# ATTI ACCADEMIA NAZIONALE DEI LINCEI

CLASSE SCIENZE FISICHE MATEMATICHE NATURALI

# Rendiconti

# ELIO CANNILLO, GIUSEPPE GIUSEPPETTI, CARLA TADINI

## The crystal structure of asbecasite

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **46** (1969), n.4, p. 457–467. Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA\_1969\_8\_46\_4\_457\_0>

L'utilizzo e la stampa di questo documento digitale è consentito liberamente per motivi di ricerca e studio. Non è consentito l'utilizzo dello stesso per motivi commerciali. Tutte le copie di questo documento devono riportare questo avvertimento.

Articolo digitalizzato nel quadro del programma bdim (Biblioteca Digitale Italiana di Matematica) SIMAI & UMI http://www.bdim.eu/

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Accademia Nazionale dei Lincei, 1969.

**Mineralogia.** — The crystal structure of asbecasite <sup>(\*)</sup>. Nota di Elio Cannillo, Giuseppe Giuseppetti e Carla Tadini, presentata <sup>(\*\*)</sup> dal Socio G. Carobbi.

RIASSUNTO. — L'asbecasite, silicato di formula Ca<sub>3</sub>(Ti, Sn) [(As<sub>3</sub>SiBeO<sub>10</sub>)<sub>2</sub>], è trigonale, con due unità stechiometriche nella cella elementare. Il gruppo spaziale è  $P_3c_1$  e le costanti reticolari sono: a = 8,36, c = 15,30 Å. La struttura cristallina è stata studiata utilizzando 695 effetti di diffrazione di raggi X registrati con la camera di Weissenberg. I parametri degli atomi sono stati ricavati con i consueti metodi di Patterson e di Fourier e raffinati col metodo dei minimi quadrati fino a un fattore di discordanza finale R = 0.054 per i riflessi osservati. Nella struttura sono presenti strati di poliedri paralleli a (0001) che si susseguono nell'ordine ...AABAAB... Lo strato A è costituito da tetraedri BeO<sub>4</sub>, SiO<sub>4</sub> e da piramidi AsO<sub>3</sub>; lo strato B da ottaedri (Ti, Sn)O<sub>6</sub> e antiprismi quadrati CaO<sub>8</sub>. Le distanze di legame e l'equilibrio elettrostatico che ne risultano presentano qualche particolarità che viene discussa.

#### INTRODUCTION.

Two new minerals, asbecasite and cafarsite, were found by Graeser in the gneiss of the Monte Leone nappe in the southern part of the Binnatal (Switzerland).

The morphological, optical, physical and chemical properties of asbecasite were fully described by Graeser himself [1] and we shall report only those features that are necessary for the following discussion. The space group is  $P_3 c_1$ , the lattice parameters are: a = 8.33, c = 15.29 Å, Z = 3. The chemical formula reported by Graeser is:

 $Ca_2Si_{1.5}Be_{0.75}Ti_{0.5}Al_{0.2}Sn_{0.1}Tl_{0.03}(AsO_3)_5$ .

More recently Strunz [2] suggested the following alternative formula (Z = 2), more consistent with the space group:

 $Ca_3(Ti,Sn)BeSi_2(O_3As_6O_{18}),$ 

but it is noteworthy that the resulting balance of the valences is unsatisfactory.

#### EXPERIMENTAL.

A specimen of asbecasite, with spherical shape (radius 0.19 mm), was used to collect the X-rays diffraction data. A redetermination of the cell dimensions carried out by means of precession and Weissenberg photographs was in accordance with the previous results, and gave the following values:

$$a = 8.36 \pm 0.02 \text{ Å}$$
  
 $c = 15.30 \pm 0.03 \text{ Å}.$ 

(\*) This work was performed in the Sezione di Pavia del Centro Nazionale di Cristallografia del C.N.R., Istituto di Mineralogia dell'Università, via Bassi 4, 27100 Pavia (Italy).

(\*\*) Nella seduta dell'8 marzo 1969.

Equi-inclinated Weissenberg photographs of the integrated reflections were taken by the multiple-film technique and CuK $\alpha$  radiation, b being the rotation axis and k ranging from 0 to 8. The intensities were measured by a Nonius microdensitometer and a total of 695 independent reflections out of about 800 present in the CuK $\alpha$  limiting sphere (87%), were examined. It was found that 122 of them could not be detected and a value of half of the minimum observable intensity was assigned to them.

Corrections were made for the absorption (linear coefficient  $\mu = 283 \text{ cm}^{-1}$  for CuKa) and for the incipient but incomplete  $\alpha_1 - \alpha_2$  separation.

#### CRYSTAL STRUCTURE DETERMINATION AND REFINEMENT.

With these data, a three-dimensional Patterson synthesis was calculated and the heavy atoms, arsenic and titanium were situated according to the equivalent points of the space group  $P\bar{3}cI$ . On this basis a three-dimensional electron density synthesis was successively obtained, on which new maxima appeared reasonably due to Ca atoms and to Si atoms, as well as possible positions for forty oxygens surrounding the heavy atoms. Finally some maxima located in a special position, with four-fold multiplicity, could reasonably be attributed to Be atoms. The above results do not agree with the chemical formula suggested by Strunz, which shows two Be atoms, instead of four, and forty-two oxygens instead of forty.

Yet, at this stage, we assumed that the structural model we had obtained could be taken as the starting point for the refinement. The refinement was carried out by means of some cycles of least squares (full matrix, isotropic temperature factors, equal weight for all the observed reflections). The structure factors were calculated by using the HFS scattering factors for neutral atoms published by Hanson, Herman, Lea and Skillman [3]. The scattering amplitudes for Ti take into account the presence of a 20 % Sn.

During the former cycles the temperature factors of three or four out of the nine independent atoms became always negative, although the R factor dropped from 0.17 to 0.08. An inspection of the Fo–Fc table revealed a conspicuous secondary extinction effect. In fact, the agreement between observed and calculated structure factors was improved by excluding from the refinement the most intense reflections. In such a way it was possible to reduce the number of negative temperature factors to only one.

