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An algorithm for producing circular and linear envelope curves in plane mechanisms consisting of complex kinematic chains with applications in digital control of machine tools. Nota I

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Meccanica applicata. — *An algorithm for producing circular and linear envelope curves in plane mechanisms consisting of complex kinematic chains with applications in digital control of machine tools* (1).
Nota I di DEMETRIO MANGERON (2),(3), MEHMET NAMIK OĞUZTÖRELI (4) e NICOLAE ORLANDEA (5), presentata (*) dal Socio B. FINZI.

To MAURO PICONE
on his 85 th birthday

RIASSUNTO. — In questa Nota gli Autori, prendendo le mosse da un loro recentissimo lavoro, presentato al XIV Congresso di Meccanica razionale ed applicata, organizzato sotto gli auspici dell'ISTAM al Regional Engineering College, Kurukshetra (Harayana), India, consacrato all'uso sistematico di Geometria tangenziale nello studio dei meccanismi e delle macchine, espongono un algoritmo concernente la generazione di curve linearmente e circolarmente inviluppate tramite meccanismi piani costituiti dalle catene cinematiche complesse e ne additano applicazioni al controllo, con le calcolatrici elettroniche, delle macchine utensili. Vari dettagli e serie di figure che illustrano la costruzione effettiva dei meccanismi corrispondenti si troveranno esposti nei prossimi fascicoli della serie IVa «Meccanica tecnica» del *Bollettino dell'Istituto Politecnico di Iaşi*.

1. In a set of their previous papers the authors introduced the *any order reduced accelerations method*, the *tangential method*, the *various numerical corps method*, the *matrix-tensor method* [1], all of interest in the theory of mechanisms and machines, and constructed different *mechanical-electrical devices*, concerning mechanism design. A part of their results was published in the *Rendiconti dell'Accademia Nazionale dei Lincei* [2] (6), and was subsequently enlarged and successfully applied to various concrete domains of mechanisms by numerous scientists, for instance, K. Ogawa and K. Yamauchi (Japan), F. Moroshkin, S. N. Kozhevnikov, J. S. Zilberman, E. G. Berger [3] (U.S.S.R.), J. J. Beggs, M. A. Chace (U.S.A.), S. Niang (Republic

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(3) The first of the authors wishes to express his sincere thanks to the University of Alberta for the invaluable conditions offered to him to develop his work in the Department of Mathematics of this University.

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(6) The authors are pleased to underline that an invaluable number of useful ideas and references pertaining both to rational and applied mechanics was derived from the well-known «Meccanica razionale» and «Problemi di Meccanica razionale», by Bruno Finzi.

of Sénégal). The authors' main results were also partially used and exposed in a recent set of monographs due, for instance, to John J. Uicker, Jr., P. A. Lebedev, F. L. Litvin [4], N. A. Bogoliubov, R. C. Bogdan [5], Chr. Pelecudi [6], R. Voinea and M. Atanasiu [7].

In what follows the authors, after giving a very short presentation of some their new results pertaining to the *linear* envelope curves generation problem, expose an algorithm for producing *circular* envelope curves in plane mechanisms consisting of complex kinematic chains with applications in digital control of machine tools.

2. We start from a set of statements, for instance the following ones:

a) The hyperplanes cutting the co-ordinate axes $x_1, x_2, \cdots, x_m$ in the points such that

$$\xi_1^{p_1}\xi_2^{p_2}\cdots\xi_m^{p_m} = a \tag{1}$$

holds, envelop the generalized "polytropic" hypersurfaces

$$x_1^{p_1}x_2^{p_2}\cdots x_m^{p_m} = a\frac{p_1^{p_1}p_2^{p_2}\cdots p_m^{p_m}}{(p_1+p_2+\cdots+p_m)^{p_1+p_2+\cdots+p_m}}\,; \tag{2}$$

b) The hyperplanes cutting the co-ordinate axes $x_1, x_2, \cdots, x_m$ in the points such that

$$\left(\frac{\xi_1}{a_1}\right)^n \pm \left(\frac{\xi_2}{a_2}\right)^n \pm \cdots \pm \left(\frac{\xi_m}{a_m}\right)^n = 1 \tag{3_{mn}}$$

holds, envelop the Lamé generalized hypersurfaces

$$\left(\frac{x_1}{a_1}\right)^{\frac{n}{n+1}} \pm \left(\frac{x_2}{a_2}\right)^{\frac{n}{n+1}} \pm \cdots \pm \left(\frac{x_m}{a_m}\right)^{\frac{n}{n+1}} = 1\,. \tag{4_{mn}}$$

