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On the relativistically-covariant effective quark model of elementary particles

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Fisica. — *On the relativistically–covariant effective quark model of elementary particles*. Nota di V. P. SHELEST, presentata (*) dal Socio G. WATAGHIN.

RIASSUNTO. — Si considera il modello relativistico degli adroni composti dai quarki «effettivi» (quasi indipendenti).

Si dimostra che l'interazione tra quarki effettivi porta un contributo soltanto agli ordini più elevati (cominciando dal terzo) nello sviluppo secondo le potenze dell'inverso della massa effettiva del quarko.

It is well known at present the spectacular success of simple quark models in the theory at elementary particles. Particularly, it is interesting feature of the situation that even very simple (additive) models of composite hadrons are in a rather good agreement with the experiment [1, 2]. At the same time there is not at present any experimental indication of the existence of free quarks. These two facts lead to the hypothesis which we are to describe in this note and which is known as the concept of " quasi-independent (effective) quarks ".

Let us consider all strongly intercating particles (mesons and baryons) as constructed of some fundamental particles, quarks and antiquarks, possessing quantum numbers which are required by symmetry considerations.

Due to the absence of experimental evidence as to the existence of free real quarks, we shall not insist on their reality, but, in what follows, we shall use the concept of quarks as a rather useful construction, which need not reflect any actual composition of hadron, but is only to serve as a simple and effective tool of investigation. Even in such cases when it seems that actual quarks are indispensable to be considered, as for example, in the case of employing the equation for hadron, where operators acting on separate quarks appear, we can treat these models as merely useful modes of writing down some suppositions, following from and supported by experimental evidence.

Having in mind this point of view, we proceed with the construction of the model of quasi-independent quarks, first investigated by P. N. Bogolubov [3]. The essence of this concept is the following. Hadrons are considered as strongly-bound states of quarks (as concerns baryons) or of quarks and antiquarks (in the case of mesons). Quarks and antiquarks are supposed to have very large masses M, which account for most of the interaction inside hadrons. This strong interaction inside hadrons constitutes some kind of self-consistent field, and quarks with effective masses $m \ll M$ are moving in this field in a quasi-independent way. There exists also some residual

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interaction (much weaker than the principal) between now quasi-independent quarks, which for some purposes may be considered as completely switched off.

In the framework of the above described concepts it is natural to characterize hadrons (for the sake of definiteness, say, baryons) by the following initial equation [4, 5]

$$\{(\hat{\partial}^{(1)} - \mathrm{M})(\hat{\partial}^{(2)} - \mathrm{M})(\hat{\partial}^{(3)} - \mathrm{M}) + \tilde{\mathrm{V}}\}\,\Psi_{\mathrm{ABC}} = 0 \tag{1}$$

where

$$\hat{\partial}^{(i)} = \sum_{\alpha} \gamma_{\alpha}^{(i)}\, l_{\alpha}^{(i)} \quad ; \quad \mathrm{A} = (i, \alpha) \quad ; \quad i = 1, 2, 3 \quad ; \quad \alpha = 1, 2, 3, 4 \tag{2}$$

mass of quark M tends to infinity, $l^{(i)}$ are momenta of quarks inside baryon, Ψ_{ABC} is the wave function of baryon, and the potential $\tilde{\mathrm{V}}$ can be expanded in series with respect to large mass of quark M:

$$\tilde{\mathrm{V}} = \mathrm{M}^3 - \mathrm{V}\mathrm{M}^2 + \cdots \tag{3}$$

Then in the second order in M^{-1} we obtain from (1)

$$[\hat{\partial}^{(1)} + \hat{\partial}^{(2)} + \hat{\partial}^{(3)} - \mathrm{V}]\,\Psi_{\mathrm{ABC}} = 0\,. \tag{4}$$

Our next step is to represent potential V in the form

$$\mathrm{V} = \sum_{(i)} m_i + \mathrm{V}' \tag{5}$$

where V' is that residual interaction between quasi-independent quarks, which was mentioned above. Indeed, when we put $\mathrm{V}' = 0$ we see that the equation (4) is obeyed when each quark in the baryon is supposed to be freely moving with effective mass m_i.

We can represent the solution of equation (4) in the form:

$$\Psi = \prod_{(i)} \left(1 + \frac{\gamma^{(i)}\, l^{(i)}}{m_i}\right) \chi^{(i)} \tag{6}$$

where $\chi^{(i)}$ are Dirac spinors of i–th quark.

Using the notation

$$\gamma_5 = i\gamma_0\,\gamma_1\,\gamma_2\,\gamma_3 \tag{7}$$

it is easy to get instead of (6) the expression

$$\Psi = 8 \prod_{(i)} \left(1 + \frac{\gamma_5^{(i)}\, \vec{\sigma}^{(i)}\, \vec{l}^{(i)}}{2\, m_i}\right) \chi^{(i)}\,. \tag{8}$$

In the non-relativistic limit we obtain, when substituting expression (8) into (4) the following result:

$$\left(\mathrm{E}_{\mathrm{Bar}} - m_{\mathrm{Bar}} - \mathrm{V}' - \sum_{(i)} \frac{\vec{l}^{(i)2}}{2\, m_i}\right) \chi_{+++} = 0 \tag{9}$$

where under χ_{+++} we understand the following function:

$$\chi_{+++} = \begin{pmatrix} \chi_1^{(1)} \\ 0 \end{pmatrix} \begin{pmatrix} \chi_1^{(2)} \\ 0 \end{pmatrix} \begin{pmatrix} \chi_1^{(3)} \\ 0 \end{pmatrix}. \tag{10}$$

It is evident that the equation (9) is the correct equation, which describes composite non-relativistic particles with total mass $m_{\mathrm{B}ar}$. But here we must consider some additional question, which concerns the expansion with respect to inverse barion mass $m_{\mathrm{B}ar}$.

