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Laura Bergamasco

Cerenkov dipole emission in high energy atmospheric showers

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SEZIONE II

(Fisica, chimica, geologia, paleontologia e mineralogia)

Fisica. — *Cerenkov dipole emission in high energy atmospheric showers* (*). Nota di LAURA BERGAMASCO, presentata (**) dal Socio G. WATAGHIN.

RIASSUNTO. — Si calcola l'effetto di emissione di dipolo in luce Cerenkov applicandolo alle coppie e^- e^+ di altissima energia degli sciami estesi. I calcoli sono fatti alle energie primarie ed alle inclinazioni che interessano i metodi di rivelazione degli EAS recentemente suggeriti.

1. – INTRODUCTION.

The detection of extensive air showers (EAS) by Čerenkov light has been until now limited to primary energies $E_0 \sim 10^{13}$–10^{16} eV. Recently S. Colgate [1] has suggested using this light for studying EAS of $\sim 10^{20}$ eV at angles nearly tangent to the earth's surface, by a combination of the coherent electromagnetic radiation at radio frequencies and by optical radiation. This method should turn out quite powerful in this range of very high energies where detection is usually not easy. Up to now in fact only two 10^{20} eV showers have been detected, one at Vulcano Range, and the other at Haverah Park.

In this note we make some observations on the intensity of the Čerenkov radiation emitted at these high energies in connection with the fact that a very energetic electron positron pair of the electrophotonic cascade (which is the main contributor) may be regarded under some conditions as a moving radiating dipole.

2. – DERIVATION OF AN EXPRESSION FOR THE ČERENKOV RADIATION BY AN $(e^+ e^-)$ PAIR.

The Čerenkov radiation of a single electron in a medium with index of refraction n may be explained through a process of driven harmonic motion. The radiation equation then is:

$$q + w^2 q = \mathrm{A} \sin [\bar{k} \cdot \bar{r} (t) + \alpha] \tag{1}$$

(*) Laboratorio di Cosmo-geofisica del C.N.R. e Istituto di Fisica Generale dell'Università – Torino.

(**) Nella seduta dell'11 gennaio 1969.

where q is the oscillation amplitude, w the angular velocity, $k = nw/c$ the wave vector and α the initial phase. From [1] we obtain the radiation emission by a single electron per unit path

$$\left(\frac{dW}{dl}\right)_e = \frac{e^2}{c^2}\left(1 - \frac{1}{\beta^2 n^2}\right)\int_{w_1}^{w_2} w \, dw. \tag{2}$$

In the case of Čerenkov radiation from a moving dipole, the second member of eq. (1) has two driving terms which may be combined into:

$$\ddot{q} + w^2 q = A\, 2 \sin\left(\bar{k}\cdot\frac{\bar{\delta}}{2}\right)\cos\left(\bar{k}\cdot\bar{r}\,(t) + \alpha\right) \tag{3}$$

where $\bar{\delta}$ is the pair separation in the rest system K of the pair, in motion with relativistic velocity $v = \beta c$ relative to the normal earth rest system K′. In this last system the separation δ undergoes a Lorentz contraction, and attains the value:

$$\delta' = \delta\left[1 - \cos\varphi\left(1 - \frac{1}{\gamma}\right)\right] \tag{4}$$

where φ is the angle of the velocity $\bar{v}$ with respect to the vector separation $\bar{\delta}$. With the help of eq. (3) and (4) we get the dipole radiation yield per unit path

$$\left(\frac{dW}{dl}\right)_d = \frac{4e^2}{c^2}\left(1 - \frac{1}{\beta^2 n^2}\right)\int_{w_1}^{w_2} \sin^2\left\{\frac{nw}{2c}\cdot\delta\left[1 - \left(1 - \frac{1}{\gamma}\right)\cos\varphi\right]\cos\left(\theta_{\check{c}} - \varphi\right)\right\} w \, dw. \tag{5}$$

The energy loss obtained in this way is free of the simplifying assumptions used in previous works [2–5], and namely: 1) non relativistic velocity of the pair 2) unique dipole orientation ($\bar{\delta}//\bar{v}$ or $\bar{\delta}\perp\bar{v}$) 3) $\bar{\delta}\to 0$ approximation. Averaging eq. (5) over all dipole orientations and carrying out the integration in the frequency range (w_1, w_2) of photomultiplier devices we find

$$\left\langle\left(\frac{dW}{dl}\right)_d\right\rangle = \frac{4e^2}{c^2}\left(1 - \frac{1}{\beta^2 n^2}\right)\left\{\frac{w_2^2 - w_1^2}{4} + \frac{w_1 \sin\left(\frac{a\delta w_1}{2}\right) - w_2 \sin\left(\frac{a\delta w_2}{2}\right)}{a\delta} + 2\,\frac{\cos\left(\frac{a\delta w_1}{2}\right) - \cos\left(\frac{a\delta w_2}{2}\right)}{a^2\delta^2}\right\} \quad \text{with} \quad a = \frac{\gamma - 1}{\gamma\beta c}. \tag{6}$$