In the meantime a new version of the least squares program became available. This permitted one to take into account the anomalous dispersion in the Fc calculation and to include the secondary extinction as a parameter in the refinement. Some further cycles of least squares allowed the complete elimination of the negative temperature factors, the reduction of the R factor to 0.054 for the observed reflections and to 0.066 for all the reflections, and the improvement of the standard deviations. The final parameters are reported in Table I. In the final calculation of the Fc's, presented in Table II, the small contributions of the imaginary components of the anomalous dispersion are neglected. The real components are those given by Cromer [4]. In the formula of the secondary extinction, given by Zachariasen [5] and rearranged in order to apply the correction to the Fc's, the terms involving the absorption were neglected. The final value of "g", secondary extinction parameter, is  $6.2 \cdot 10^{-6}$ .

| Atoms    | x/a          | y/b          | z/c           | В         |
|----------|--------------|--------------|---------------|-----------|
| As       | 0.01840 (14) | 0.29632 (14) | 0.10361 (6)   | 0.35 (3)  |
| (Ti, Sn) | о            | 0            | 1/4           | 0.66 (5)  |
| Si       | 2/3          | 1/3          | 0.07320 (28)  | 0.20 (6)  |
| Са       | 0.59205 (31) | 0.59205 (31) | 3/4           | 0.38 (4)  |
| O(I)     | 0.89326 (97) | 0.77841 (97) | 0.17534 (41)  | 0.54 (10) |
| O(2)     | 0.53703 (96) | 0.72118 (96) | 0.17392 (42)  | 0.56 (10) |
| O(3)     | 0.59836 (97) | 0.46997 (97) | 0.11721 (43)  | 0.78 (11) |
| O(4)     | 1/3          | 2/3          | 0.03008 (85)  | I.06 (20) |
| Be       | 1/3          | 2/3          | 0.13011 (126) | 0.22 (29) |

 TABLE I.

 Final atomic parameters and their standard deviations (in parentheses)

| Table | I | I. |
|-------|---|----|
|-------|---|----|

#### Structure factors of asbecasite.

Reflections marked with an asterisk were unobservably weak;

in this case Fo derives from  $0.5\ I_{\rm min}\,.$ 

| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $ | h l  | 10F0   | 10F <sub>c</sub>   | h   | I  | юF <sub>o</sub>   | 10Fc  | h   | Z  | юFo   | 10F <sub>c</sub>   | h  | I   | 10F0  | юF <sub>c</sub>              |
|---|--|--|--|---|--|---|---|---|--|---|--------------------|--|---|---|------------------------------|
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 1016<br>1992<br>1987<br>1515<br>1292<br>1151<br>1292<br>1151<br>1292<br>495<br>495<br>390<br>1292<br>1151<br>1292<br>495<br>495<br>390<br>1292<br>1151<br>1292<br>1151<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1292<br>1155<br>1295<br>1155<br>1295<br>1155<br>1295<br>1155<br>1295<br>1305<br>586<br>641<br>1130<br>1305<br>586<br>641<br>1130<br>1305<br>586<br>641<br>1130<br>1305<br>586<br>641<br>1130<br>1303<br>586<br>1303<br>586<br>1303<br>587<br>332<br>586<br>1303<br>588<br>1303<br>588<br>1303<br>588<br>1303<br>588<br>1303<br>588<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>1303<br>130<br>130 | $\begin{array}{c} 10F_{c} \\ \hline \\ $ | A           2           3           3           3           3           3           3           3           2           2           2           2           3           3           3           3           3           3           3 | $\begin{array}{c} & & \\ & -2 \\ & -4 \\ & -4 \\ & -6^* \\ & -6^* \\ & -10 \\ & -10 \\ & -12 \\ & -14^* \\ & -14^* \\ & -14^* \\ & -18 \\ & -1$ | 10F<br>692<br>784<br>1678<br>183<br>855<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1375<br>1474<br>1474<br>1474<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498<br>1498 | $\begin{array}{c} {}_{10}{\rm F_{c}} \\ \hline \\ -739 \\ 324 \\ 1729 \\ -156 \\ -846 \\ 1960 \\ -9160 \\ 1468 \\ 1960 \\ -9160 \\ -1261 \\ 2655 \\ 339 \\ -229 \\ -226 \\ 687 \\ 673 \\ -528 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ 2076 \\ -528 \\ -518 \\ -$ | 3         4         5         5         5         5         5         5         5         5         5         5 | 2<br>14<br>-14<br>$16^{*}$<br>$-16^{*}$<br>-18<br>$0^{*}$<br>$-2^{*}$<br>4<br>-4<br>-6<br>8<br>-8<br>$-10^{*}$<br>$-2^{*}$<br>4<br>-4<br>-6<br>$-10^{*}$<br>$-10^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-10^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-2^{*}$<br>$-2^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-10^{*}$<br>$-2^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*}$<br>$-4^{*$ | 10F <sub>0</sub><br>1064<br>1034<br>577<br>286<br>302<br>1404<br>121<br>1115<br>943<br>819<br>1395<br>858<br>871<br>548<br>1511<br>123<br>558<br>1027<br>1101<br>293<br>640<br>1825<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017<br>1017 | IOF <sub>c</sub>   | λ<br>5555555555555555555666666666666666666 | /<br>8<br>-10<br>-10<br>-12<br>14<br>-14<br>-14<br>-16<br>-16<br>-16<br>-2<br>-2<br>-2<br>-4<br>-4<br>-4<br>-6<br>-6<br>8<br>-10<br>-10<br>-12<br>-12<br>-12<br>-12<br>-12<br>-12<br>-12<br>-12 | 10F <sub>0</sub><br>1363<br>153<br>990<br>1333<br>125<br>900<br>365<br>1406<br>875<br>406<br>51<br>1221<br>157<br>175<br>231<br>1918<br>400<br>183<br>208<br>924<br>454<br>678<br>133<br>125<br>135<br>125<br>157<br>157<br>157<br>157<br>157<br>157<br>157<br>15 | 10F <sub>c</sub><br>1354<br> |
|   | 2 0<br>2 2   | 325<br>1411  | 256<br>1487  | 3<br>3<br>3   | -10<br>12<br>-12   | 765<br>272  | 1123<br>778<br>212  | 5   | 4<br>6*<br>6   | 1775<br>128<br>821  | 1832<br>30<br>—809 | 7<br>7<br>7                                | 2<br>4<br>4   | 351<br>498<br>980   | —290<br>—455<br>—984         |

35. — RENDICONTI 1969, Vol. XLVI, fasc. 4.