The authors designed, for $m = 2$ and $m = 3$, various new useful mechanisms enlarging their previous set presented during the Socialist Republic of Romania Academy Jubilee Session (Bucharest, July, 1960) and the IIIrd International Congress on Mechanisms and Machines (Miskolc, September, 1963). For instance, the authors show, in their paper to appear in the *Bulletin of the Polytechnic Institute of Jassy*, a parabola *linear* enveloping mechanism corresponding to $m = 2$, $n = 1$, and an ellipse evolute *linear* enveloping mechanism corresponding to $m = 2$, $n = 2$.

It is worthwhile to mention here that a design for a *linear* envelope curves device was developed and subsequently a profile grinding machine constructed by the third author in [8].

3. A desmodromic mechanism with k degrees of freedom is considered, as in fig. 1. Let $q_1(t), q_2(t), \cdots, q_k(t)$ be known parameter functions of time that determine the position of the mechanism at a given instant. Let $q_s(t)$ $(s = 1, 2, \cdots, k)$ be the generalized Lagrangean co-ordinates. Let (C)

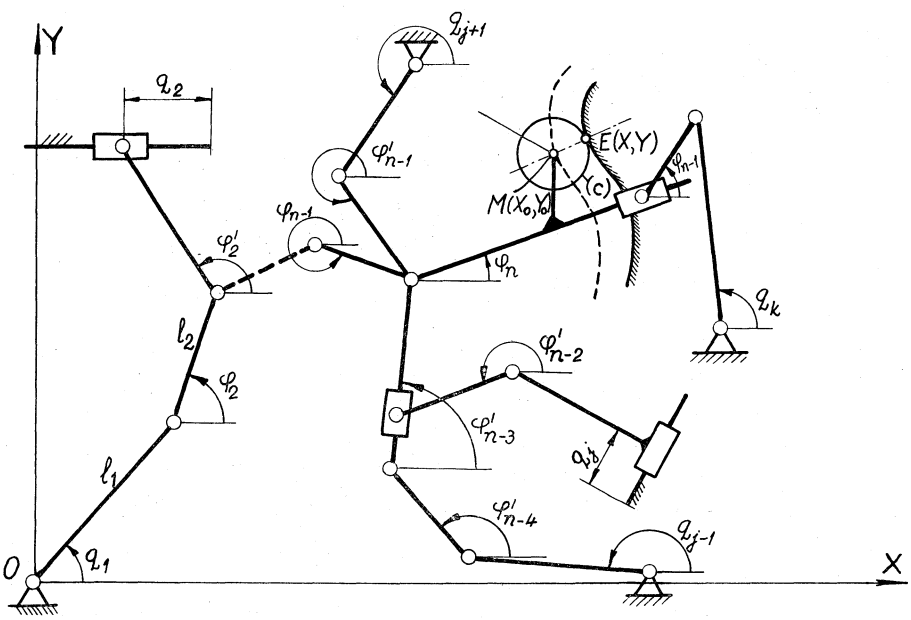

Fig. 1.

be a circle of radius r invariably connected to the element n of the mechanism and let

(5) $$(X - X_0)^2 + (Y - Y_0)^2 = r^2$$

be its equation in a fixed system XOY. According to [9] the expressions of the co-ordinates X_0 and Y_0 of the centre M of (C) may be written in the form

(6) $$X_0 = \sum_{i=1}^{n} l_i \cos \varphi_i \quad , \quad Y_0 = \sum_{i=1}^{n} l_i \sin \varphi_i ,$$

where $l_i = l_i\,[q_1(t), q_2(t), \cdots, q_k(t)]$ and $\varphi_i = \varphi_i\,[q_1(t), q_2(t), \cdots, q_k(t)]$ represent respectively the length l_i of the element i of the mechanisms and the orientated angle φ_i formed by the axis of the element i with the base vector $\vec{i}$. Aftpr differentiating the relation (5) with respect to time and obtaining explicit expressions for X and Y, taking into account (6), there result the parametric equations of the circle envelope

