)$.

5. Cases of guidance and driving of spatial mechanisms

5 a. *Guidance on fixed surfaces.*—In figg. 7–12 the most usual open kinematic chains are presented, that give the guidance on fixed surfaces of the points belonging to the central element of the spacial mechanisms with

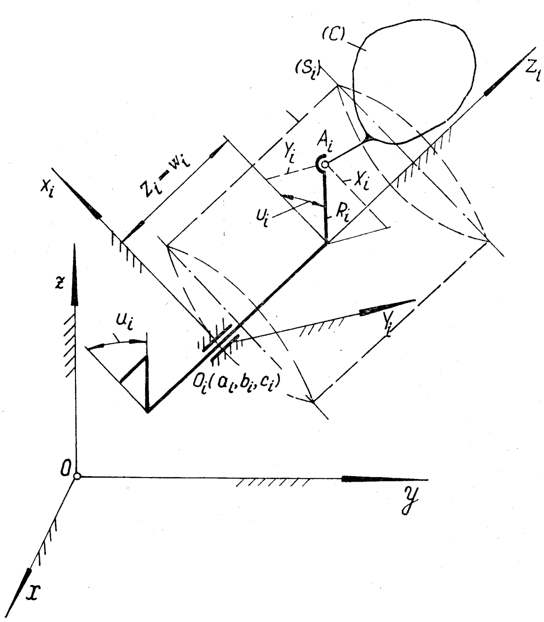


Fig. 7 *a*.

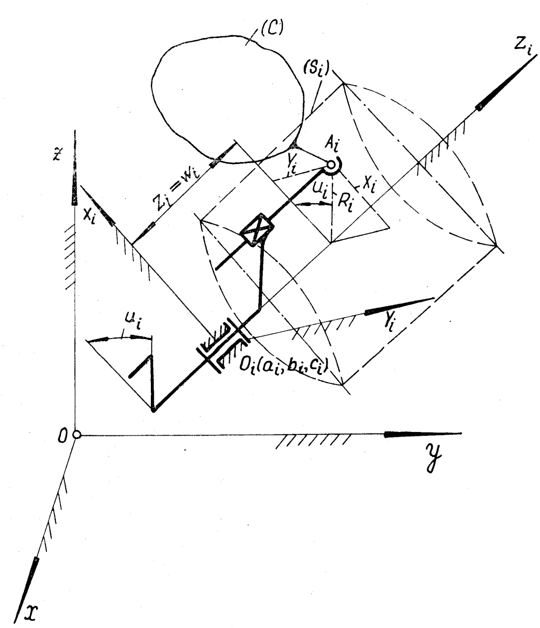
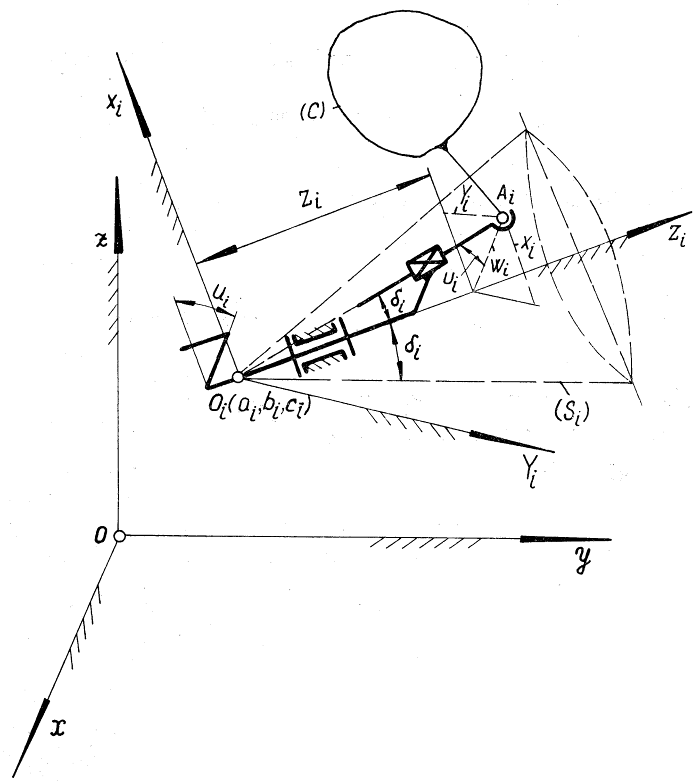


