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# Rendiconti

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## The common fixed points for many parameters

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<http://www.bdim.eu/item?id=RLINA_1969_8_47_3-4_173_0>

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**Topologia.** — *The common fixed points for many parameters.* Nota [(*)] di WŁODZIMIERZ HOLSZTYŃSKI, presentata dal Socio B. SEGRE.

SUNTO. — Si dimostra che, se X è uno spazio di Hausdorff compatto tale che il prodotto $X \times I^n$ di X per il cubo ad $n$ dimensioni $I^n$ gode della proprietà del punto fisso, allora per ogni applicazione continua $f: X \times I^n \to X$ esiste un $x \in X$ tale che $\dim \{t \in I^n : f(x, t) = x\} \geq$ $\geq n$–dim X.

The aim of this short Note is to prove the following.

THEOREM. – *Let* X *be a Hausdorff compact space such that the product* $X \times I^n$ *of* X *and the finite-dimensional cube* $I^n$, *has the fixed point property. Then, for any continuous mapping* $f: X \times I^n \to X$, *there exists* $x \in X$ *such that*

$$\dim \{t \in I^n : f(x, t) = x\} \geq n - \dim X.$$

*Proof.* – Let $f: X \times I^n \to X$ be a continuous mapping. We put

$$F = \{(x, t) \in X \times I^n : f(x, t) = x\} \tag{i}$$

and

$$p(x, t) = x \quad , \quad q(x, t) = t \tag{ii}$$

for any $(x, t) \in F$. Then the mapping $q: F \to I^n$ given by (ii) is universal (see [1]), i.e., for any mapping $g: F \to I^n$ there exists a pair $(x, t) \in F$ such that

$$g(x, t) = q(x, t) = t.$$

Indeed, let $g(x, t) \neq t$ for any $(x, t)$ from F and let $g': X \times I^n \to X$ be a continuous extension of $g$. Then the mapping $h: X \times I^n \to X \times I^n$ given by

$$h(x, t) = (f(x, t), g'(x, t)),$$

has not a fixed point. This contradicts the assumption that $X \times I^n$ has the fixed point property. Thus $q: F \to I^n$ is a universal mapping and, consequently (see [1]), $\dim F \geq n$.

Next, by the generalized Hurewicz theorem (see [2]), there exists an $x$ in X such that $\dim p^{-1}(x) \geq n - \dim X$, where $p: F \to X$ is a mapping given by (ii). For such an $x$ the theorem holds.

A part of the above theorem admits the following generalization.

(*) Pervenuta all'Accademia il 25 luglio 1969.

PROPOSITION. – *Let* $X \times Y$ *be a normal space with the fixed point property and* $Y$ *be an* AR–*space. For a continuous mapping* $f : X \times Y \to X$, *we put*

$$F = \{(x, t) \in X \times Y : f(x, t) = x\}.$$

*Then the projection* $q : F \to Y$, *defined by* $q(x, t) = t$, *is a universal mapping.*

## REFERENCES.


[1] W. HOLSZTYŃSKI, *Une generalisation du theorème de Brouwer sur les points invariants*, « Bull. Acad. Polon. Sci. », *12*, 603–606 (1964).

[2] B. PASYNKOV, *On Hurewicz formula*, Vestnik Mosc. Univ., 4 (1965), 3–5 (Russian).