Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

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On the decomposition of E-points condition

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **49** (1970), n.3-4, p. 198–199.

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<http://www.bdim.eu/item?id=RLINA_1970_8_49_3-4_198_0>

Teoria dei giochi. — *On the decomposition of E–points condition.* Nota (*) di EZIO MARCHI, presentata dal Socio B. SEGRE.

RIASSUNTO. — Si danno condizioni generali che assicurano la proprietà generale per l'esistenza di un vasto gruppo di punti di stabilità su giochi.

Following the nomenclature in [1], we have that if for any $\sigma_N \in \Sigma_N$ there is an $\tau_N \in \Sigma_N$ such that

$$A_i(\tau_{d(i)}, \sigma_{N-d(i)}) = \max_{s_{d(i)} \in \Sigma_{d(i)}} A_i(s_{d(i)}, \sigma_{N-d(i)})$$

then Theorem 1 assures the existence of an E–positive equilibrium point for the game Γ_E when the functions A_i are quasi-concave in $\sigma_{d(i)} \in \Sigma_{d(i)}$.

Let $\mathfrak{M} = \{M_1, \cdots, M_r\}$ be the largest partition induced by the sets $d(i)$. Thus, $d(i) = \sum_{k \in S_i} M_k$ where S_i is the set of indices in $\varphi = \{1, \cdots, r\}$ such that $M \subset d(i)$.

PROPOSITION 1: *The payoff functions are given by*

$$A_i(\sigma_{d(i)}, \sigma_{N-d(i)}) = \sum_{k \in S_i} A_i^k(\sigma_{M_k}, \sigma_{N-d(i)}).$$

Then, if for any $\sigma_N \in \Sigma_N$, $k \in \varphi$ *there is an* $\tau_{M_k} \in \Sigma_{M_k}$ *such that*

$$A_j^k(\tau_{M_k}, \sigma_{M-d(i)}) = \max_{s_{M_k} \in \Sigma_{M_k}} A_j^k(s_{M_k}, \sigma_{N-d(i)})$$

for all $j \in \alpha(k) = \{j \in N : M_k \subset d(j)\}$, *the condition holds true.*

Let τ_N be the joint strategy obtained from these τ_{M_k}'s. Hence, τ_N is the desired plan.

If N_k indicates the set $\bigcup_{l \in \alpha(k)} (N - d(l))$ and $\mathfrak{N}_k = \{N_k^1, \cdots, N_k^{r_k}\}$ is the largest partition for N_k, let φ_k be the set of indices $\{1, \cdots, r_k\}$ and $T_{k,j} = \{l \in \varphi_k : N_k^l \subset N - d(j)\}$.

PROPOSITION 2: *For all* $k \in \varphi$ *and* $j \in \alpha(k)$

$$A_j^k(\tau_{M_k}, \sigma_{N-d(i)}) = \sum_{l \in T_{k,j}} A_j^{k,l}(\tau_{M_k}, \sigma_{N_k^l}).$$

Then, if for all $k \in \varphi$ *and* $l \in \varphi_k$ *and any* $\sigma_{N_k^l} \in \Sigma_{M_k^l}$ *there is an* $\tau_{M_k} \in \Sigma_{M_k}$ *such that for all* $j \in \beta_{k,l} = \{p \in N : N_k^l \subset N - d(p)\}$:

$$A_j^{k,l}(\tau_{M_k}, \sigma_{N_k^l}) = \max_{s_{M_k} \in \Sigma_{M_k}} A_j^{k,l}(s_{M_k}, \sigma_{M_k^l}),$$

the condition in Proposition 1 *is satisfied.*

(*) Pervenuta all'Accademia il 14 ottobre 1970

As a simple example take $d(i) = \{i, i+1\}$ if $i \leq |N| - 1$ and $d(|N|) = \{|N|\}$. Hence $M_k = \{k\}$, $k = 1, \cdots, N$, $\alpha(k) = \{k, k+1\}$ when $k \leq N - 1$ and $\alpha(|N|) = |N|$. $N_k = N - \{k\}$, $N_k^1 = \{1, \cdots, k-2\} \cup \cup \{k+2, \cdots, |N|\}$, $N_k^2 = \{k-1\}$ and $N_k^3 = \{k+1\}$.

The maximum property in the last proposition is very weak when $A_j^{j,1} \equiv 0$ and $A_j^{j+1,3} \equiv 0$. Similarly, this is the case when $d(|N|) = \{1, |N|\}$. When we have $d(i) = \{i, i+1, \cdots, i+s\}$ for a fixed s, where $i \leq |N| - s$ and $d(i) = \{i, \cdots, |N|\}$ for the remaining i's, the examination is analogous.

A sufficient condition for the maximum requirement when we have two matrices $\| a_{ij}^1 \|$ and $\| a_{ij}^2 \|$ is that the undominated faces of the polihedra formed by the rows vectors be parallel.

The propositions indeed allow us to decompose the condition for all E–points in further studies.

REFERENCE

[1] E. MARCHI, *E-Points of Games*, Proc. N.A.S., Vol. LVII, No. 4, pp. 878–882, April, 1967.