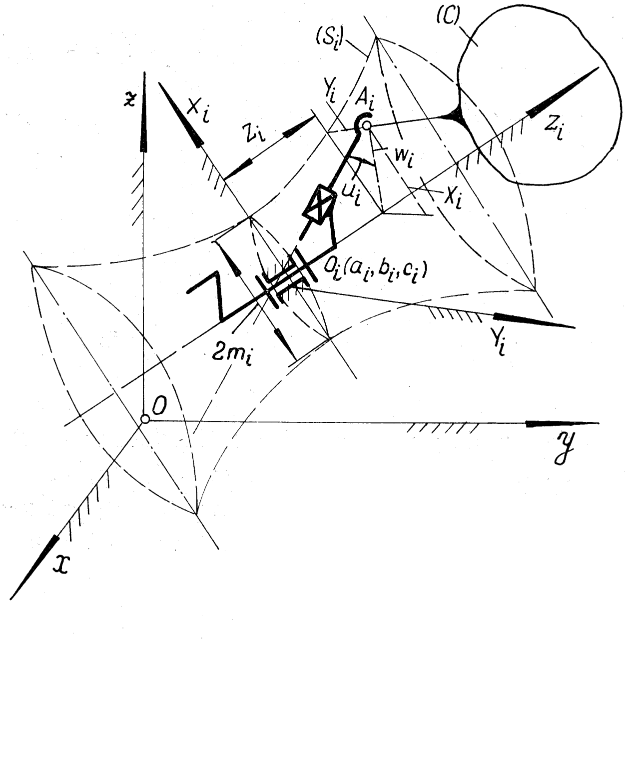
Fig. 7 *b*.

guidance, as well as the driving variants of these mechanisms. One remarks that various studies concerning mechanisms of this kind are to be found in [14]–[15]. In Table I below the cartesian co-ordinates of the guided points A_i are determined using a canonic system $O_i X_i Y_i Z_i$ in terms of the curvilinear co-ordinates on the guiding surface (S_i).



(C)
X_i
Z_i
A_i
Y_i
X_i
Z_i
w_i
u_i
δ_i
δ_i
u_i
z
$O_i(a_i, b_i, c_i)$
(S_i)
Y_i
O
y
x

Fig. 8.



(C)
(S_i)
Y_i
A_i
z
X_i
Z_i
Z_i
w_i
u_i
X_i
$O_i(a_i, b_i, c_i)$
Y_i
$2m_i$
O
y
x

Fig. 9.

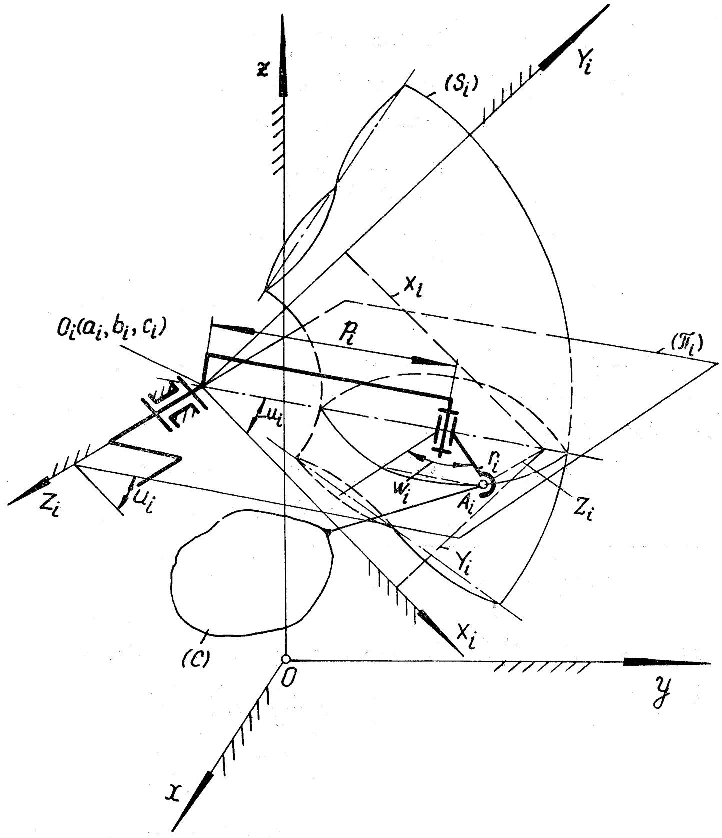

z
Y_i
(S_i)
x_i
$O_i(a_i, b_i, c_i)$
R_i
(Π_i)
u_i
r_i
w_i
A_i
Z_i
Z_i
u_i
Y_i
X_i
(C)
0
y
x

Fig. 10.