(7) $$X = X_0 \pm r \sqrt{\frac{\left(\sum_{j=1}^{k} \frac{\partial Y_0}{\partial q_j}\,\dot{q}_j\right)^2}{\left(\sum_{j=1}^{k} \frac{\partial X_0}{\partial q_j}\,\dot{q}_j\right)^2 + \left(\sum_{j=1}^{k} \frac{\partial Y_0}{\partial q_j}\,\dot{q}_j\right)^2}} ,$$

$$Y = Y_0 \pm r \sqrt{\frac{\left(\sum_{j=1}^{k} \frac{\partial X_0}{\partial q_j}\,\dot{q}_j\right)^2}{\left(\sum_{j=1}^{k} \frac{\partial X_0}{\partial q_j}\,\dot{q}_j\right)^2 + \left(\sum_{j=1}^{k} \frac{\partial Y_0}{\partial q_j}\,\dot{q}_j\right)^2}} ,$$

where the derivatives in the radicals

$$\sum_{j=1}^{k} \frac{\partial X_0}{\partial q_j} \dot{q}_j = \sum_{i=1}^{n} \left[\cos \varphi_i \sum_{j=1}^{k} \frac{\partial l_i}{\partial q_j} \dot{q}_j - l_i \sin \varphi_i \sum_{j=1}^{k} \frac{\partial \varphi_i}{\partial q_j} \dot{q}_j \right] \tag{8}$$

and

$$\sum_{j=1}^{k} \frac{\partial Y_0}{\partial q_j} \dot{q}_j = \sum_{i=1}^{n} \left[l_i \cos \varphi_i \sum_{j=1}^{k} \frac{\partial \varphi_i}{\partial q_j} \dot{q}_j + \sin \varphi_i \sum_{j=1}^{k} \frac{\partial l_i}{\partial q_j} \dot{q}_j \right] \tag{9}$$

are the projections V_{X_0} and V_{Y_0} of the velocity of the center M of (C) in the directions of the co-ordinate axes respectively. Taking into account (5), (8) and (9), (7) yield the *circular* envelope equations in the most general case of plane mechanisms.

4. The practical problem concerning an algorithm for producing circular and linear envelope curves in plane mechanisms consisting of complex kinematic chains is formulated in the following way:

Given a continuous curve Γ

$$X = X(p) \quad , \quad Y = Y(p) \, , \tag{10}$$

possessing at least first and second derivatives on the interval $[p_1, p_2]$, it is required to envelope it by means of a family of circles (or of straight lines respectively) (C) of radius $r \leq \rho_{\min}$, where $\rho_{\min}$ is the minimum radius of curvature of Γ on $[p_1, p_2]$. Taking into account the fact that the normals to the enveloped curve Γ pass through the corresponding center of the envelope circle, the equations of the equidistant curve Γ' to Γ may be written as follows:

$$X_0(p) = X(p) \mp r \frac{Y_p}{\sqrt{X_p^2 + Y_p^2}} \quad , \quad Y_0(p) = Y(p) \pm r \frac{X_p}{\sqrt{X_p^2 + Y_p^2}} \, , \tag{11}$$

where X_p and Y_p denote the derivatives of $X(p)$ and $Y(p)$ with respect to p.

The necessary and sufficient conditions expressing the fact that the given curve (10) in enveloped by the circle (C) are

$$\sum_{i=1}^{n} l_i \cos \varphi_i = X(p) \mp r \frac{Y_p}{\sqrt{X_p^2 + Y_p^2}} \quad , \quad \sum_{i=1}^{n} l_i \sin \varphi_i = Y(p) \pm r \frac{X_p}{\sqrt{X_p^2 + Y_p^2}} \, . \tag{12}$$

Writing the equations (12) in the form

$$F_1(q_1, q_2, \cdots, q_k, p) = 0 \quad , \quad F_2(q_1, q_2, \cdots, q_k, p) = 0 \, , \tag{13}$$

one observes that, for a given p, only two generalized coordinates may be determined, while the remaning $k-2$ of them can be chosen by adding some supplementary conditions of optimality, or some other conditions of interest in the considered concrete problem, such as to satisfy the existence of the implicit function theorem. Thus, if one calculates q_1 and q_2 and chooses the other $k-2$ Lagrangean co-ordinates to satisfy the inequality

$$\frac{\partial F_1^*}{\partial q_1}\frac{\partial F_2^*}{\partial q_2}-\frac{\partial F_1^*}{\partial q_2}\frac{\partial F_2^*}{\partial q_1}\neq 0, \tag{14}$$

one obtains

$$F_1^*(q_1, q_2, p)=0 \quad , \quad F_2^*(q_1, q_2, p)=0, \tag{15}$$

or $q_1=q_1(p)$ and $q_2=q_2(p)$.

