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On the range straggling of ultra-relativistic muons

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Fisica. — *On the range straggling of ultra-relativistic muons* (*).

Nota (**) di CARLO CASTAGNOLI, PIO PICCHI, RENATO SCRIMAGLIO e GIUSEPPE VERRI, presentata dal Socio G. WATAGHIN.

RIASSUNTO. — Si calcolano le fluttuazioni del percorso di muoni ultrarelativistici tenendo conto anche delle interazioni nucleari.

Dopo aver discusso brevemente i processi principali della perdita di energia a seconda del loro carattere fluttuante o no, si calcola con un Montecarlo la probabilità di sopravvivenza P (R, E) di un muone di energia iniziale E alla profondità R. Si segue il comportamento di 12.000 mesoni fino alla profondità massima di 10^4 metri di H_2O equivalente.

Infine si calcola il rapporto tra le intensità integrali verticali dei muoni sottoroccia tenendo conto o meno delle fluttuazioni.

I risultati sulle intensità sono confrontati in fig. 5 con quelli di altri Autori ottenuti con metodi analitici o numerici e con differenti approssimazioni fisiche. Le differenze che si osservano risultano abbastanza consistenti da dare sensibili effetti sulle abituali deduzioni dello spettro muonico alle alte energie dalle curve intensità–profondità.

Si dànno infine in fig. 6 le relazioni percorso–energia per muoni tenendo conto oppure no delle fluttuazioni: queste ultime abbassano sensibilmente il range–medio.

1.—INTRODUCTION.

Many authors [1, 7] have recently dealt with the problem of the range straggling of ultra-relativistic muons of cosmic rays in connection with the last sets of measures made at great depths underground [7, 8]. These calculations have been carried out with analytical methods, numerical methods and with Monte Carlo methods, reaching quite different results, also for the introduction of several physical approximations. The most serious of them seem to be: 1) to neglect the nuclear interactions of muons [4, 5, 6, 7]; 2) to assume a complete screening in the calculation of bremsstrahlung and pair production; 3) a different subdivision between either processes with or without straggling; 4) when using the Monte Carlo method, a different degree of accuracy in the physical simulation of the problem.

In connection with a preceding work [9] in this note we shall deal with the straggling problem without any of these limitations and paying particular attention to the energy-losses due to nuclear effects.

2.—FUNDAMENTAL PROCESSES.

The energy loss (dE/dx) of muons may occur through:

a) processes without straggling, that is continuous ones, where the transferred energy E′ is always a small fraction of the particle energy E_μ (ionization, pair production);

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b) processes with straggling, that is discontinuous ones, where the particles lose a great fraction of their energies in each single interaction (bremsstrahlung, nuclear effects). We assume the straggling of pair production less

