Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

Wolfgang Otto Ulrici, Georg Süssmann

Non-Local Shell Model Parameters for the Nuclear Ground State

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **41** (1966), n.3-4, p. 183–188.

Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA_1966_8_41_3-4_183_0>

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Accademia Nazionale dei Lincei, 1966.

Fisica. — *Non-Local Shell Model Parameters for the Nuclear Ground State.* Nota (*) di WOLFGANG OTTO ULRICI (**) e GEORG SÜSSMANN (**), presentata dal Corrisp. M. AGENO.

RIASSUNTO. — Il presente lavoro ha lo scopo di cercare di determinare i parametri di un modello a strati non locale, che forniscono le corrette energie di legame dei protoni, il raggio di carica del nucleo nonché il suo difetto di massa. Si noti in particolare che quest'ultima proprietà nucleare è in generale fortemente sottostimata dai metodi di calcolo che si fondano su un modello a strati locale.

The attempt was made to determine the parameters of a non-local shell model by fitting the binding energies of the protons and the charge radius of the nucleus, together with its mass defect. The latter has been strongly underestimated in calculations using a local shell model.

The nucleons move in a non-local potential U,

$$U\psi(\vec{r}) = \int d^3\vec{r}'\, K(\vec{r}, \vec{r}')\, \psi(\vec{r}') . \tag{1}$$

The kernel is supposed to be separable [1, 2],

$$K(\vec{r}, \vec{r}') = U_\sigma\left(\frac{\vec{r}+\vec{r}'}{2}\right) \delta_\beta(\vec{r}' - \vec{r}) \tag{2}$$

with the local potential $U_\sigma(\vec{r})$ and the distribution function $\delta_\beta(\vec{r})$ of the non-locality range β, which is normalized to

$$\int d^3\vec{r}\, \delta_\beta(\vec{r}) = 1 . \tag{3}$$

We choose δ_β to be a Gaussian,

$$\delta_\beta(\vec{r}) = e^{-\vec{r}^2/\beta^2}/\pi^{3/2}\beta^3 . \tag{4}$$

This ansate reproduces very well the energy dependence of the optical model of elastic scattering in the energy range between 4 and 21 MeV [2].

The potential used is spherically symmetric; thus we can treat only spherical nuclei in an adequate manner. For protons we add to the non-local potential a local Coulomb potential generated by a Fermi charge distribution [3]

$$\rho(r) \sim \Theta_{a_c}(r_c A^{1/3} - r) \tag{5}$$

(*) Pervenuta all'Accademia il 18 settembre 1966.

(**) Institut für Theoretische Physik der Universität, Frankfurt am Main.

with r_c and a_c given experimentally. For $U_\sigma(r)$ we take

$$U_\sigma(r) = U_0(r) + \sigma \frac{r_0^2}{r} \frac{\partial}{\partial r} U_0(r) \vec{s} \cdot \vec{l} \tag{6}$$

with

$$U_0(r) = -V_0 \left(1 - \frac{\tau}{A} \vec{t} \cdot \vec{T}\right) \Theta_a (r_0 A^{1/3} - r) \tag{7}$$

Θ_a is given by

$$\Theta_a(r) = 1/(1 + e^{-r/a}) . \tag{8}$$

The first term describes the so-called Woods–Saxon potential with the radius $R_0 = r_0 \cdot A^{1/3}$. The spin-orbit term is the simplest invariant (with respect to rotations and inversions) proportional to $\vec{p}$, $\vec{s}$ and $\vec{\nabla} U$, namely $\vec{\nabla} U (\vec{p} \times \vec{s})$. It happens to be similar to the "Thomas term", but most of it should be of different origin. The factor r_0^2 is introduced for dimensional reasons. The potential depth V_0 is corrected for the two different kinds of nucleons by the symmetry term $V_0 \cdot \tau/A \cdot \vec{t} \cdot \vec{T}$ [4].

The Schrödinger equation is solved numerically in the local energy approximation (LEA) [5].

From the charge distribution

$$\rho(r) = \frac{1}{4\pi} \sum_{\substack{\text{proton} \\ \text{levels}}} n_i \psi_i(r)^2 , \tag{9}$$

n_i being the number of nucleons in the i–th level ε_i, ψ_i the corresponding radial wave function, we get the equivalent radius

$$r_{eq} = \frac{5}{3} \left[\frac{4\pi}{Z} \int_0^\infty dr \, r^4 \rho(r) \right]^{1/2} \cdot A^{-1/3} . \tag{10}$$

The mass defect is obtained from the formula

$$D = -\frac{1}{2} \sum_{\substack{\text{all nucleon} \\ \text{levels}}} n_i (\varepsilon_i + t_i) ; \tag{11}$$

$$t_i \equiv \langle i | t | i \rangle = \frac{\hbar^2}{2m} \int_0^\infty dr \left[l_i (l_i + 1) \psi_i(r)^2 + r^2 \psi_i'(r)^2 \right] \tag{12}$$

is the mean value of the kinetic energy in the single particle state i.