## TABLE II (continued).

| h   | 1         | юFo           | 10Fc  | h   | I       | 10F0 | 10Fc       | h   | I       | юFo         | юFc  | h   | l   | 10F0       | юF <sub>c</sub> |
|-----|-----------|---------------|-------|-----|---------|------|------------|-----|---------|-------------|------|-----|-----|------------|-----------------|
| 7   | 6         | 1353          |       | 2   | 17      | 337  | 372        | 5   | 2*      | 161         |      | 8   | 2   | 400        |                 |
| 7   | 6         | 1434          | -1389 | 2   |         | 451  | -457       | 5   | - 3     | 1177        |      | 8   |     | 278        | 270             |
| 7   | 8         | 1016          | 006   | 2   | 18*     | 67   | 26         | 5   |         | 1466        | 1307 | 8   | 2   | 1182       |                 |
| 7   | 8         | 1386          | 1386  | 2   | —18     | 360  | 405        | 5   | 4       | 1135        | 1001 | 8   | 3   | 1100       | 1068            |
| 7   | IO        | 571           | 561   | 3   | 0       | 612  | 681        | Š   |         | 547         | 527  | 8   | 3   | E01        | E 27            |
| 7   | 10        | 908           | 940   | 3   | I       | 520  | 537        | 5   | ,<br>5  | 636         | 586  | 8   |     | 408        | 415             |
| 7   | 12        | 266           | 252   | 3   | —- I    | 1542 | -1517      | 5   |         | 155         |      | 8   | 5   | 578        | 567             |
| 7   | -12       | 578           | 603   | 3   | 2       | 442  | 413        | 5   | Ğ       | 471         | 417  | 8   |     | 351        |                 |
| 8   | 0         | 366           | 310   | 3   | 2*      | 118  | —11<br>—11 | 5   | 6       | 370         | 331  | 8   | 6   | 810        | 856             |
| 8   | 2         | 1064          |       | 3   | 3*      | 173  | 153        | 5   | 7       | 953         | 898  | 8   | 6   | 486        | 402             |
| 8   | 2         | 344           | 348   | 3   | 3       | 337  | 329        | 5   | 7       | 1015        |      | 8   | 7   | 503        | 571             |
| 8   | 4*        | 71            | 56    | 3   | 4       | 987  |            | 5   | 8*      | 197         | -176 | 8   |     | 560        |                 |
| 8   | 4         | 468           | 473   | 3   | 4       | 775  |            | 5   | 8*      | 102         | 206  |     | . 1 | 5-9        | 54*             |
| . 8 | 6*        | 71            | 6     | 3   | 5       | 1034 | 1060       | 5   | 9       | 531         | 499  |     | h   | 2          | 1               |
| 8   | 6         | 508           | -470  | 3   | 5       | 446  | 436        | 5   | 9       | 867         |      |     |     |            |                 |
| 8   | 8         | 824           | 848   | 3   | 6       | 895  | 927        | 5   | IO      | 827         | -802 | 2   | 0   | 362        | 273             |
| 8   | 8*        | 71            | 72    | 3   | 6       | 518  | 545        | 5   | -10     | 567         | 558  | 2   | ı*  | 130        | 171             |
| 8   | IO        | 135           | 122   | 3   | 7       | 783  | -75I       | 5   | II      | 859         | 856  | 2   | 2*  | 188        | 196             |
| 8   | 10        | 439           | 446   | 3   | 7       | 809  | 828        | 5   | II      | 734         | 730  | 2   | 3   | 1240       | 1363            |
| 9   | 0         | 185           | 140   | 3   | 8       | 836  | 835        | 5   | 12      | 500         | 491  | 2   | 4   | 1165       | 1244            |
| 9   | 2         | 707           | —761  | 3   | 8       | 710  | 748        | 5   | 12      | 251         | 221  | 2   | 5*  | 134        | 108             |
| 9   | 2         | 340           | 390   | 3   | 9       | 442  | -434       | 5   | 13      | 783         | 779  | 2   | 6*  | 139        | —51             |
| 9   | 4*        | 43            | —18   | 3   | 9       | 820  | 799        | 5   | —13     | 1003        | 1055 | 2   | 7   | 1022       | -1055           |
| 9   | 4         | 837           | 868   | 3   | IO      | 1125 | 1105       | 5   | 14      | 391         | 390  | 2   | 8   | 323        | 255             |
|     |           |               |       | 3   | -10     | 413  | 405        | 5   | 14      | 538         | 534  | 2   | . 9 | 667        | 678             |
|     | h         | I             | l     | 3   | II      | 379  |            | 5   | 15      | 378         | 379  | 2   | 10  | 1055       | -1083           |
|     |           |               |       | 3   | —11*    | 138  | 74         | 5   | 15      | 195         | 189  | 2   | II  | 922        | 955             |
| I   | 0         | 472           | 492   | .3  | 12*     | 186  | 200        | 6   | 0*      | 130         | 118  | 2   | 12  | 297        | 294             |
| I   | I         | 1025          | 1045  | 3   | -12     | 666  | 667        | 6   | r       | 339         | 304  | 2   | 13  | 509        | 501             |
| I   | 2*        | 108           | 50    | 3   | 13      | 1005 | 1041       | 6   | — 1     | 890         | 854  | 2   | 14  | 630        | 645             |
| I   | 3         | 2147          | 2043  | 3   | -13     | 443  | 457        | 6   | 2*      | 173         | 93   | 2   | 15* | 107        | 40              |
| Ι   | - 4       | 984           | 1055  | 3   | I4      | 527  | 594        | 6   | $2^{*}$ | 136         | 146  | 2   | 16  | 514        | 535             |
| I   | 5         | 927           | 960   | 3   | 14      | 602  | 623        | 6   | 3*      | 147         | 122  | 2   | 17  | 807        | 851             |
| I,  | 6         | 680           | 633   | 3   | 15      | 213  | 184        | 6   | 3       | 675         | 618  | 2   | 18  | 634        | 687             |
| I   | 7         | 1344          | 1407  | 3   | 15      | 74I  | 784        | 6   | 4*      | 160         | -139 | 3   | 0   | 395        |                 |
| I   | 8         | 702           | 704   | 3   | 16      | 526  | 557        | 6   |         | 249         | 24 I | 3   | I   | 595        | 619             |
| I   | 9         | 1198          | 1177  | 3   | 16*     | 88   | 39         | 6   | 5       | 863         | 812  | 3   | I   | 928        |                 |
| I   | 10        | 1027          | 1078  | 3   | 17*     | 109  | 67         | 6   | 5       | 236         | 221  | 3   | 2   | 600        | 606             |
| I   | II        | 1524          | 1572  | 3   | -17*    | 75   | 35         | 6   | 6       | 564         | 550  | 3   | -2  | 1730       | —1650           |