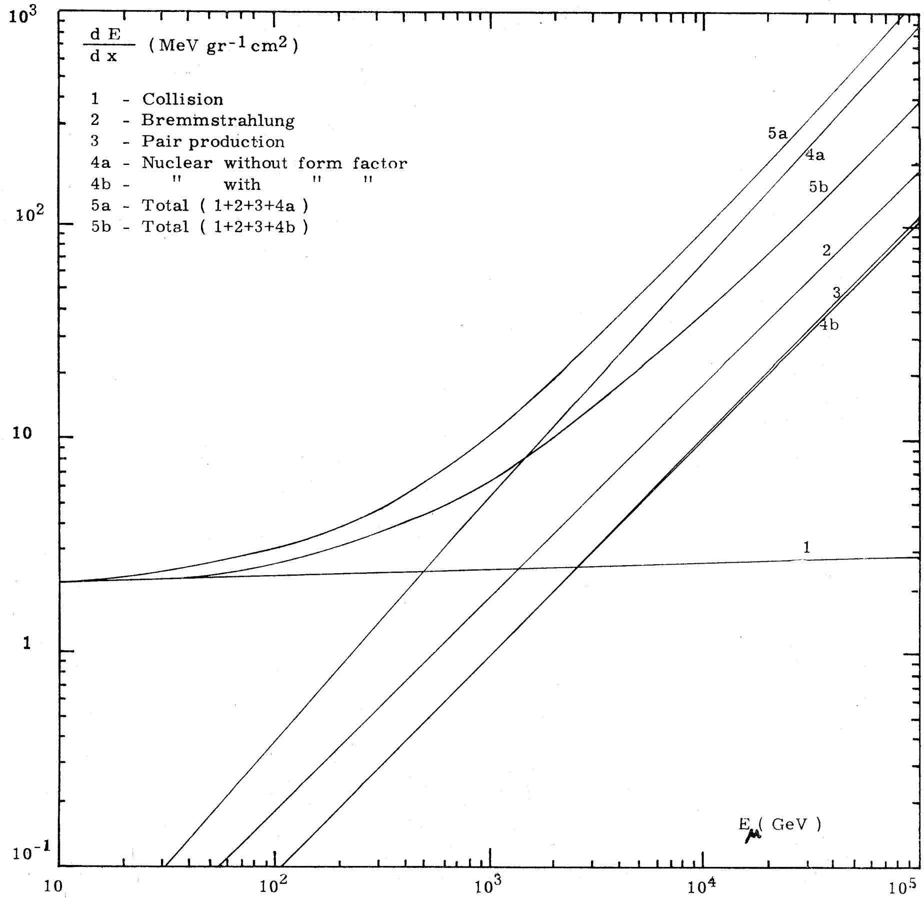

Fig. 1.

than that of bremsstrahlung because over a wide range of transferred energy and in conditions of complete screening the respective cross section can be expressed by

$$\Phi_{\text{p.p.}}\,(\mathrm{E}\,,\,v) \sim dv/\mathrm{E}\,v^3 \quad ; \quad \Phi_{\text{brem.}}\,(\mathrm{E}\,,\,v)\,dv \sim dv/v$$

where $v = \mathrm{E}'/\mathrm{E}$. Moreover an accurate calculation has shown that for $\mathrm{E}_\mu \approx 30$ GeV the ratio of $\sigma_{\text{p.p.}}$ for energy transfers of ≈ 75 MeV and ≈ 750 MeV is about 20.

When integrating over $(d\mathrm{E}/dx)_{\text{tot}}$ to compute [9] the range-energy relation R (E), mean values are implicitely assumed to make sense even for the straggling processes. But this is not correct at great depths (of the order of 1 shower unit > 500 GeV $= 4000$ m. H_2O e.).

We shall use for the cross section of processes such as ionization, bremsstrahlung and pair production, the expressions already discussed in previous works [9, 10] of ours.

For nuclear energy losses we showed in a previous work [11] how the differential cross section for inelastic interactions can be correctly written as

$$d^2 \sigma_\mu (E , \varepsilon , \tau)/d\varepsilon \, d\tau = N (E , \varepsilon , \tau) \, \sigma_\gamma (\varepsilon) \, [\Lambda/\Lambda + r]^2 \tag{1}$$

where $N (E , \varepsilon , \tau)$ is the equivalent photon spectrum evaluated by Kessler; $\sigma_\gamma (\varepsilon)$ is the cross section for photoproduction by real photons, and the squared