The medium weight and heavy nuclei the equivalent radii of which are known from electron scattering experiments [6] are ^{32}S, ^{40}Ca, ^{51}V, ^{59}Co, ^{115}In, $^{121,123}Sb$, ^{197}Au, ^{208}Pb and ^{209}Bi. We know their mass defects [7], and for $A \leqslant 60$ the binding energies of the $2s$ protons are determined by $(p, 2p)$–experiments [8]. For the other nuclei we estimated the magnitude of the binding energy of the last proton from its separation energy. The last information is given only for odd–A nuclei, therefore we have not studied ^{208}Pb.

TABLE I.

Non-local shell model parameters, and comparison of calculated quantities with experimental ones.

τ and σ were kept fixed at 2.0 and 0.55, respectively.

NUCLEUS	r_0 (*fm*)	a (*fm*)	β (*fm*)	V_0 (MeV)	Proton binding energies (MeV)			Mass defect (MeV)		Equivalent radius (*fm*)	
					State	calc.	exp.	calc.	exp.	calc.	exp.
$^{32}_{16}S$	1.04	0.65	1.38	137	2 *s* 1/2	8.7	8.8	280	272	1.298	1.30
					1 *d* 5/2	16.5	16.1				
					1 *p* 1/2	35.0	33.5				
					1 *p* 3/2	43.5					
					1 *s* 1/2	76.0	70–80				
$^{40}_{20}Ca$	1.07	0.70	1.53	137	1 *d* 3/2	8.9	8.4	339	342	1.320	1.32
					2 *s* 1/2	11.0	11.1				
					1 *d* 5/2	19.1	19.0				
					1 *p* 1/2	38.2	36.8				
					1 *p* 3/2	45.4					
					1 *s* 1/2	76.8	70–80				
$^{51}_{23}V$	1.07	0.50	1.73	157	1 *f* 7/2	2.8	8.1	449	446	1.251	1.25
					2 *s* 1/2	14.6	14.7				
$^{59}_{27}Co$	1.07	0.47	1.78	157	1 *f* 7/2	4.5	9.5	475	517	1.254	1.27
					2 *s* 1/2	15.8	13.2				
$^{115}_{49}In$	1.07	0.65	1.56	149	1 *g* 9/2	9.2	9.3	970	979	1.188	1.19
$^{121}_{51}Sb$	1.15	0.65	1.30	114	1 *g* 7/2	5.0		1011	1026	1.199	1.20
					2 *d* 7/2	5.5	5.8				
$^{123}_{51}Sb$	1.17	0.65	1.30	112	2 *d* 5/2	6.7		1036	1042	1.202	1.20
					1 *g* 7/2	6.8	6.6				
$^{197}_{79}Au$	1.08	0.65	1.67	157	2 *d* 3/2	5.2	5.8	1510	1559	1.179	1.18
$^{209}_{83}Bi$	1.17	0.65	1.41	118	1 *h* 9/2	3.8	3.8	1647	1640	1.203	1.20

In order to fit the experimental values we had to determine six parameters: V_0, σ, τ, β, r_0 and a. The spin-orbit coupling constant σ was roughly fixed to 0.55 from the splitting of the *d*-levels in ^{40}Ca. A variation of σ has approximately no effect on the values of the mass defect and of the equivalent radius. The isospin parameter τ was kept [9] equal to 2. The parameters V_0, β, and r_0 were determined for several a, and it turned out that if a was too high or too low, the position of the *d*-center relative to the *s*-level in ^{40}Ca or the level order versus mass number for ^{121}Sb and ^{123}Sb was wrong. Thus we chose a to be about 0,65 to 0,70 *fm*; this choice agrees with fits to nucleon-nucleus scattering [2].

This method did not work for ^{51}V and ^{59}Co: with $a = 0.65\,fm$, the radius parameter r_0 went to 0.90 *fm*. Thus we tried it the other way round and determined a, β, and V_0 for $r_0 = 1.07\,fm$ ($= r_0$ of the charge distribution). We do not think this had much success.

The results are given in Table I.

Compared with the parameters found in optical model studies [2] ($\beta = 0.85\,fm$, $V_0 = 71$ MeV) and fits to the binding energies of the last bound nucleons only [9] ($\beta = 0.90\,fm$, $V_0 = 76$ MeV), our values essentially indicate a remarkably larger nonlocality β of 1.30 to 1.50 *fm*, and correlated with it a potential depth of 110 to 140 MeV. These parameters are consistent with those determined in a similar attempt [10] using the equivalent radii, the proton knock-out reaction data, and the electron scattering results for light and medium weight nuclei as the data to be fitted.