| I   | 12*       | 135           | 72    | 3   | 18      | 179  | 173        | 6   | 6*      | 129         | 82   | 3   | 3   | 1303       | 1373            |
| I   | 13        | 898           | 941   | 3   | 18      | 536  | 595        | 6   | 7       | 236         | 176  | 3   | 3   | 1013       | 1040            |
| I   | 14        | 872           | 916   | 4   | 0       | 432  | 400        | 6   | -7      | 379         | 364  | 3   | 4   | 1743       | 1642            |
| r   | 15        | 259           | 224   | - 4 | I       | 669  | 674        | 6   | . 8     | 210         | 180  | 3   | 4   | 937        | 969             |
| I   | 10        | 400           | 513   | 4   | —- I ,  | 727  | 744        | 6   | 8       | 337         | 337  | 3   | 5   | 246        | 248             |
| I   | 17        | 1240          | 1390  | 4   | 2*      | 169  | 201        | 6   | 9       | 287         | 243  | 3   | 5*  | 188        | 166             |
| I   | 18        | 520           | 507   | . 4 | -2      | 265  | 267        | 6   | -9      | 550         | 544  | 3   | 6*  | 136        | 48              |
| I   | 19        | 151           |       | 4   | 3       | 955  | 954        | 6   | IO      | 727         | 740  | 3   | 6   | 632        | 640             |
| 2   | 0         | 500           |       | 4   | 3       | 606  | 604        | 6   | 10      | 184         |      | 3   | 7   | 1132       | -1125           |
| 2   | I         | 1019          |       | 4   | 4       | 485  | 481        | 0   | II      | 696         | -687 | 3   | 7   | 1118       | 1116            |
| 2   | -1        | 035           | 054   | 4   | 4<br>*  | 419  | 422        | 0   | 11      | 224         | -219 | 3   | 8   | 598        | 561             |
| 2   | 2         | 60.           |       | 4   | 5       | 199  | 210        | 0   | 12      | 325         | 315  | 3   | 8   | 683        | 664             |
| 2   | 2         | 004           | 737   | 4   | 5"      | 170  | 150        | 0   | 12      | 310         | 302  | 3   | 9   | 301        | -343            |
| 2   | 3         | 352           |       | 4   | 0<br>4* | 282  | 174        | 0   | 13      | 548         | 574  | 3   | 9   | 462        | 462             |
| 2   | 73        | 074           | 914   | 4   | 0       | 150  | 70         | 0   | -13.    | 82          | 01   | 3   | 10  | 1325       | 1323            |
| 2   | 4         | 410           | 477   | 4   | 7       | 1040 | -001       | 7   |         | 000         | 031  | 3   | 10  | 939        | 933             |
| 2.  |           | 929           |       | 4   |         | 003  | 700        | 7   | 1       | 505         | -455 | 3   | II  | 030        | 079             |
| 2   |           | 429           | 300   | 4   | 0*      | 352  | 315        | 1 2 | 1       | 510         | -473 | 3   |     | 952        | 948             |
| ~   | 6         | 5/1           | 305   | 4   |         | 139  |            | . 7 | 2       | 547         | 523  | 3   | 12  | 728        | 722             |
| 2   | 6         | 043           | 015   | 4   | 9       | 150  | 91         | 7   | 2       | 291         | 240  | 3   | 12  | 374        | 305             |
| 2   |           | 2/9           |       | 4   | 9       | 405  | 423        | 1 7 | 3       | -6.         | 575  | 3   | 13  | 001        | 047             |
| 2   |           | 1294          | 1344  | 4   | 10      | 530  |            | 7   |         | 104         | -100 | 3   | -13 | 575        | 586             |
| 2   | Q*        | 1203          |       | 4   |         | 570  | 575        | 7   | 4       | 133         | 05   | 3   | 14* | 102        | . 71            |
| 2   | 8*        | 139           | . 107 | 4   | 11      | 551  | 531        | 7   |         | 150         | 137  | 3   | 14" | 143        |                 |
| ~   |           | 139           | 700   | 4   |         | 473  | 440        | 7   | 5       | 300         | 204  | 3   | 15" | 131        | -153            |
| 2   | 0*        | 141           | 133   | . 4 |         | 309  | 350        | 1   | 5       | 057         | 820  | 3   |     | 420        | 397             |
| 2   | 70        | 1004          |       | 4   | 12      | 607  |            | 1 / | 6       | /30         | /25  | 3   | 10  | 500        | 510             |
|     | -10       | 202           | 1200  | 4   | 13      | 607  | 608        | 17  | 0       | 354         | 325  | 3   |     | 735        | 735             |
| õ   | 10        | 675           |       | 4   | 13      | 001  | 000        | 17  | *       | 134         | 107  | 3   | 17  | 057        | -717            |
| 20  | *         |               | 002   | 4   | 14      | 329  | 311        | 7   | 0*      | 07          | 45   | 3   | 17  | 572        | 000             |
| 2   | ***       | - 143         | 90    | 4   |         | 324  | 291        | 7   | 0       | 125         |      | 4   | 0   | 415        | 383             |
| 2   | ал<br>то  | 130           | 42    | 4   |         | 300  | 330        | 1 7 |         | 353         | 324  | 4   | 1   | 757        | 703             |
| 2   | T 2       | 450           | 440   | 4   | 15      | 207  | 202        |     | 9       | 055         |      | 4   | 1   | 1210       | 1202            |
| 2   |           | 822           | 844   | 4   |         | 240  | 235        | 1 7 | ***     | 319         | -273 | 4   | - 2 | 147        | 23              |
| 2   | ±3<br>∓∡* | - 107         | . 044 | 4   | 10      | 419  | 430        | 1 7 | 10      | 01          | 15   | 4   | 2   | 794        | 774             |
| 2   |           | 121           |       | 4   |         | 212  | 503        | 1 7 |         | 77          | 24   | 4   | 3   | 070        | 027             |
| 2   | -4<br>TE  | = 1/2<br>= 80 | 607   | 4   | -1/     | 310  | -1040      |     |         |             |      | 4   | 3   | 753        | 710             |
| 2   |           | 500           |       | , D | -<br>-  | 1104 |            |     |         | 544<br>E 46 | 550  | 4   | 4   | 347        | 332             |
| 2   | +5        | 109           | 244   | 2   |         | 1190 | 54         | R   |         | 202         |      | 4   | 4   | 400<br>8-6 | 301             |
| 2   |           | 331           | 344   | 2   | -1      | 154  | /01        | 8   | 1<br>TT | 300         |      | 4   | 5   | 070        |                 |
| 2   |           |               | ~+    | 0   | ~       | 540  | 3-+        | 1   | -       | ~/0         | -54  | . 4 | 5   | 320        | 323             |