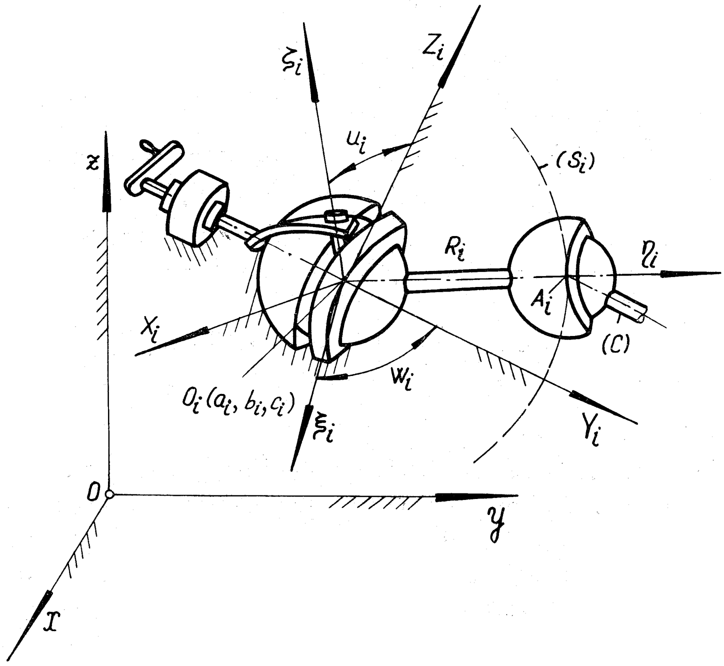

ζ_i
Z_i
z
u_i
(S_i)
R_i
η_i
X_i
A_i
(C)
w_i
$O_i(a_i, b_i, c_i)$
ξ_i
Y_i
0
y
x

Fig. 11.

TABLE I

Cases of guidance and driving variants.

The guiding surfaces and equations with respect to the $O_i X_i Y_i Z_i$ system	The co-ordinates of A_i guided point with respect to the $O_i X_i Y_i Z_i$ system	Figure
1. Straight circular cylinder $X_i^2 + Y_i^2 - R_i^2 = 0$	$X_i = R_i \cos u_i$, $Y_i = R_i \sin u_i$, $Z_i = w_i$	7a, 7b
2. Straight circular cone $X_i^2 + Y_i^2 - Z_i^2 \operatorname{tg}^2 \delta_i = 0$	$X_i = w_i \cos u_i$, $Y_i = w_i \sin u_i$, $Z_i = w_i \operatorname{ctg} \delta_i$	8
3. Rotation hyperboloid of one sheet $n_i^2 (X_i^2 + Y_i^2) - m_i^2 Z_i^2 - m_i^2 n_i^2 = 0$	$X_i = w_i \cos u_i$, $Y_i = w_i \sin u_i$, $Z_i = \frac{n_i}{m_i} \sqrt{w_i^2 - m_i^2}$	9
4. Torus $(X_i^2 + Y_i^2 + Z_i^2 + p_i^2 - r_i^2)^2 - 4 p_i^2 (X_i^2 + Y_i^2) = 0$	$X_i = (p_i + r_i \sin w_i) \cos u_i$ $Y_i = (p_i + r_i \sin w_i) \sin u_i$ $Z_i = r_i \cos w_i$	10
5. Sphere $X_i^2 + Y_i^2 + Z_i^2 - R_i^2 = 0$	$X_i = - R_i \cos u_i \cos w_i$ $Y_i = R_i \sin w_i$ $Z_i = R_i \sin u_i \cos w_i$	11
6. Plane $Z_i = 0$	$X_i = w_i \cos u_i$, $Y_i = w_i \sin u_i$, $Z_i = 0$	12

5 b. *Guidance on fixed curves.*—The guidance on fixed curves is obtained by combination of two open guiding chains on surfaces presented in Table I and figg. 7–12. In figg. 13–15 three simple guiding cases of points A_i on a circle, a straight cylindrical helix and a straight line are presented. In that cases the corresponding co-ordinates x_i, y_i, and z_i can be expressed in terms of the curvilinear co-ordinates u_i of the guiding curves (C_i).