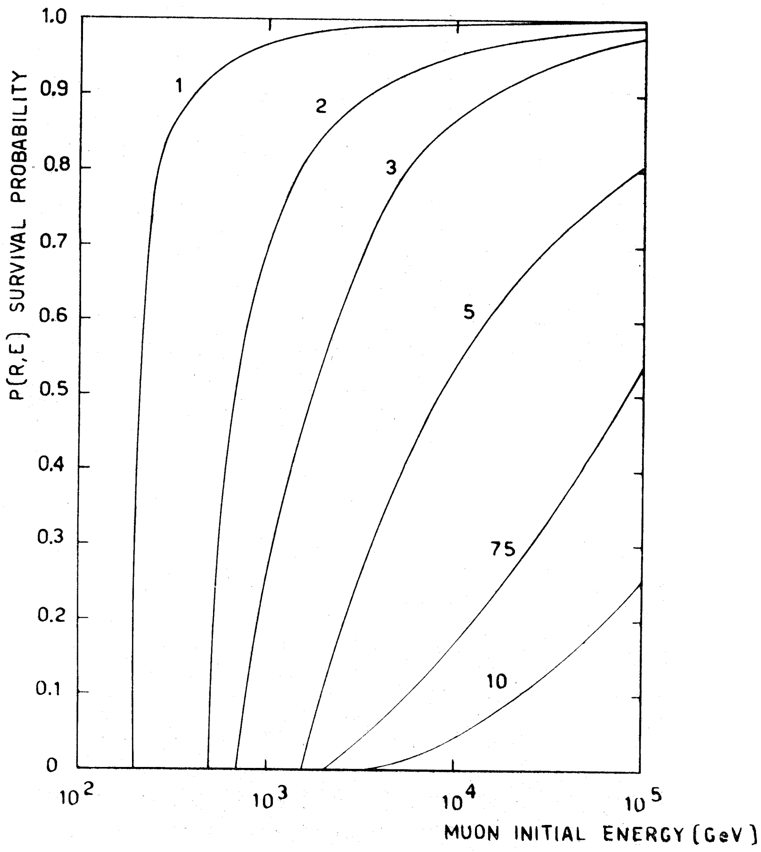

Fig. 2.

term is a factor of electromagnetic structure of Hofstadter type. We showed the good agreement of expression (1) with the available data with $\Lambda = 0.365$ and $\sigma_\gamma^0 = 3.7 \times 10^{-28}$ cm²/nucleon as determined experimentally by Castagnoli et al. [12] through measures on photostars.

The value of dE/dx we computed for the four processes of collision, bremsstrahlung, pair production and nuclear interaction and the total loss dE/dx is shown in fig. 1.

It is easily seen how $(dE/dx)_{nucl}$ turns out to be actually the same as $(dE/dx)_{p.p.}$ and about the half of $(dE/dx)_{brem.}$.

Thus, it is not justified in the evaluation of the straggling to take into account only this last process and to neglect the nuclear one, as some authors do.

Let

$$j(E)\,dE = k_j\,E^{-(\gamma+1)}\,dE \tag{2}$$

be the differential energy spectrum of muons at s.l.

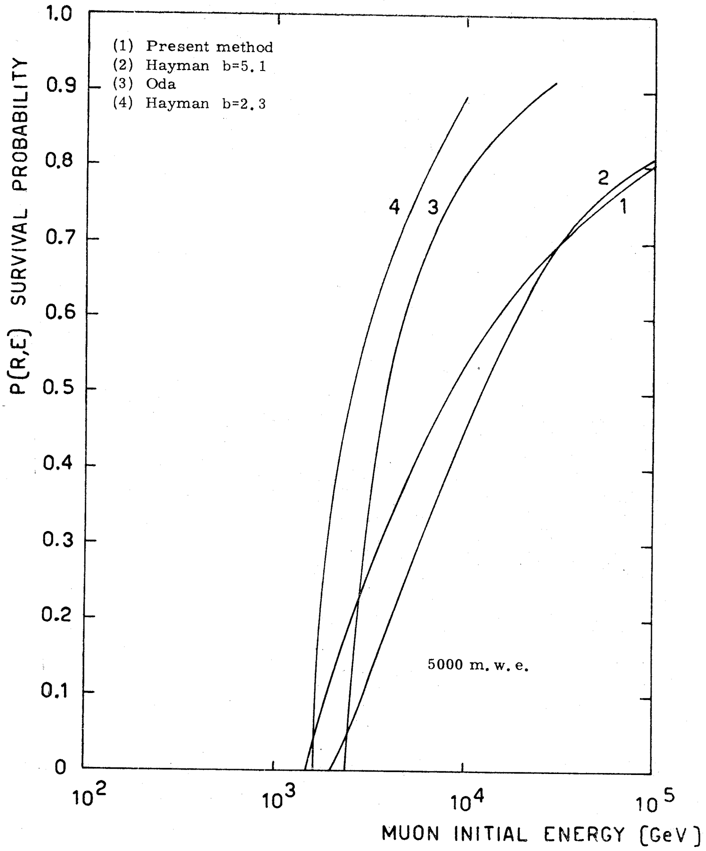

Fig. 3.