### TABLE II (continued).

|    |              |                        |                  |     |                  |             |                 |     |            |      |            |      | -      |      |              |
|----|--------------|------------------------|------------------|-----|------------------|-------------|-----------------|-----|------------|------|------------|------|--------|------|--------------|
| h  | 1            | юFo                    | тоF <sub>c</sub> | h   | l                | 10F0        | юF <sub>c</sub> | h   | 2          | 10F0 | 10Fc       | h    | l      | 10F0 | 10Fc         |
| 4  | 6*           | 186                    | 178              | 6   | 9                | 879         | 908             | 4   | 8          | 601  | 565        | 7    |        | 75   |              |
| 4  | 6            | 785                    | 759              | 6   | 9                | 300         | 282             | . 4 | 8          | 1007 | 060        | 1 7  | 4*     | - =8 |              |
| 4  | 7            | 1372                   | 1247             | 6   | 10               | 853         |                 | 4   | 0          | 627  | 600        | 7    |        | 258  |              |
| 4  | -7           | 1482                   |                  | 6   | 10               | 239         |                 | 4   |            | 100  | 186        | 7    | 5      | 305  | 337          |
| 4  | 8            | 397                    | 358              | 6   | II               | 837         | 859             | 4   | 10         | 675  | 638        | 1 7  |        | 393  | 214          |
| 4  | 8*           | 142                    |                  | 6   |                  | 943         | -975            | 4   | —10        | 1033 | 1062       | 1 1. | 5      | 5-4  | . 3*4        |
| 4  | 9            | 439                    | 428              | 6   | 12*              | 60          | 66              | 4   | II         | 302  | 254        |      | h      | 4    | 1            |
| 4  | 9            | 615                    | 592              | 6   | 12               | 222         | 218             | 4   |            | 40T  | 376        |      |        | 4    | v            |
| 4  | 10*          | 131                    | 38               | 7   | 0                | 210         |                 |     | T2         | 222  | 160        | 1.   | 0      | 770  | 604          |
| 4  | 10           | 561                    |                  | 7   | . т*             | 88          | 31              |     | -12        | 207  | 262        | 4    | ,<br>, | 507  |              |
| 4  |              | 332                    | -306             | 7   | I                | 587         | 583             | T A | T2         | 105  |            | 4    |        | 307  | 551          |
| 4  | II           | 572                    | 501              | 7   | 2*               | 164         | 101             | T A |            | 200  |            | 4    | 2      | 307  | 301          |
| 4  | 12           | 688                    | 604              | 7   | 2                | 021         |                 | 4   | ± 3<br>T 4 | 309  | 253        | 4    | 3      | 207  | 109          |
| 4  |              | 443                    |                  | 7   | ~<br>~*          | 172         |                 | 4   | 14         | 10/1 | -1100      | 4    | 4      | 420  | 370          |
| 4  | 7.2          | 850                    | 870              |     |                  | 1/2         |                 | 4   |            | 1017 | -1062      | 4    | 5      | 303  | 327          |
| 4  |              | \$62                   | = = 8            |     | 3                | 223<br>F 18 |                 | 5   | 0          | 140  | -100       | 4    | 0      | 503  |              |
| 4  | - 3          | 107                    | 131              | 1   | 4                | 540         | 511             | 5   | -*         | 702  | 710        | 4    | 7      | 010  | 559          |
| 4  |              | -9/<br>TOT             |                  |     | 4                | 300         | 470             | 5   | 1          | 201  | -210       | 4    | .8°    | 102  | 34           |
| 4  | -++<br>T E   | 202                    | -39              | 1 / | 5                | 374         |                 | 5   | 2          | 1139 | 1088       | 4    | 9      | 326  | 309          |
| 4  |              | 292                    | 200              | 7   | 5                | 310         | 311             | 5   | -2*        | 145  | 95         | 4    | 10     | 279  | 262          |
| 4  | 15           | 070                    |                  | 7   | 6                | 405         | 400             | 5   | 3          | 501  | 483        | 4    | II     | 237  | 186          |
| 4  |              | 325                    | 340              | 7   | 0                | 331         | 324             | 5   | <u> </u>   | 644  |            | 4    | 12*    | 67   | 35           |
| 4  | 10           | 442                    | 450              | 7   | 7                | 122         | 127             | 5   | 4*         | 171  | 87         | 4    | 13     | 425  | 400          |
| 5  | 0            | 771                    | 775              | 7   | 7^               | 132         |                 | 5   | 4          | 1265 | 1211       | 5    | 0      | 651  | 637          |
| 5  | 1            | 729                    | 009              | 7   | 81               | 64          | 44              | 5   | 5          | 472  | -417       | 5    | r      | 360  | 323          |
| 5  |              | 202                    | 235              | 7   | 8                | 500         | 479             | 5   | 5          | 390  | -328       | 5    | I      | 203  | 152          |
| 5  | 2            | 530                    | -489             | 7   | - 9              | 252         | 280             | 5   | 6          | 387  | 361        | 5    | 2      | 796  | 736          |
| 5  | 2            | 483                    | 419              | 7   | -9               | 284         | 308             | 5   | 6          | 370  | 337        | 5    | 2*     | 110  | 9            |
| 5  | 3            | 353                    | 286              | 8   | 0                | 688         | 680             | 5   | 7          | 665  | 646        | 5    | 3      | 631  | -572         |
| 5  | 3            | 518                    | 474              | 8   | 1*               | 86          | 27              | 5   | 7          | 537  | 502        | 5    | 3      | 814  | 742          |
| 5  | 4            | 951                    | 897              | 8   | I                | 570         | 622             | .5  | 8          | 905  | 892        | 5    | 4      | 355  | 355          |
| 5  | -4           | 002                    | 528              | 8   | - 2              | 247         | 245             | 5   | - 7        | 441  | -488       | 5    |        | 1022 | 963          |
| 5  | 5            | 446                    | -436             | 8   | 2                | 641         | 598             | 5   | - 9*       | 97   | 64         | 5    | .5*    | 103  | 12           |
| 5  |              | 269                    | 210              | 8   | 3                | 429         | -467            | 5   | -9         | 592  | 558        | 5    | -5*    | 103  | -40          |