Because the most rational kinematic chain, by means of which mechanisms with two degrees of freedom can be designed, pertains, taking into account the Assur classification [10], to the second class group of the second order, one develops in what follows a synthesis of the required mechanisms with two degrees of freedom.

5. Let us consider the equidistant curve Γ' and a pole $P(-a,-b)$ (fig. 2). Let

$$R=\sqrt{(X_0+a)^2+(Y_0+b)^2} \tag{16}$$

be the modulus of the vector radius $\overrightarrow{PM}$ of a point $M\in\Gamma'$. Taking into account two conveniently dimensioned vectors $\overrightarrow{A}$ and $\overrightarrow{B}$, one obtains $A+B\geq R_{max}$ and $A-B\leq R_{min}$, or $B\geq\frac{1}{2}(R_{max}-R_{min})$ and $R_{max}-B\leq$ $\leq A\leq B+R_{min}$, and, consequently, to obtain convenient dimensions for the moduli of $\overrightarrow{A}$ and $\overrightarrow{B}$, the magnitudes R_{max} and R_{min} must be determined.

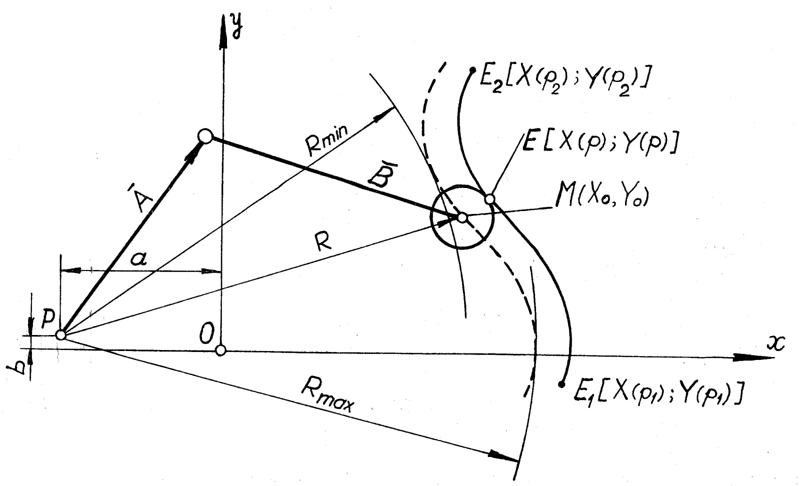

Fig. 2.

Solving the equation

$$[(X_0+a)^2+(Y_0+b)^2]^{-1/2}\left[(X_0+a)\frac{dX_0}{dp}+(Y_0+b)\frac{dY_0}{dp}\right]=0 \tag{17}$$

with respect to p and assuming that only a portion of the considered curve Γ is enveloped, the following two possible cases may occur:

a) If the values of the roots of (17) are included within the limit values p_1 and p_2 corresponding to the extremities $E'_1(p_1)$ and $E'_2(p_2)$ of Γ', we shall calculate the values of R for these roots and, comparing them to the values of R corresponding to p_1 and p_2, one determines the requested R_{max} and R_{min};

b) If between the values p_1 and p_2 there is any one root of the equation (17), it results that R_{max} and R_{min} are obtained by calculating the values of R for p_1 and p_2.

Obviously, when the curve Γ is continuous and closed, the values of p_1 and p_2 can be determined taking into account exclusively the roots of the equation (17).

The values of p_1 and p_2 can also be determined using graphical methods [11] without influencing the mechanism's working accuracy.

As the movement of the vector $\vec{B}$ determines the movement of the plane of the connecting rod, one can chose another vector $\vec{C}$, rigidly connected to $\vec{B}$, in the top of which one fixes a *rotoid couple*. Consequently, the vectors $\vec{A}$ and $\vec{C}$ will determine a second class group, or a *dyad* PNM (fig. 3).

