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# A new approach to the red-shift law, as a consequence of a new theory of relativity. Nota I

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**Astronomia.** — *A new approach to the red-shift law, as a consequence of a new theory of relativity.* Nota I (*) di GIUSEPPE ARCIDIACONO e NICOLA VIRGOPIA, presentata dal Corrisp. M. CIMINO.

1. INTRODUCTION.—One of the more interesting problems of modern Cosmology, is the theoretical determination of the relation between the distances and the apparent radial velocities of the extragalactic nebulae and their comparison with the observational data.

All the many model universes up to now studied, provide for a velocity-distance relation of the type:

$$\mathbf{V} = \mathrm{H}(t)\,\boldsymbol{r} \tag{1.1}$$

$\mathrm{H}(t)$ being the Hubble's parameter.

Since the velocity of light is finite, it follows that the regions of the universe (i.e. the galaxies) are observed in different instants, according their distances and, in consequence, the expansion velocity of the universe is variable with time. For the largest distances therefore, a deviation from the linearity in the velocity-distance law should be observed which, in turn, should allow a choice among the many models proposed. Unfortunaltely the present observed data being inadequate, it is impossible to make a choice.

As it is known, General Relativity fixes, through the Einstein's gravitational equations, a link between the geometrical structure of the space-time continuum and its material and energetic content. This theory gives an extensive series of model universes, namely the Einstein's static models [1], with uniform distribution of matter at low density, the De Sitter's model [2] with zero density, and the non-static models of Friedmann [3]. These last models, according to the mean density of the matter, may expand indefinitely, contract or oscillate.

At the present time cosmological theories are mainly of two types. In the "evolutionary type" as for instance the Lemaître's theory, the universe originated from hyperdense matter (or primitive atom) and evolved in time with an indefinite expansion. "The steady-state theory", proposed by Bondi and Gold [4], is based on the validity of the "perfect cosmological principle" according which the universe presents the same aspect from whatever point of space and time it is observed. This theory further developed by Hoyle [5] was enriched by the assumption that hydrogen atoms are created continously in order to supply new matter to keep constant the space density of galaxies which, in an expanding universe, necessarily falls as the time goes on.

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The comment of Sciama [6] according whom the general theory of relativity is in agreement with an infinite number of model universes, and not only with the model of the stady-state theory, presents us with finding theories less wide than those of Einstein in such a manner as to find an unique model for the universe.

One theory of this type is the " Kinematic Relativity " constructed by Milne [7] which, based on the cosmological principle (1), postulates a high degree of uniformity and of symmetry for the observed universe, and the validity of the Hubble's law $\mathbf{V} = \boldsymbol{r}/t$.

In this theory the universe can be described in two different but equivalent ways: if the scale of the " kinematic time " $t$ is used, the universe is limited in time and the galaxies undergo recession. Using instead the scale of " dinamic time " $\tau$, the galaxies are not in recession, the past is infinite and space too is infinite. These two time scales are linked by the following relation:

$$\tau = t_0 + t_0 \log (t/t_0) \tag{1.2}$$

$t_0$ being the present time on the kinematic scale.

This theory even though it shows some interesting aspects, is not very convincing both because it professes to be an alternative to general relativity without proposing new gravitational equations, and because, according the Einstein's judgment [8], it does not rests on a solid basis.

A new type of relativity, recently proposed by L. Fantappiè [9], allows to study the cosmological problem starting in a different way. In this theory infact the Lorentz's group of special relativity is perfected into a new group wich represents the motions into itself of De Sitter's space-time at constant curvature. But, while the De Sitter's universe is obtained through the methods of general relativity, i.e. starting from the Einstein's gravitational equations under the hypotesis of empty space, the main merit of Fantappiè lies in having obtained the De Sitter's universe indipendently from general relativity, but through a reasoning based on group theory. Fantappiè's relativity, under the assumption of a four dimensional space-time and a ten parameter group is, from this point of view, the only possible improvement of special relativity to which it reduces as the radius $r$ of the space-time tends to infinity.