| 5  | 6            | 588                    | -566             | 8   | 3                | 308         | 299             | 5   | 10         | 590  | 522        | 5    | 6      | 805  | 780          |
| 5  | 6            | 447                    | -438             |     |                  |             |                 | 5   | 10         | 666  | 623        | 5    | 6      | 413  | 316          |
| 5  | 7*           | 128                    | 5                |     | h                | 3           | 1               | 5   | II         | 511  | 400        | 5    | 7      | 716  | 650          |
| 5  | -7*          | 146                    | 105              | 1.1 |                  |             |                 | 5   |            | 198  | -172       | 5    |        | 632  | 574          |
| 5  | . 8*         | 140                    | 55               | 3   | 0                | 1881        | 1752            | 5   | 12         | 153  | 74         | 5    | 8      | 560  | 480          |
| 5  | 8            | 473                    | 422              | 3   | I                | 512         | 544             | 5   | -12        | 545  | 545        | 5    | 8      | 416  |              |
| 5  | 9            | 274                    | 243              | 3   | 2*               | 113         | 42              | 5   | 13         | 323  | 352        | 5    | 0      | 421  | . 074<br>001 |
| 5  | 9            | 310                    | 242              | 3   | 3                | 153         | -04             | 5   | -13        | 543  | 507        | 5    | 0      | 227  | -240         |
| 5  | 10           | 779                    | -710             | 3   | 4                | 865         | 801             | 6   | 0          | 105  |            | 5    | 10     | 1000 |              |
| 5  | 10           | 906                    | 839              | 3   | 5                | 301         | 252             | 6   | I          | 547  | 501        | 5    |        | 67   | 26           |
| 5  | 11           | 300                    | 274              | 3   | 6                | 543         | -460            | 6   | r          | 575  |            | 6    |        | 455  | 400          |
| 5  | II           | 213                    | 184              | 3   | 7                | 545         | -400            | 6   | 2*         | 120  | 05         | 6    | т      | 604  | 680          |
| 5  | 12           | 596                    | 564              | 3   | 8*               | 167         | 03              | 6   |            | 475  | 447        | 6    |        | 200  |              |
| 5  | -12          | 483                    | 4.59             | 3   | 0                | 205         | 178             | 6   | 2          | 473  | 447        | 6    |        | 299  | 204          |
| 5  | 13           | 180                    |                  | 3   | 10               | 473         | 423             | 6   |            | 80   | 99         | 6    | -      | 78 - | 242          |
| 5  |              | 374                    | -336             | 3   | 11*              | 126         |                 | 6   |            | 80   |            | 6    | 2      | 105  | 140          |
| 5  | 14*          | 60                     |                  | 3   | 12               | 303         | 252             | 6   |            | 600  | =20<br>=20 | 6    | 3      | 115  | 90           |
| 5  |              | 167                    | -133             | 3   | 13               | 417         | 406             | 6   |            | 282  | 300        | 6    | 3      | 6-6  | -620         |
| 6  | 0            | 701                    | 751              | 3   | -5<br>T4         | 676         |                 | 6   |            | 101  |            | 6    | 4      | 050  |              |
| 16 | 1*           | 126                    | 114              | 2   | - <del>-</del> - | 212         |                 | 6   | 6          | 404  | 329        | 6    | 4      | 145  | - 40         |
| 6  | — T          | 66 T                   |                  |     | -5               | 1406        | 292             | 6   | 6          | 240  | 202        | 6    | 5      | 502  | 500          |
| 6  | 2            | 212                    | 104              |     | T                | 610         | - 347           | 6   | . 0        | 403  | 3/1        | 6    | 5      | 033  | 779          |
| 6  | 2            | 811                    |                  |     | T                | 228         | 337             | 6   | /          | 600  |            | 6    | 0      | 100  |              |
| 6  | 3            | 1206                   | 7/7<br>T 2 2 2   | 4   | 2                | 330         | 303             | 6   | 6          | 092  | 043        | 6    | 0      | 439  |              |
| 6  | 3            | 1170                   | -1164            | 4   |                  | 199         |                 | 6   | 0          | 192  | 170        | 0    | 7      | 297  | 219          |
| 6  |              | 801                    | 770              | 4   | 5                | 405         |                 | 6   | 0          | 545  | 541        |      | 7      |      | 7            |
| 6  |              | 556                    | 1/9              | 4   | د<br>*           | 337         | -300            | 6   | 9          | 235  | 195        |      | п      | 5    | ı            |
| ĕ  | *            | 550                    | 495              | 4   | 3                | 602         |                 | 2   | 9          | IOQ  | -108       |      |        |      |              |
| 6  | 3            | 686                    | 640              | 4   | 4                | 089         |                 | 0   | 10         | 455  |            | 5    | 0      | 412  | 453          |
| 6  | 6            | 000                    | 049              | 4   | 4                | 1012        | 900             | 0   | 10         | 153  | -146       | 5    | I      | 471  | 554          |
| 6  |              | 250<br>20 <sup>9</sup> | 201              | 4   | 5                | 292         |                 | 7   | 0          | 853  | 846        | 5    | 2      | 390  | 400          |
| 6  | -0           | 330                    | 317              | 4   | -5               | 743         | -713            | 7   | I          | 290  | 246        | 5    | 3      | 557  | 635          |
| 6  | <u></u>      | 0 <u>9</u> 2           |                  | 4   | 0                | 994         | 905             | 7   | I          | 406  | 426        | 5    | 4      | 676  | 769          |
| 6  | 6            | 007                    | 097              | 4   | 0                | 913         | —895            | 7   | 2*         | 79   | 67         | 5    | 5      | 222  | 223          |
| 6  | Q            | ×45                    |                  | 4   | 7                | 472         | -430            | 7   | -2         | 483  | 444        | 5    | 6      | 497  | 558          |
| Ŭ, |              | 349                    | 340              | 4   | 7                | 204         | 214             | 7   | 3          | 335  |            | 5    | 7      | 402  | 541          |
|    | - in station |                        |                  | 1   |                  |             |                 | 1   |            |      |            | 1    |        |      |              |