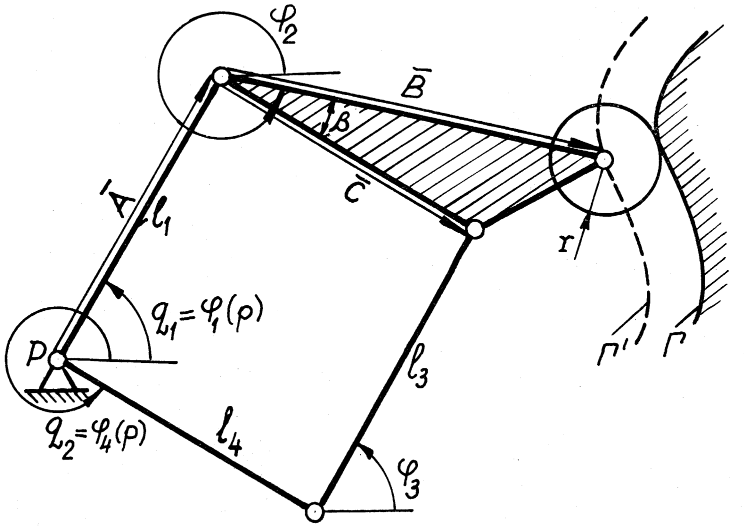

Fig. 3.

By adjoining another second class group (or another dyad), identical to the first and such that a parallelogram is formed (fig. 3), we obtain a mechanism with two degrees of freedom, whose Lagrangean co-ordinates will be

$$q_1 = q_1(p) \quad \text{and} \quad q_2 = q_2(p).$$

For this mechanism the relations (12) become

$$
(18) \qquad \begin{cases} l_1 \cos \varphi_1 + l_2 \cos \varphi_2 = X(p) \pm r \dfrac{Y_p}{\sqrt{X_p^2 + Y_p^2}} , \\ l_1 \sin \varphi_1 + l_2 \sin \varphi_2 = Y(p) \mp r \dfrac{X_p}{\sqrt{X_p^2 + Y_p^2}} . \end{cases}
$$

To this system one can add the relations (fig. 3)

$$
\varphi_4 = \varphi_2 - \beta \quad , \quad \varphi_1 = \varphi_3 .
$$

If the circle (C) (fig. 3) is considered as representing a grinding wheel, the mechanism, conveniently dimensioned, may constitute a manufacturing device for a profile grinding machine (fig. 4). The realization of the motion of the drive shafts is made possible in our case using small step by step motors (4, fig. 4). These small motors may be run by impulses whose frequencies

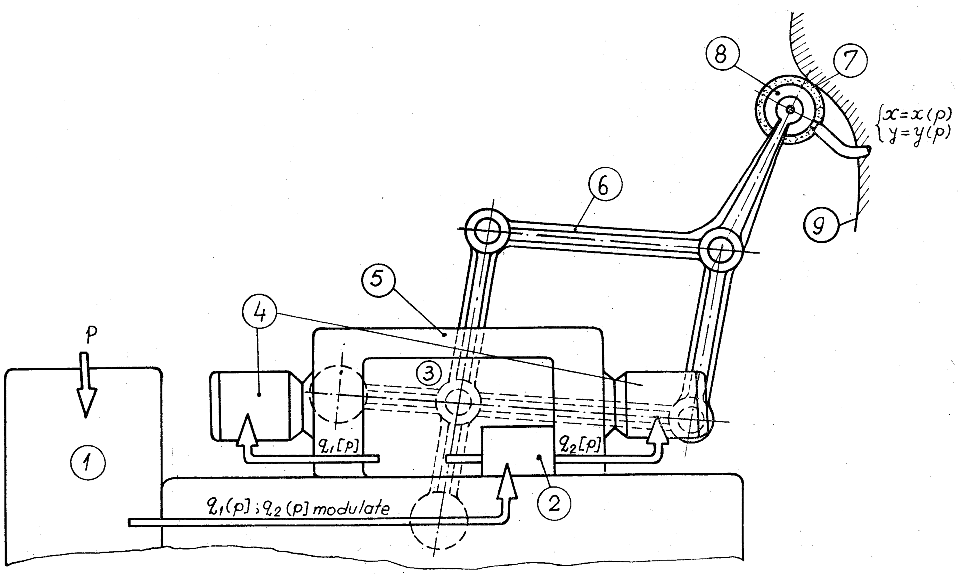

Fig. 4.