We next consider the ratio $R = \left(\frac{dW}{dl}\right)_d \Big/ 2\left(\frac{dW}{dl}\right)_e$ to have an information on the behaviour of the pair Čerenkov emission in function of the value of the separation δ (fig. 1). For $\delta = 0$, the electron and positron radiative fields interfere destructively, that is there is no dipole emission ($R = 0$). In the limit $\delta \to \infty$ the e^+ and e^- radiate independently ($R = 1$). We arbitrarily assume $R = 0.8$ as upper limit for dipole emission: this corresponds to the condition $\delta \leq 6 \times 10^{-5}$ cm.

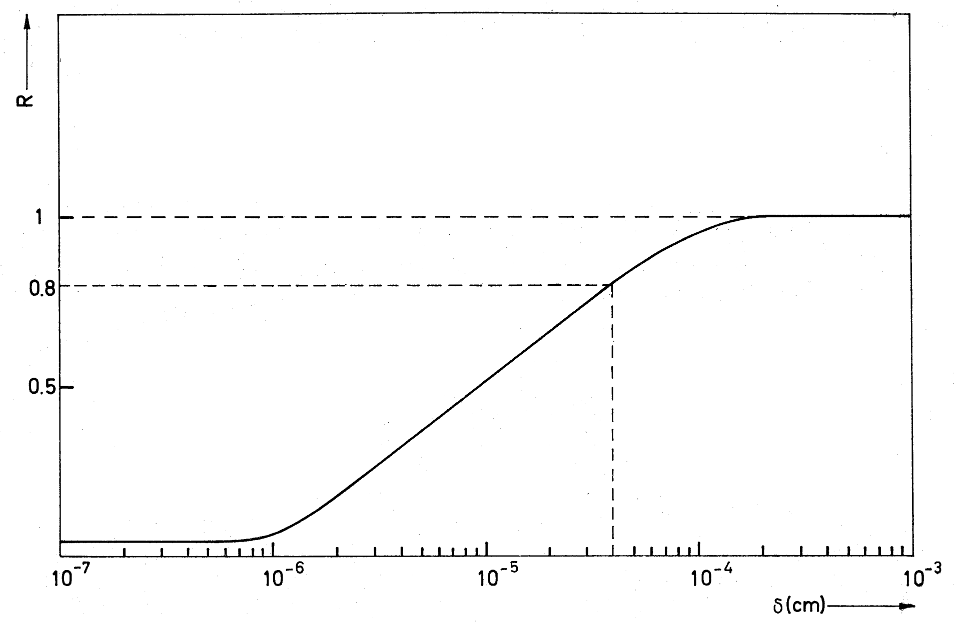

Fig. 1. - Ratio $R = \left(\frac{dW}{dl}\right)_d \Big/ 2\left(\frac{dW}{dl}\right)_e$ against the separation δ of the $e^+ e^-$ pair.

3. – ESTIMATE OF PAIR SEPARATION δ.

We consider electron positron pairs in EAS inclined on the vertical of an angle θ. Let q be the production quote, x the vertical distance and $l = x/\cos\theta$ the effective path travelled.

Assuming energy equipartition the separation of a pair with energy E_γ (MeV) and opening angle $\varphi = \frac{2}{E}$ at a distance l (cm) from the production quote is $s = \frac{2}{E_\gamma}$ [6].

A second more important contribution comes from the multiple scattering in atmosphere. The probability that an electron in traversing a distance l in atmosphere suffers a lateral displacement between y and $y + dy$ is Gaussian with [7]

$$\sigma = \frac{1.05\, ln\,(150\, E_\gamma)}{E_\gamma^2} \int_0^l l^2 \rho(l)\, dl. \tag{7}$$

The expression for the atmosphere density $\rho(l)$ is obtained by fitting the NASA experimental points [8], and is given by

$$\rho(l) = \rho(q) \exp(1.38 \cdot 10^{-6}\, l \cos\theta). \tag{8}$$

The total separation δ at l is then:

$$\delta = \sqrt{4y^2 + s^2 - 4yc\cos\psi} \tag{9}$$

Averaging over the two parameters ψ and y, we find the mean square value:

$$(10) \qquad \langle\delta^2\rangle = \frac{2\,l^2 + 0.35\, ln\,(150\,E_\gamma)\,\rho\,(q)\int_0^l l^2 \exp\,(1.38.10^{-6}\,l\cos\theta)\,dl}{E_\gamma^2}\,.$$

This result has been satisfactorily compared with a Montecarlo calculation: in fig. 2 we report for instance the results for pairs created at ~ 15 km by photons of primary energy $E_\gamma \sim 10^{14}$ eV in vertical showers.