CRYSTAL STRUCTURE DESCRIPTION AND DISCUSSION.

Fig. 1 shows a clinographic projection of the trigonal cell. As one can see the most prominent feature of the structure consists of layers of polyhedra parallel to (0001). Two kinds of layers are present: layer A, formed



Fig. 1. – Clinographic projection of the trigonal cell (with a rotation angle  $\vartheta$  of 18°16' about the triad axis and an elevation angle  $\varphi$  of the same axis of 6°20').

by BeO<sub>4</sub> tetrahedra, SiO<sub>4</sub> tetrahedra and AsO<sub>3</sub> pyramids, and layer B, formed by (Ti, Sn)O<sub>6</sub> octahedra and CaO<sub>8</sub> square antiprisms. Two A layers, placed upon each other, are inverted with respect to a symmetry centre, and they are connected by a common vertex of the BeO<sub>4</sub> and SiO<sub>4</sub> tetrahedra, so forming (BeSiO<sub>7</sub>) groups. Layer B acts as a bridge between pairs of these double A layers.

In order to explain better the structural model, the polyhedra contained in limited slabs of the unit cell, projected on a plane normal to the c axis are presented in fig. 2–3–4.

Fig. 2 shows the projection of the cell slab which includes the A layer of the  $BeO_4$ ,  $SiO_4$  and  $AsO_3$  polyhedra. The two tetrahedra  $BeO_4$  and  $SiO_4$ 



Fig. 2. – Projection of the cell slab with -0.03 < z/c < 0.20, showing the A layer of polyhedra.



Fig. 3. – Projection of the cell slab with 0.11 < z/c < 0.38, showing the B layer of polyhedra.

have the beryllium and silicon atoms on a triad axis; owing to their position, three vertices of each tetrahedra are occupied by equivalent oxygens (O(2) for BeO<sub>4</sub> and O(3) for SiO<sub>4</sub>); the fourth oxygen O(4), common to both tetrahedra, is aligned along the three-fold axis with the Be and Si atoms. The arsenic is situated on the vertex of a sort of trigonal pyramid and the remaining three vertices are occupied by independent oxygens (O(1), O(2), O(3)) belonging respectively to the titanium octahedron and, as said above, to the beryllium and silicon tetrahedra.



Fig. 4. – Projection of the cell slab with -0.03 < z/c < 0.38, showing the connections between the antiprism of calcium and the other polyhedra.

The B layer of the remaining groups of polyhedra is represented in fig. 3. The (Ti, Sn) atom, lying on  $\overline{3}$ , co-ordinates six equivalent oxygens O(1) in a nearly regular octahedron, whereas the Ca atom, which lies on a diad axis, is connected with eight oxygens and forms a coordination polyhedron that could be described as a square antiprism. Each CaO<sub>8</sub> polyhedron is always joined to four equivalent Ca polyhedra by means of four vertices occupied by O(2) and is also connected with a common edge to a (Ti, Sn) octahedron.

It is possible to understand more clearly the various connections between the antiprism of calcium and the other polyhedra from fig. 4, consisting of the overlying of fig. 2 and 3. The connections occur by edges as well as by vertices. In fact the CaO<sub>8</sub> antiprism, besides the edge O(I)-O(I) in common

[108]

with the octahedra, has two more edges O(2)—O(2) in common with two BeO<sub>4</sub> tetrahedra, and also it shares two vertices O(3) with respectively a tetrahedron SiO<sub>4</sub> and a pyramid AsO<sub>3</sub>.

From the structural arrangement of the atoms, the chemical formula results:

$$Ca_6As_{12}(Ti, Sn)_2 Si_4Be_4O_{40}$$
.

Owing to the presence of the anionic group (SiBeO<sub>7</sub>) the formula could be modified as follows:

Ca<sub>3</sub>As<sub>6</sub>(Ti,Sn)[O<sub>13</sub> SiBeO<sub>7</sub>].

On the other hand, if one regards the layer arrangement of the polyhedra and by assigning an anionic character to the A layer, the previous formula could be written:

$$Ca_3(Ti, Sn)[(As_3SiBeO_{10})_2];$$

consequently two of these formula units are present in the unit cell. However only a new chemical analysis could definitively confirm the results of the structural study, but the amount of material at our disposal is not enough to permit a new analysis.

The bond lengths and angles.—The (Ti, Sn) octahedron is nearly regular; the Ti-O(1) distance between the cation and surrounding oxygens measures 1.970 Å.

The eight fold coordination polyhedron of Ca forms a very irregular square antiprism, whose bases have sides of different lengths ranging from 2.5 Å to 3.2 Å and the Ca—O distances from 2.433 to 2.560 Å.

The polyhedron of arsenic is not regular: in each «trigonal» pyramid the distance As—O(3) (1.846 Å) is somewhat longer than the two others (As–O(1) 1.787 Å and As–O(2) 1.757 Å). The same coordination of As<sup>3+</sup> has been found in finnemanite [6].