are to be proportional with the values of φ_1 and φ_2, obtained from the laws of motion (18). The designed profile grinding machine (fig. 4) works as follows:

The relations (18) transcribed in a codified system, corresponding to one of the current ALGOL or FORTRAN languages, are introduced in a digital computer (1, fig. 4), programmed to compute for the series of values given to p the values of $q_1(p)$ and $q_2(p)$, and, subsequently, to convert them into impulses of the corresponding frequencies. The latter, after being inscribed on a tape recorder, are transmitted into the device or may even reach the amplifier (3, fig. 4) directly. The signals are further put out and received by the above mentioned small motors and transformed into corresponding angular velocities. Each of the motors (4, fig. 4) runs a duplex worm gear, to avoid random looseness, which is set up in a reducing train of gears (5, fig. 4). The worm gears will set in motion the drive shafts (6, fig. 4) of the designed machine tool, whereas the grinding wheel (8, fig. 4), powered by a turbine (7, fig. 4), will envelope the requested profile (9, fig. 4). Due to the use of a reducing train of gears the roughness error due to the generally permanent transient runing of the electric motors is less than 10^{-5} mm for the usual transmission ratios of the worm gears.

We shall not comment further on the dynamic analysis of the designed machine tool.

Finally, the system (18) will be solved and transcribed into ALGOL code.

6. For a given value of p, the relations (18) may be written

$$l_1 \cos \varphi_1 = a - l_2 \cos \varphi_2 \quad , \quad l_1 \sin \varphi_1 = b - l_2 \sin \varphi_2 \tag{19}$$

and, subsequently, one obtains

$$a \frac{2 \tan \frac{1}{2} \varphi_2}{1 + \tan^2 \frac{1}{2} \varphi_2} + b \frac{1 - \tan^2 \frac{1}{2} \varphi_2}{1 + \tan^2 \frac{1}{2} \varphi_2} = t\,, \tag{20}$$

where $t = \frac{1}{2}(a^2 + b^2 + l_2^2 - l_1^2)/l_2$. After some transformations, there results $(t - b) \tan^2 \frac{1}{2} \varphi_2 - 2a \tan \frac{1}{2} \varphi_2 + t - b = 0$, and using $t - b = g$, the former yields $q \tan^2 \frac{1}{2} \varphi_2 - 2a \tan \frac{1}{2} \varphi_2 + q = 0$. Hence

$$\varphi_2 = \arctan (a \pm \sqrt{a^2 - q^2})\,. \tag{21}$$

In the same way one obtains φ_1.

7. ALGOL code programming (7).

Procedure. Solve Syst $(n, p, l_1, l_2, r, x, y)$. Result: (φ_1, φ_2).

Value p, l_1, l_2, r, n; *integer* n; *real* l_1, l_2, r;

real procedure x, y; *array* φ_1, φ_2, p; *begin real* DENOM a, b, c, q, t, u; *integer* i, j; *for* $j = 1$ *stop* 1 *until* n *yes*; *begin* $t : p[j]$.

DENOM $i = sqrt(y^2(t) + x^2(t))$;

for $i = 1, 2$ *yes*;

begin $a : x(t) + (-1)^i \times r \times y(t)$ /DENOM;

$b : y(t) + (-1)^{i-1} \times r \times x(t)$ /DENOM;

$q : (a^2 + b^2 + l_2^2 - l_1^2)/(2 \times l_2) - b$;

$u^2 : a^2 - q^2$;

(7) The authors are pleased to underline here M. Picone's world wide contributions to the development and progress of computational methods [12].

if $u \geq 0$ *then* $u : sqrt(u)$ *else yes* E;
$\varphi_2\,[p\,,\,i] : = 2 \times \text{arc tan}\ (a - u)$
$c : (a - l_2 \times \sin \varphi_2\,[t\,,\,i])/l_1$;
u: $1 - c^2$
if $u \geq 0$ *then* $u : sqrt(u)$ *else yes* E;
$\varphi_1\,[t\,,\,i] : 2 \times \text{arc tan}\ (1 - u)$;
E: *end*
end
end.

In one of our next Notes we shall expose an algorithm for producing *plane* envelope surfaces in space mechanisms consisting of complex kinematic chains with applications in digital control of machine tools.

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