Dealing with the energy losses as continuous ones, that is neglecting the straggling, each range R is coupled in the relation R(E) with an energy E_0 for which the integral underground intensity in the vertical direction at depth R_0 is given by

$$\bar{I}(R_0) = \int_{E_0}^{\infty} j(E)\,dE\,. \tag{3}$$

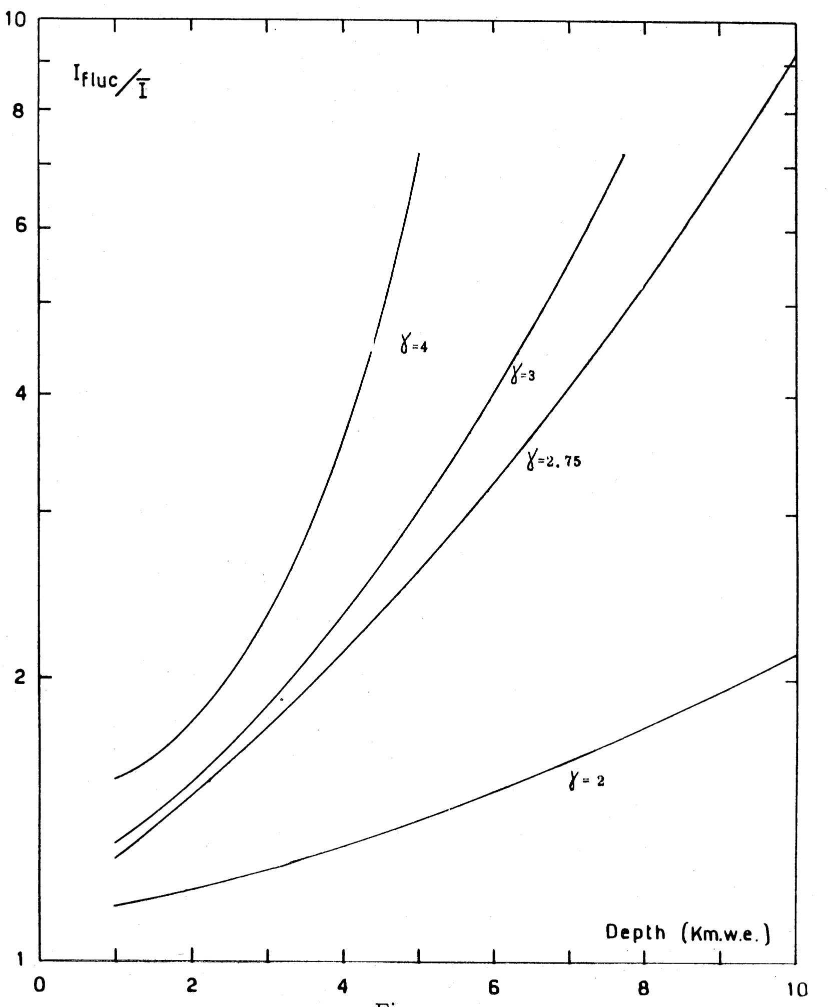

Fig. 4.

If, on the contrary, we deal with the losses as discontinuous ones and call P (R, E) the probability of survival of a particle at depth R, the intensity at depth R_0 will be:

$$I(R_0) = \int_{E_{min}}^{\infty} j(E)\, dE\, P(R_0, E) \tag{4}$$

where E_{min} is the energy loss due only to the continuous processes by a particle which survives just up to depth R_0.

The purpose of our work is to evaluate the ratio

$$I(R_0)/\bar{I}(R_0). \tag{5}$$

Therefore we shall compute with the Montecarlo method the probability of survival P (R, E).

3.—CALCULATION OF P (R, E) WITH THE MONTECARLO METHOD.

The computer simulates mathematically the passage of muons through the rocks. The chosen depth unit is $\Delta R = 25$ m.w.e. Given a muon of initial energy E_μ, its energy loss is computed altogether for the continuous processes previously defined and for the discontinuous ones.