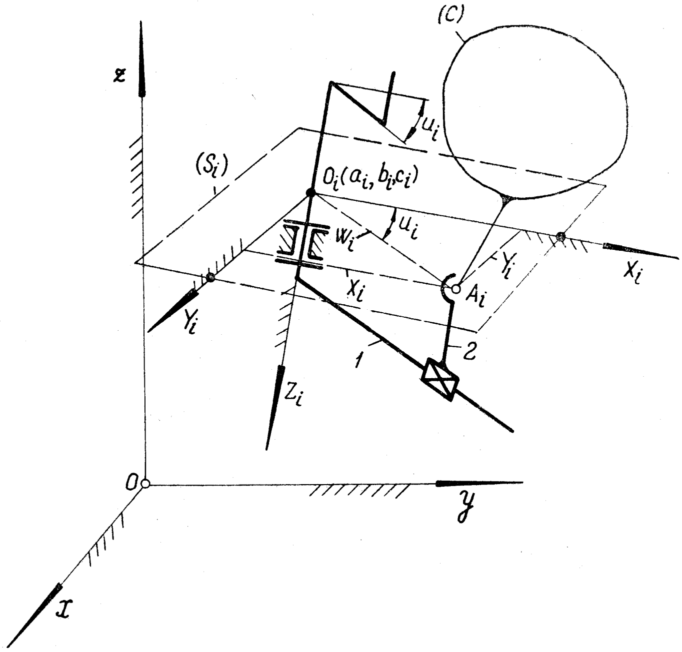

(C)
z
u_i
(S_i)
$O_i(a_i, b_i c_i)$
w_i
u_i
Y_i
X_i
A_i
X_i
Y_i
2
1
Z_i
O
y
x

Fig. 12.