In order to perform such expansion it is necessary to consider $m_{\mathrm{B}ar}$ as a rather large quantity to make reasonable this expansion. Note that the subsequent taking into consideration of the terms at this expansion corresponds to the including of relativistic corrections at higher orders.

But when adopting such an ideology, we note that it is necessary to put

$$V' = \frac{W}{2\,\overline{m}^{(i)}} \tag{11}$$

where

$$\overline{m}^{(i)} = \frac{1}{3}\sum_{(i)} m^{(i)} = \frac{1}{3} m_{\mathrm{B}ar} \tag{12}$$

and W is some finite quantity. Indeed, only in such case can we guarantee that kinetic energy of our system (baryon) is of the same order of magnitude as its potential energy, and not negligible in comparison with it, because this latter situation can lead to appearance of non-renormalizable solutions of Schröedinger equation.

But now we can see that our residual interaction enters into consideration only in the third order with respect to inverse hadron mass. It can be checked if we begin to introduce additional terms to our approximate solution of initial equation (4). Namely, we shall now represent baryon wave function in the form

$$\Psi = \Psi_{+++} + \Psi_{-++} + \Psi_{+-+} + \Psi_{++-} + \cdots \tag{13}$$

and try to find the components of the type Ψ_{-++} which have following spinor structure:

$$\chi_{-++} = \begin{pmatrix} 0 \\ \chi_2^{(1)} \end{pmatrix} \begin{pmatrix} \chi_1^{(2)} \\ 0 \end{pmatrix} \begin{pmatrix} \chi_1^{(3)} \\ 0 \end{pmatrix}. \tag{14}$$

From our equation it follows immediately that

$$(\Psi_{\mathrm{ABC}})_{-++} = i\,\frac{\left(\vec{\sigma}^{(1)}\,\vec{l}^{(1)}\right)}{2\,\overline{m}_i + \dfrac{W}{2\,\overline{m}_i}}\,\Psi_{+++} - \frac{\gamma_5^{(1)}\,\vec{\sigma}^{(1)}\,\vec{l}^{(1)}}{2\,\overline{m}_i^2}\,\Psi_{\substack{+\cdots\\+++}} \;. \tag{15}$$

But, expanding the expression inside brackets, we get that it takes the form

$$(\Psi_{\mathrm{ABC}})_{\substack{-++\\+-+\\++-}} \cong i\left(\frac{1}{2\,\overline{m}_i} - \frac{W}{8\,\overline{m}_i^3}\right)\left(\sum_{(i)} \vec{\sigma}^{(i)}\,\vec{l}^{(i)}\right)(\Psi_{\mathrm{ABC}})_{\substack{+\cdots\\+++}} \tag{16}$$

and, therefore, our previous assertion is valid. Contributions from the components of the type Ψ_{--+} etc. include terms containing W even in higher orders of m^{-1} expansion.

In our paper (4) we constructed matrix elements of baryon currents and checked (in the first order of m^{-1}) the validity of algebra of currents relations for them. But, as follows from the considerations of this note, pratically it means dealing with a model of quasi-independent quarks without any residual interaction. Results obtained in this way will be, therefore, of kinematical character. It concerns not only our calculations but also that made by Gell-Mann and collaborators [6, 7] as far as order not higher than m^{-2} was considered.

So we constructed the relativistically covariant matrix elements of currents, taken in the free effective quarks brackets and succeeded in showing that the algebra at currents both for meson [8, 9] and baryon [10] cases takes place for this case.

But it means that the algebra at currents is proved up to the second order (inclusive) in the expansion on inverse hadron mass.

The potential acting between effective quarks, which enters into consideration beginning from the third order at this expansion is to be postulated by other model suppositions of ours and could violate the algebra at currents, as is indicated by some calculations of Gell-Mann, Horn and Weyers [11].

So it seems promising not to insist on the validity of algebra of currents approach in all orders in the above-mentioned expansion but to construct some reasonable potentials, to indicate then the possibile classes of them by comparison with experiment. This program is to be developed by us elsewhere.

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REFERENCES.

[1] E. M. LEVIN and L. L. FRANKFURT, « JETP Letters », *2*, 105 (1965).
[2] M. J. LIPKIN and F. SCHEK, « Physics Rev. Letters », *16*, 71 (1965).
[3] P. N. BOGOLUBOV, « JINR Preprint », Dubna, P–2569 (1966).
[4] V. P. SHELEST, « Preprint ITP », Kiev, ITP–67–47 (1967).
[5] V. P. SHELEST. « Preprint ITP », Kiev, ITP–67–18 (1967).
[6] M. GELL–MANN, « Preprint CALT », 68–102 (1966).
[7] R. DASHEN and M. GELL–MANN, « Physics Rev. Letters », *17*, 340 (1966).
[8] V. P. SHELEST, « Preprint ITP », Kiev, ITP–67–51 (1967).
[9] V. P. SHELEST, « Reports of the A.S. of USSR », 1968, to be pubbl.
[10] V. P. SHELEST, « Preprint ITP », Kiev, ITP–67–52 (1967).
[11] M. GELL–MANN, D. HORN and J. WEYERS, *Paper presented at the International Conference on Particle Physics in Heidelberg*, Sept. 1967.