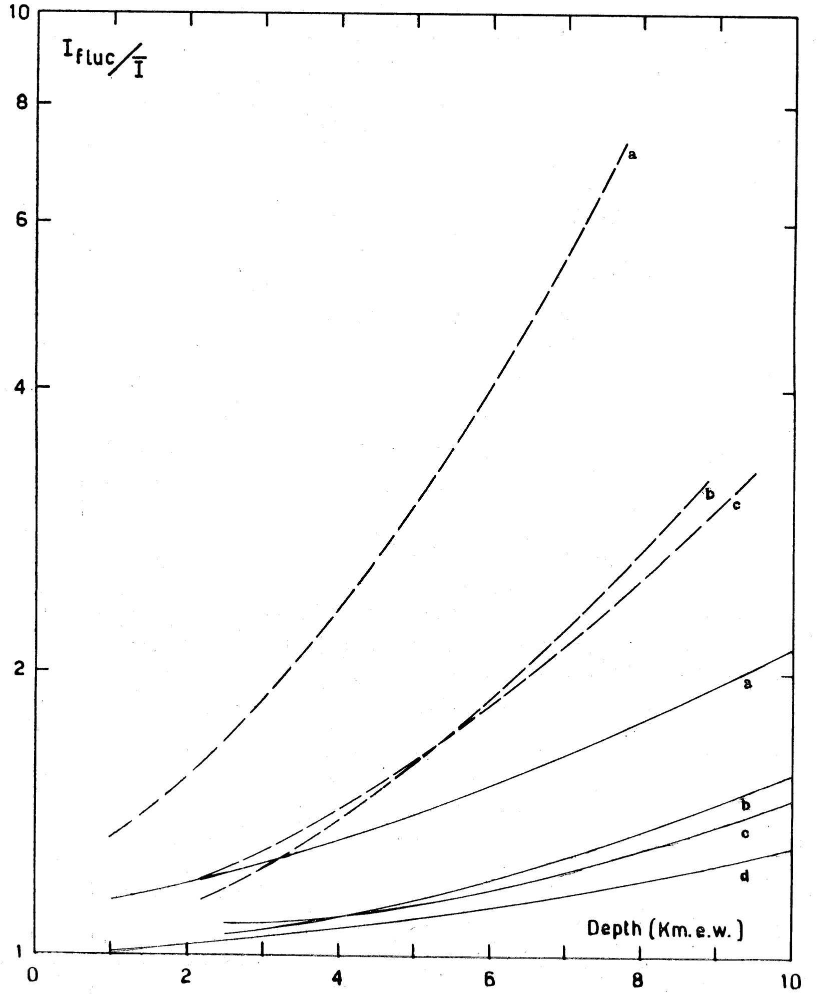

Fig. 5.

We have a discontinuous event in the layer ΔR_n (the n^{th} layer beginning from the one where the last discontinuous event occurred) as soon as the following condition is satisfied:

$$n\,[P_1(E_1) + \cdots + P_i(E_i) + \cdots + P_n(E_n)] \geq \ln 1/r \tag{6}$$

where r is a decimal number chosen at random uniformly among $0 \cdots 1$ by the computer; $P_i(E_i)$ is the probability of a discontinuous event in the layer ΔR_i; $E_i = E_{i-1}$, E_{i-1} being the energy loss due to the continuous processes in R_i.

The discontinuous event is due to bremsstrahlung if:

$$\frac{\sum_{r=1}^{n} P'_i(E_i)}{\sum_{r=1}^{n} P_i(E_i)} \geq r' \tag{7}$$

and is due to a nuclear interaction in the opposite case.

In eq. (7) r' is a new number drawn at random uniformly among 0 and 1 and the apex stands for the probability of energy losses for bremsstrahlung in ΔR_i. The energy lost by a muon in a discontinuous event is:

$$\int_0^{E'} P(E, v)\, dE = r \int_0^{E_0} P(E, v)\, dv$$

where $P(E, v)$ is the probability for a muon of energy E of losing the energy fraction v in a discontinuous event. The initial muon is followed up to when its energy becomes ≈ 1 GeV. In this way we followed the behaviour of 12,000 muons (namely 2000 muons for each energy $E_\mu = 10^5, 10^6, 5 \times 10^6, 10^7, 5 \times 10^7, 10^8$ MeV) down to a maximum depth of 10,000 m.w.e.

4.—Results and conclusions.