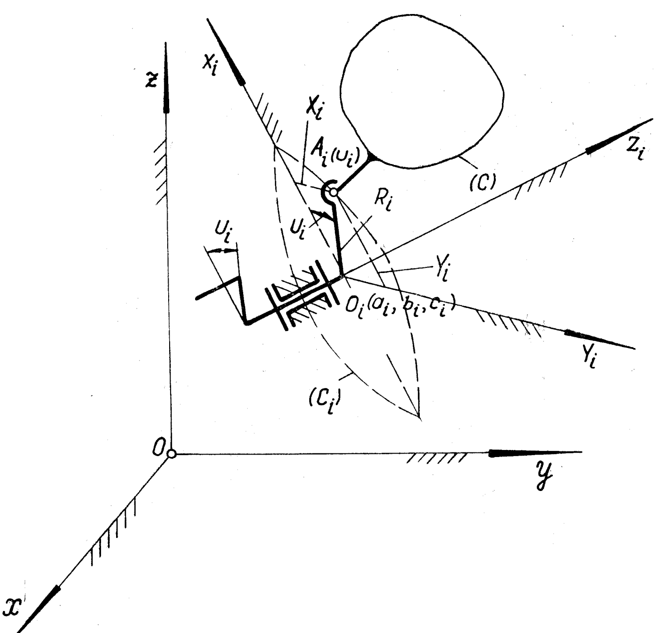
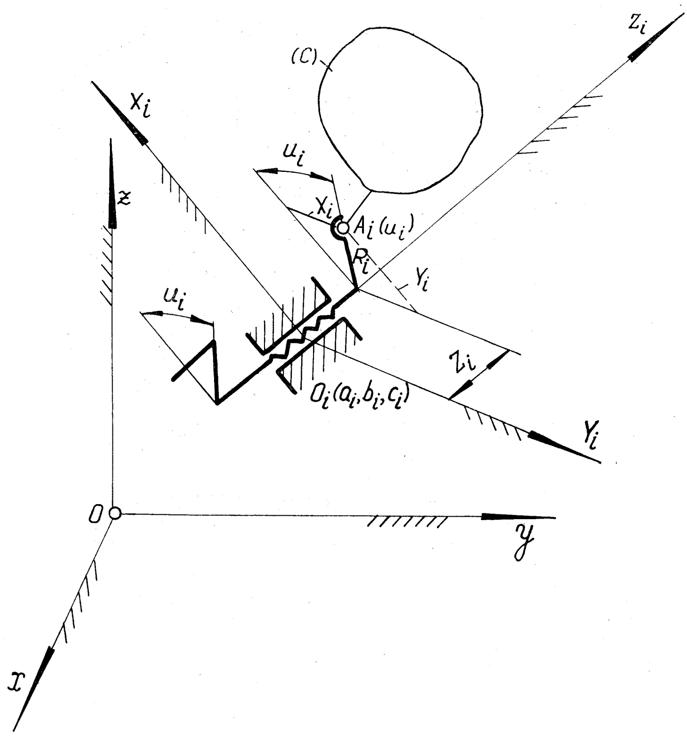
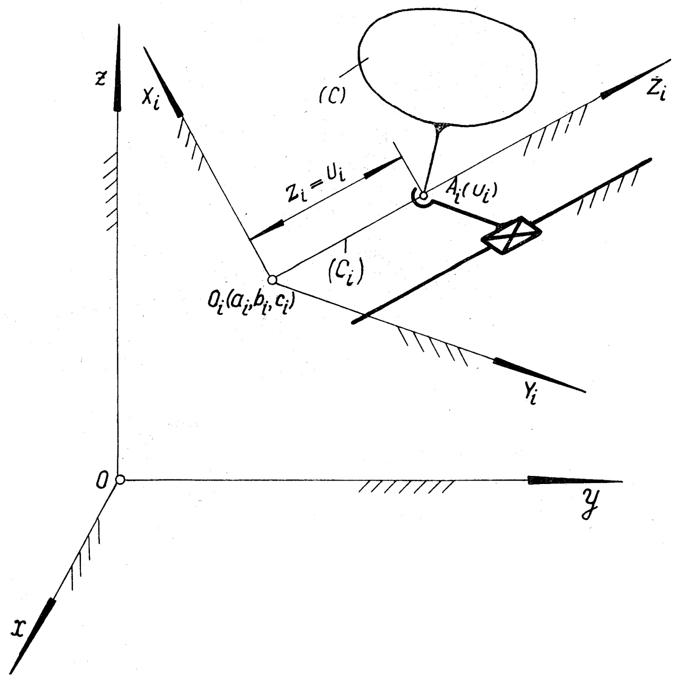

z
X_i
X_i
$A_i(u_i)$
Z_i
(C)
R_i
u_i
u_i
Y_i
$O_i(a_i, b_i, c_i)$
Y_i
(C_i)
O
y
x

Fig. 13.



Z_i
(C)
X_i
u_i
x_i
$A_i(u_i)$
R_i
Y_i
z
u_i
z_i
$O_i(a_i, b_i, c_i)$
Y_i
O
y
x

Fig. 14.



z
X_i
(C)
Z_i
$z_i = u_i$
$A_i(u_i)$
(C_i)
$O_i(a_i, b_i, c_i)$
Y_i
O
y
x

Fig. 15.

6. REMARKS

6 a. On the basis of cases presented in figg. 7–15 we can achieve a series of spatial mechanisms with one, two, and three degree of mobility, with central element guided in five, four, and three points. The analysis of spatial mechanisms with guidance of points of central element on various surfaces will be published in future issues of the *Bulletin of the Polytechnic Institute of Jassy.*

6 b. The speed and acceleration equations can be obtained from direct and repeated differentiation with respect to time of position equations established above. One obtains linear equations for the unknowns $\frac{du_i}{dt}$, $\frac{dw_i}{dt}$, and $\frac{d^2 u_i}{dt^2}$, $\frac{d^2 w_i}{dt^2}$ respectively. The speeds and accelerations of the guided points A_i follow by differentiating the relations of the form (2.2a.1) and (2.2c.1) with respect to time. Having the speeds and accelerations of three points belonging to the central element of the considered mechanisms, their angular speeds and angular accelerations can be easily determined using the formulas given in a set of very recent Romanian Monographs on Mechanisms [16]–[18].

6 c. The results obtained in this paper are to be used by the third of the authors in his doctoral dissertation under Prof. D. Mangeron in relation with various dynamic problems concerning spatial mechanisms with guidance, as well as with some synthesis problems of spatial mechanisms having a large number of degrees of freedom.

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