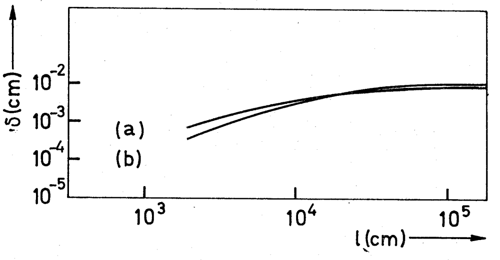

Fig. 2. – Pair separation δ against the distance travelled l: (*a*) our calculation, (*b*) Montecarlo calculation.

4. – Discussion of results.

We estimate the separation δ for electron positron pairs of energy up to 10^{20} eV. We considered three probable quotes of production $q = 15$, 20, 25 km and several angles of inclination on the vertical ($0^\circ \leq \theta \leq 87$). The results of fig. 3 and 4 show the way the dipole emission effect increases with the primary energy, the production quote and the angle of inclination of the $e^+ e^-$ pair.

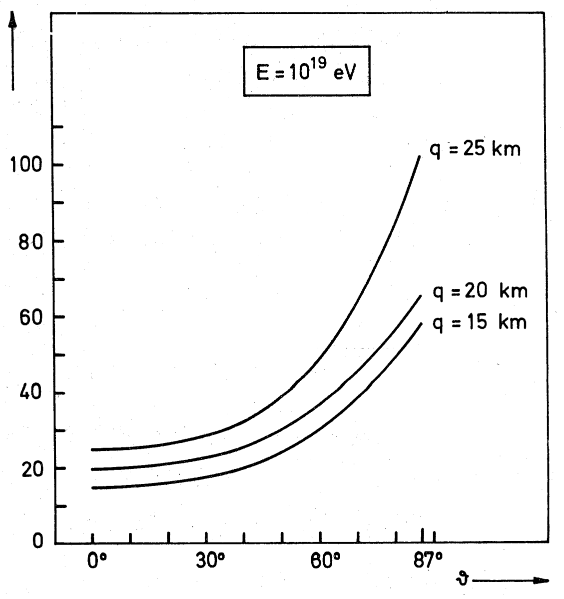

Fig. 3. – Distance travelled by the $e^+ e^-$ emitting as dipole against the angle of inclination to the vertical for 3 values of production quote (primary energy $E = 10^{19}$ eV).

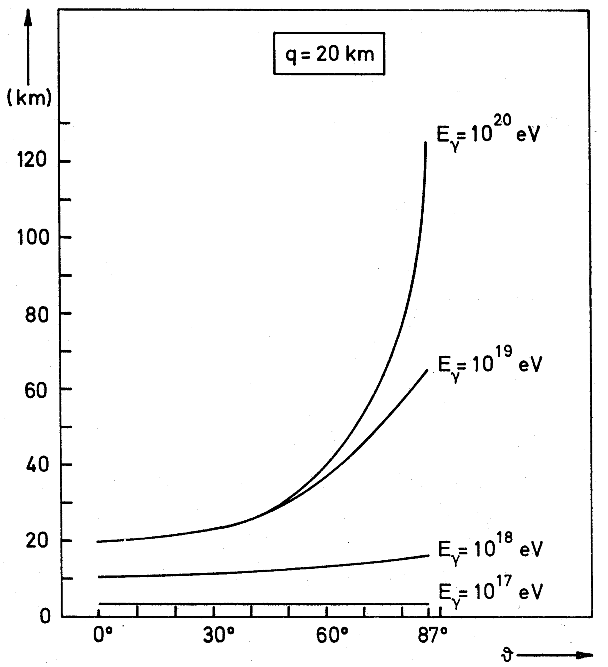

Fig. 4. – Distance travelled by the e^+ e^- pair emitting as dipole against the angle of inclination to the vertical for 4 va'ues of primary energy (production quote $q = 20$ km).

Our results show that the dipole effect is emphasized in the detection of EAS with Colgate's method, which actually requires showers with primary energy $E_p \geq 10^{20}$ eV and very large angles of inclination: yet it is found effective only in the first stages of the cascade, calling for $E_\gamma \geq 10^{17}$ eV.

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