The results obtained in such a way for the probability P (R, E) are shown in fig. 2 for different values of $E_\mu = 10^5, 10^6, 5 \times 10^6, 10^7, 5 \times 10^7, 10^8$ MeV. In fig. 3, for a depth of 5000 m.w.e., our curve is directly compared with the ones obtained by other authors with the different approximations and calculation methods summarized in Table I.

Table I.

Authors	Method	Z	A	Radiation	Nuclear	p.p.	Total
Nishimura [4]	Analytical	12.9	26.1	$2.2 \cdot 10^{-6}$ E	—	$1.70 \cdot 10^{-6}$ E	$3.9 \cdot 10^{-6}$ E
Hayman [3]	Montecarlo	11	22 (1)	$1.05 \cdot 10^{-6}$ E	$0.5 \cdot 10^{-6}$ E	$0.75 \cdot 10^{-6}$ E	$2.3 \cdot 10^{-6}$ E
			(2)	$1.8 \cdot 10^{-6}$ E	$1.7 \cdot 10^{-6}$ E	$1.6 \cdot 10^{-6}$ E	$5.1 \cdot 10^{-6}$ E
Oda [6]	Numerical	12.9	26.9	$2 \cdot 10^{-6}$ E	—	$1.78 \cdot 10^{-6}$ E	$3.78 \cdot 10^{-6}$ E
Present work	Monte Carlo	12	24	$1.84 \cdot 10^{-6}$ E	form. [1]	$0.99 \cdot 10^{-6}$ E	—

The evaluation whose foundation most approaches ours is the one carried out by Hayman et al. [3] who make use of a Montecarlo method and take account of nuclear energy losses. The difference from our results is due: 1) to a different estimate of $(dE/dx)_{nucl.}$; 2) to a different simulating method.

With our P (R , E) and equations (2), (3) we obtained for the ratio $I(R_0)/\bar{I}(R_0)$ the values shown in fig. 4 for different values of γ. In fig. 5 we plot for a direct comparison the results obtained by the other authors for $\gamma = 2$ and $\gamma = 3$.

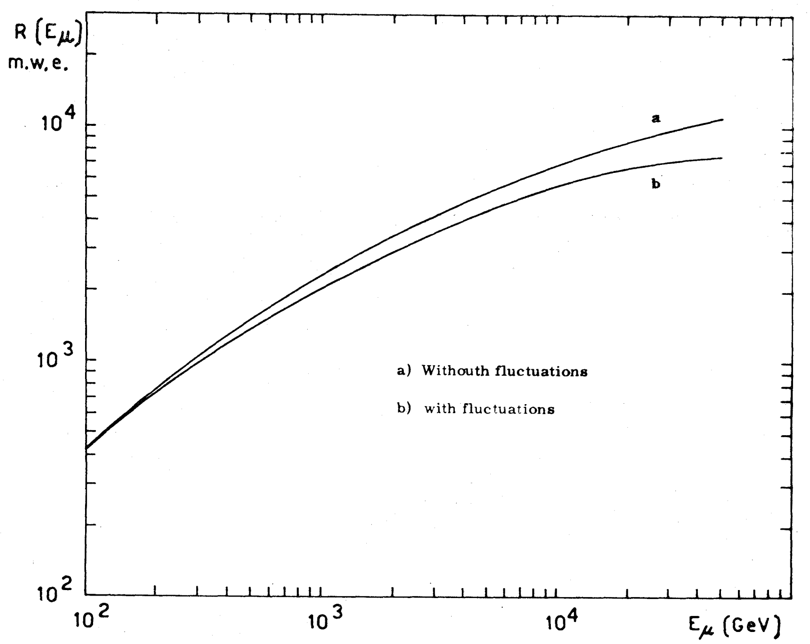

Fig. 6.

As may be seen, there are sensible differences between our results and the ones obtained by other authors. They can influence the usual deduction of the energy spectrum of muons which has the intensity-depth curves as starting point.

Finally in fig. 6 we report the results on R (E) obtained by inserting the value of (dE/dx) from fig. (1) in:

$$R(E) = \int dE/(dE/dx)_{tot.}$$

and the R_{str} (E) obtained from curves similar to the ones shown in fig. 4 by deriving from these distributions the mean range for every value of E_μ. We can therefore see how the straggling sensibly lowers the mean range.

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