Our values need not be a contradiction to the optical model parameters: they indicate a certain dependence of β on the energy. In fig. 1 we see the

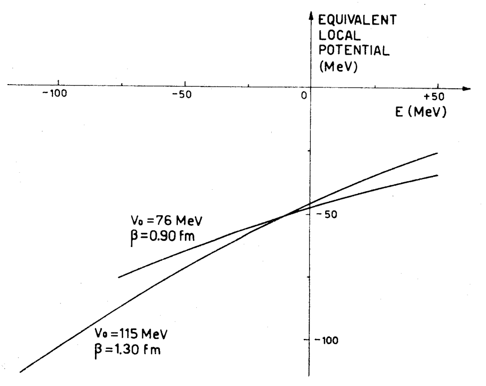

Fig. 1. – Approximate energy dependence of the local potential equivalent to a non-local one [5] using a Gaussian non-locality.

energy dependence of the equivalent local potential [2–5] using a Gaussian non-locality distribution. The true energy dependence seems to be somewhat like the $\beta = 1.30\,fm$ one for low energies and like the $\beta = 0.90\,fm$ one for high energies. Non-locality distributions other than Gaussian should be able to reproduce this behaviour.

For ^{51}V , ^{59}Co , ^{115}In, and ^{197}Au, the parameters β and V_0 are extremely high. A possible explanation is most easily seen for ^{51}V, where the *f* 7/2 level is poorly fitted: the experimental one lies much deeper than the calculated one. This is a characteristic of a spherical model treating non-spherical nuclei. Accordingly we should conclude that these nuclei are non-spherical. The model seems to be rather sensitive for deviations from sphericity. In our calculations we found it necessary to increase V_0 and β in order to keep the mass defect if the separation energy had to be lowered.

A hint for the principal correctness of the parameters is that the 1*s* proton levels of ^{40}Ca and ^{32}S come out according with the (*e* , *e′ p*)–data of the Rome-Frascati group [11], another, that the 2s neutron level of ^{40}Ca [12], too, is well fitted (see Table II). The model is not able to predict the energies of the levels above the Fermi energy. The reason may be seen from fig. 1: the level distances in that region are too large becausse of the strong energy dependence of the potential. Again this indicated an energy dependence of the parameter β.

TABLE II.

Neutron levels of ^{40}Ca.

The parameters are the same as in Table I.

NUCLEUS	r_0 (*fm*)	*a* (*fm*)	β (*fm*)	V_0 (MeV)	Neutron binding energies (MeV)		
					State	calc.	exp.
$^{40}_{20}Ca$	1.07	0.70	1.53	137	1 *d* 3/2	16.0	15.6
					2 *s* 1/2	18.1	18.2
					1 *d* 5/2	26.2	21.9
					1 *p* 1/2	46.0	
					1 *p* 3/2	53.2	
					1 *s* 1/2	85.4	

This work is partly sponsored by the Deutsche Forschungsgemeinschaft. We thank the Deutsches Rechenzentrum, Darmstadt, for the use of the IBM 7090 computer. It is a pleasure to one of us (W.U.) to thank the Istituto Superiore di Sanità, Rome, for a scholarship and for the use of the computer facilities; he acknowledges gratefully the warm hospitality enjoyed there.

REFERENCES.

[1] W. E. FRAHN and R. H. LEMMER, « Il Nuovo Cimento », *5*, 1564 (1957); *6*, 664 (1957).

[2] F. PEREY and B. BUCK, « Nucl. Phys. », *32*, 353 (1962).

[3] M. CROISSIAUX, R. HOFSTADTER, A. E. WALKER, and M.R. YEARIAN, « Phys. Rev. », *121*, 283 (1961).

[4] A. M. LANE, « Nucl. Phys. », *35*, 676 (1962); L. A. SLIV and Yu. I. KHARITONOV, « Phys. Lett. », *16*, 176 (1965).

[5] F. G. PEREY and D.S. SAXON, « Phys. Lett. », *10*, 107 (1964); W. E. FRAHN, « Nucl. Phys. », *66*, 358 (1965); H. FIEDELDEY, « Nucl. Phys. », *77*, 149 (1966).

[6] R. HOFSTADTER, « Rev. Mod. Phys. », *28*, 214 (1956).

[7] J. H. E. MATTAUCH, W. THIELE and A. H. WAPSTRA, « Nucl. Phys. », *67*, 1 (1965).

[8] M. RIOU, « Rev. Mod. Phys. », *37*, 375 (1965).

[9] H. MELDNER, G. SÜSSMANN and W. ULRICI, « Z. Naturforschg. », *20 a*, 1217 (1965).

[10] A. SWIFT and L.R.B. ELTON, to be published and L.R.B. ELTON, private communication.

[11] U. AMALDI Jr., G. CAMPOS VENUTI, G. CORTELLESSA, G. FRONTEROTTA, A. REALE, P. SALVADORI and P. HILLMAN, « Phys. Rev. Lett. », *13*, 341 (1964), and private communication on preliminary results with ^{40}Ca.

[12] C. D. KAVALOSKI, G. BASSANI, N. M. HINTZ, « Phys. Rev. », *132*, 813 (1963); H. EJIRI, Y. SAJI, Y. ISHIZAKI, M. KOIKE, K. MATSUDA, I. NONAKA, K. YAGI, Y. NAKAJIMA and E. TANAKA, « J. Phys. Soc. of Japan », *21*, 14 (1966).