Also beryllium and silicon tetrahedra are not perfectly regular: the two distances Be–O(2) and Si–O(3) are respectively 1.668 and 1.654 Å, but the O(4) forms distances noticeably shorter than the preceding ones, 1.530 Be–O(4) and 1.580 Å Si–O (4).

The bond angles are listed in Table III.

TABLE III.

Interatomic angles (°) and their standard deviations (in parentheses).

| O(2)BeO(4)  | 113°42′ (50′)  | O(I)AsO(2)     | 102°50′(20′  |
|-------------|--|----------------|--------------|
| O(2)BeO(2)' | 104°56′(1°30′)   | O(1)AsO(3)     | 94°55′(21′)  |
| O(3)SiO(4)  | 114° 1′ (17′)  | O(2)AsO(3)     | 900 6'(21')  |
| O(3)SiO(3)' | 104°34′ (49′)  |                |              |
|             | n ann an Shailtean Chailtean<br>Chuirtean Shailtean Ann an Shailtean | O(I)TiO(I)'    | 1780 2'(38') |
| SiO(3)As    | 127°38′ (24′)  | O(I)TiO(I)''   | 91°39′(38′)  |
| SiO(4)Be    | 180°   | O(I)TiO(I)'''  | 89°45′(38′)  |
|             |  | O(I)TiO(I)'''' | 88°52′(38′   |

The balance of charges.—Some remarks can be made on the electrostatic equilibrium of the crystal structure. As it can be seen in Table IV, in which are reported the bond lengths, ordered according to their electrostatic polarity, the balance of the electrostatic charges on the oxygens atoms is not seemingly satisfied: the sum of the positive charges is 1.92 on O(1), 2.00 on O(2), 2.25 on O(3), and 1.50 on O(4).

#### TABLE IV.

Bond lengths (Å) ordered according to their electrostatic polarity (standard deviations in parentheses).

|    | Neutral oxygen atoms O(1) and O(2)                   | Overbonded oxygen<br>atoms O(3) | Underbonded oxygen<br>atoms O(4) |  |  |  |  |
|----|--|---------------------------------|----------------------------------|--|--|--|--|
| т; | (6) T:O(1) , cTo (6)                                 |                                 |                                  |  |  |  |  |
| Ca | (c) $IIO(1) = 1.070$ (c)<br>(c) $CaO(1) = 2.433$ (c) | (2) CaO $(3)$ 2.471 $(7)$       |                                  |  |  |  |  |
|    | (2) $CaO(2)$ 2.560 (7)                               | () () ()                        |                                  |  |  |  |  |
|    | (2) CaO(2)' 2.435 (9)                                |                                 |                                  |  |  |  |  |
| As | (I) AsO(I) I.787 (7)                                 | (I) AsO(3) I.846 (7)            |                                  |  |  |  |  |
|    | (I) AsO(2) I.757 (8)                                 |                                 |                                  |  |  |  |  |
| Be | (3) $BeO(2)$ I.668(12)                               |                                 | (I) BeO(4) I.530(23)             |  |  |  |  |
| 51 |  | (3) SiO(3) I.654 (9)            | (I) SiO(4) I.580(I3)             |  |  |  |  |

Actually a close relationship can be observed between the concentration of either positive or negative charge on the oxygen atoms, and the lengths of the cation-oxygen distances, that are respectively long or short; that is, the Pauling-Zachariasen [7] rule is well verified in this crystal structure.

The electronic mechanism that underlies such variable bond lengths can be different for the different cations, and, as Pant [8] observes, in a single cation-oxygen distance more than one mechanism could act for achieving the valency balance.

In the case of the SiO<sub>4</sub> system, the d-p  $\pi$  double bond theory put forward by Cruickshank [9] is consistent with the data presented here. In fact, the Si-O(4)-Be angle of 180°, imposed by a triad, favours a very good  $\pi$ -overlapping. On the other hand the O(3) atoms that connect silicon and arsenic can share their p-electrons with both the latter atoms, since also arsenic has d-orbitals available; then the double bond  $\pi$ -character of the Si-O(3) (As) bond is lower than the one of the Si-O(4) (Be). The Si-O(4) (Be) distance is very short (1.580 Å) whereas the Si-O(3) (As) is very long (1.654 Å); then, without excluding the possibility for other electronic mechanisms, the Cruick[111]

shank d-p  $\pi$  theory can be accepted as a work hypothesis for the SiO<sub>4</sub> system. Of course this mechanism cannot be put forward to explain the variability of the lengths of the BeO<sub>4</sub> system.

Aknowledgements.—The Authors are very grateful to Dr. Stefan Graeser who made this study possible by sending them a specimen of asbecasite.

#### References.

- [1] GRAESER S., Asbecasit und cafarsit, zwei neue Mineralien aus dem Binnatal, «Schweiz. Mineral. Petrog. Mitt. », 46, 367 (1966).
- [2] STRUNZ H., New mineral names, «Am. Min», 52, 1583 (1967).
- [3] HANSON H. P., HERMAN F., LEA J. D. and SKILLMAN S., HFS atomic scattering factors, «Acta Cryst. », 17, 1040 (1964).
- [4] CROMER D. T., Anomalous dispersion corrections computed from self-consistent field relativistic Dirac-Slater wave functions, «Acta Cryst.», 18, 17 (1965).
- [5] ZACHARIASEN W. H., The secondary extinction correction, «Acta Cryst. », 16, 1139 (1963).
- [6] GABRIELSON O., The crystal structure of finnemanite, reported in «Structure Reports for 1957», 21, 389 (1964).
- [7] ZACHARIASEN W. H., The crystal structure of monoclinic metaboric acid, «Acta Cryst.», 16, 385 (1963).
- [8] PANT A. K., A reconsideration of the crystal structure of β-Na<sub>2</sub>Si<sub>2</sub>O<sub>5</sub>, «Acta Cryst.», B 24, 1077 (1968).
- [9] CRUICKSHANK D. W. J., The role of 3d-orbitals in  $\pi$  bonds between (a) silicon, phosphor, sulphur or chlorine and (b) oxygen or nitrogen, « J. Chem. Soc. », 5486 (1961).