By a suitable development of this theory [10], we can obtain a new cosmology of the same type as that of Milne, but more logical and complete. Besides, if the Maxwell's equations are improved in such a way as to be invariant against the Fantappiè's group, one obtains more general equations which, at the relativistic limit ($r \to \infty$) reduce to the usual electromagnetic equations and to the relativistic hydrodinamics of perfect incompressible fluids [11]. One obtain in such a way a more consistent formulation of the

(1) Note that this cosmological principle is less restrictive than the one given by Bond and Gold in the sense that this last postulates also time symmetry.

relativistic magnetofluidodinamics based on a theory of classical type. Moreover to Fantappiè's relativity corresponds a new general projective relativity [12], of the same kind as the unitary theory of Kaluza–Klein, in wich the curvature and torsion tensor are held together in the same tensor.

The aim of the present work is to derive a velocity-distance relation withouth applying general relativity, but starting from Fantappiè's group, that is following an analogous procedure to the one used in special relativity to obtain the velocity addition law.

As it is known, a strictly linear red-shift law can be obtained starting from the metric of De Sitter's space-time. But, by a suitable transformation of this metric, one can obtain laws which are linear only in a first approximation, so that " the same universe may appear in many different guises according to the coordinates used···, (Synge [13]) ". Using instead the method of group, as in the present paper, the red-shift law is obtained in a single way. It must be noticed that although Fantappiè's relativity appears as a spontaneous means for obtaining a relativistic generalization of the hydrodinamical and electromagnetic-fenomena, and therefore a more consistent formulation of the relativistic magnetofluidodinamics, the gravitational problem remains open and outside of this scheme.

2. RELATIVISTIC COSMOLOGY.—In relativistic cosmology the validity of the cosmological principle is assumed and, therefore, the isotropy and homogeneity of physical space is established, i.e. the description of phenomena is the same for every observer in each region of the universe. Under this hypotesis a " cosmic time " $t$ may be introduced and the space-time metric takes the simpler form [14]:

$$ds^2 = c^2\,dt^2 - \mathrm{R}\,(t)\,du^2 \tag{2.1}$$

where $\mathrm{R}\,(t)$ is an arbitrary function, and

$$du^2 = (dr^2 + r^2\,d\theta^2 + r^2 \operatorname{sen}^2\theta\,d\varphi^2)/(1 + kr^2/4)^2 \tag{2.2}$$

represents the metric of a tridimensional riemannian space. The scale-factor $\mathrm{R}\,(t)$ which is a function of cosmic time $t$, describes the evolution of the universe and may be explicitly determined by (2.1) together with the Einstein's gravitational equations:

$$\mathrm{G}_{ik} - \frac{1}{2}\mathrm{G}g_{ik} + \lambda g_{ik} = -\varkappa\mathrm{T}_{ik} \qquad (i\,,k = 1\,,2\,,3\,,4). \tag{2.3}$$

In this way the differential equations of the cosmological problem from which $\mathrm{R}\,(t)$ can be determined are given by:

$$\left\{\begin{aligned} \varkappa\rho &= -\lambda + 3\,\frac{k+\dot{\mathrm{R}}^2}{c^2\,\mathrm{R}^2} \\ \frac{\varkappa p}{c^2} &= \lambda - \frac{2\,\mathrm{R}\ddot{\mathrm{R}} + \dot{\mathrm{R}}^2 + k}{c^2\,\mathrm{R}^2} \end{aligned}\right. \tag{2.4}$$

where $\varkappa$ is the Einstein's gravitational constant linked to the newtonian constant $\omega$ by: $\omega = c^4 \varkappa / 8\pi$; $p$ is the hydrodinamical pressure of matter and radiation, $\rho$ is the density of matter and energy, $\lambda$ is the cosmological term, $k$ is a constant and may take the values $+1, 0, -1$ according as space is spherical, euclidean or hyperbolic, i.e. $k/R^2$ is the space-riemannian curvature.

In order to obtain the velocity-distance law, valid for every model universe we observe that if $\boldsymbol{u}$ is the distance parameter between two events $P_0$ and P, measured by the metric (2.2), the space-time interval at instant $t$ is given by the following " distance ":

$$\boldsymbol{r}(t) = R(t)\,\boldsymbol{u} \tag{2.5}$$

which, in general, is a function of time. It follows that the velocity with which $\boldsymbol{r}$ varies is:

$$\mathbf{V} = \frac{d\boldsymbol{r}}{dt} = \dot{R}(t)\,\boldsymbol{u} = \dot{R}(t)\,\boldsymbol{r}/R(t) \tag{2.6}$$

that is, the " apparent velocity " of P against $P_0$ is proportional to the distance:

$$\mathbf{V} = H(t)\,\boldsymbol{r} \tag{2.7}$$

where we have set:

$$H(t) = \dot{R}/R\,. \tag{2.8}$$

Since the Hubble's parameter $H(t)$ is a function of cosmic time, we can consider its variation:

$$\frac{dH}{dt} = (\ddot{R}R - \dot{R}^2)/R^2 = \frac{\ddot{R}}{R} - H^2$$

and, if we introduce the so-called " deceleration parameter " $q$ [15]:

$$q = -\ddot{R}/RH^2 \tag{2.9}$$

we can write:

$$\frac{dH}{dt} = -H^2(q+1)\,. \tag{2.10}$$

The equations (2.4) must be integrated taking the following initial conditions:

$$\left(\frac{\dot{R}}{R}\right)_{t=t_0} = H_0 \quad , \quad \left(\frac{\ddot{R}}{R}\right)_{t=t_0} = -q_0 H_0^2$$

which may be determined from observations; $H_0$ and $q_0$ being the values computed at present time. The experimental values of $q_0$ have been given by Humason, Mayall and Sandage [16] by means of the relation $[m, z]$, i.e. apparent bolometric magnitude-red shift, which, corrected for the first order terms of $z$ ($z = c\Delta\lambda/\lambda$), is given by:

$$m_{\text{bol}} = 5 \log z + 1.086\,(1 - q_0)\,z + \cdots$$

from which $q_0$ may be determined through the values of $m_{bol}$ and $z$ given by the observations. It has been found that $q_0$ is in the range o |—| 3, with the best approximation given by $q_0 = 1 \pm 1/2$. The present value of $q_0$ is generally taken equal to 75 Km/sec $10^6$ parsecs, from which one derives the age of the universe $H^{-1} \approx 1.3 \; 10^{10}$ years.

3. NEWTONIAN COSMOLOGY.—One of the big difficulties of newtonian cosmology was the postulated staticity of the universe (i.e. no large-scale motions in the universe) together with the cosmological principle.

The work of Milne and McCrea [17], re-examining the newtonian cosmology, showed its equivalence, in many respects, to the relativistic cosmology. Moreover, its mean results and features could be obtained with a simpler formalism and of more simple interpretation than the ones used in relativity. In the newtonian cosmology of Milne and McCrea the validity of the cosmological principle is postulated; the observer O moves with the substratum which is idealized as the streaming of a uniform fluid. The observer lies at the origin of a system of coordinates and observes the physical propierties at a generic point P at time $t$. Let V, $\rho$ , $p$ , OP $= \boldsymbol{r}$, be the velocity, the density, the pressure and the distance of P. We then have (2):

$$\mathbf{V} = \mathbf{V}(\boldsymbol{r}, t) = \mathrm{H}(t)\,\boldsymbol{r} \quad , \quad \rho = \rho(t) \quad , \quad p = p(t) \tag{3.1}$$

where H , $\rho$ , $p$ are only function of $t$. If the first relation of (3.1) is regarded as the equation of motion of a particle, it may be integrated and gives:

$$\boldsymbol{r} = \mathrm{R}(t)\,\boldsymbol{r}_0 \tag{3.2}$$

where R $(t)$ satisfies the conditions

$$\frac{\dot{\mathrm{R}}}{\mathrm{R}} = \mathrm{H}(t) \quad , \quad \mathrm{R}(t_0) = 1 \tag{3.3}$$

the first of which coincides with (2.8) and $\boldsymbol{r}_0 = \boldsymbol{r}(t_0)$ , $t_0$ being the actual cosmic time (i.e. the age of the universe).

It can be shown that, if for $\mathbf{V}$, $\rho$ , $p$, the equation of continuity and the Euler's equations of motion are satisfied, then a differential equation for R $(t)$, similar to the one of the relativistic cosmology, is found.

It is interesting to note that even if the newtonian cosmology is not quite satisfactory it, nevertheless, shows that the behaviour of the whole universe can be characterized by only one function R $(t)$ wich satisfies a certain differential equation. Milne and McCrea have also shown that the newtonian theory gives not only excellent results in the local description of the gravitational field (i.e. when the velocities are small compared with the velocity of light), but also on the cosmic scale.

(2) For a more satisfactory discussion see H. BONDI, *Cosmology*, Cambridge 1961.

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