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Paolo Salmon

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The Contribution of Aldo Andreotti to the Study of Algebra in Italy Around 1950

PAOLO SALMON (*)

1. – Some reasons for the lack of interest in Algebra shown by several Italian mathematicians before 1945

The Italian school of algebraic geometry counted famous scholars since the second half of 19^{th} Century. First of all Luigi Cremona and Eugenio Bertini became famous, followed by Corrado Segre and Giuseppe Veronese, then by the famous triad of Guido Castelnuovo, Federigo Enriques and Francesco Severi. To the names just mentioned, among others, those of their talented students Beniamino Segre, Giacomo Albanese, Oscar Chisini, Fabio Conforto can be added. One may look at the recent interesting article *G. Castelnuovo* and *F. Severi*: two personalities and two letters, by D. Babbit - J. Goodstein, which appeared in *Notices of the American Mathematical Society*, vol. 56, n. 7, 2009.

The objects of research in algebraic geometry are known as algebraic varieties, defined as zero loci of polynomials, thus making use of very basic algebraic concepts, therefore elementary algebra is applied to geometry. As a consequence, a particular attention to algebra, as a basic mathematical discipline, appeared in our country. The introduction of an institutional course of algebra in the first year of the Degree in Mathematics at the Faculty of Sciences in Italian universities has occurred only in 1961, adapting so very late Italy to other, particularly European nations. This may seem to be a contradiction with the international success obtained by the Italian school of algebraic geometry.

In Italy, however, at least until the first half of the last century, there has been some reluctance to give prominence and importance to the algebraic foundations, especially by the algebraic geometry scholars. In fact, according to the common feeling of the mathemathical environment, achieving important results in algebraic geometry by Italian scholars was due to an extraordinary intuition and wise use of algebraic-geometric tools in which the algebraic aspects, necessary to give rigor to the proofs, was systematically

(*) Professore emerito - Università di Bologna.

underestimated. It can be said that sometimes the proofs exhibited by the Italian algebraic geometry scholars were not quite accurate in all details and this often happened because using more or less openly intuition rather than investigating algebraic facts. On that purpose, in 1932, there was a controversy between Severi and Emmy Noether who was an excellent algebraist in Germany, interested in algebraic geometry. Also Oscar Zariski, a mathematician who in some ways could be considered as a student or collaborator of the Italian algebraic geometry scholars, noticed some gaps in the proofs given by some Italian geometers and proposed to find a remedy for them by writing, around 1940, a few papers on the foundations of commutative algebra with the purpose of providing the tools for the full rigor needed in the proofs of various results.

Of course in Italy there were also scholars in purely algebraic disciplines (i.e. not necessarily related to geometrical issues) as, for example, Gaetano Scorza, a professor at the University of Rome and a well recognized master of several topics, including the theory of algebraic groups, subject of a well known and widespread book. One can also mention Scorza's pupil Guido Zappa, who first studied at the Scuola Normale Superiore in Pisa, then became an assistant professor in Rome and finally full professor in Naples and Florence. Together with Zappa, there was in Rome an other algebraist, Lucio Lombardo Radice (assistant professor and then professor). Also Mario Fiorentini (later professor in Ferrara), interested in algebra and algebraic geometry, had contacts with mathematicians at the university of Rome.

We can say, in general, that a mathematician who wanted to enter the Italian Academic world and then start a university career was almost always forced to apply for teaching courses in basic analysis or geometry at the Faculty of Science, Engineering or Architecture. So the scholars of algebra had often to accept (before 1961) some compromises, writing articles with a geometric content in order to have a chance to be selected as assistants or full professors of geometry; this is the case, for example, of Zappa, who taught geometry until 1961.

The reasons we have just given in order to explain the long delay to introduce in Italy a basic algebra course are probably the most important and are substantially independent from any political considerations. But we can add that the national pride and the so-called *Italianity* during the long period of the Fascist domination may have contributed to delay the needed recognition to a discipline more cultivated abroad than in Italy. We must also recall the so-called *racial laws* of 1938: all Jewish students and teachers were obliged to leave their positions in schools or universities. Finally, we may mention the involvement of Italy in the Second World War, which forced the entire population to confront urgent and pressing problems as well as tragic situations until the liberation in April 1945.

2. – The first interest in algebra by some Italian mathematicians after 1945. The particular contribution of Aldo Andreotti

Starting from 1946, when the situation was almost normal inside the Italian universities after the war disasters, some Italian mathematicians began to show an interest in algebra and also criticisms and complaints about the absence of an elementary course of algebra in the Degree of Mathematics in the Italian universities.

One of the most significant examples was Beniamino Segre, a scholar of algebraic geometry strictly related to his teachers G. Castelnuovo, F. Enriques and (especially) F. Severi. B. Segre had to leave the teaching of geometry in Bologna in 1938, as a consequence of the racial laws, and to emigrate to England, teaching there at different universities, better understanding the limitations of the Italian school of algebraic geometry. It was no longer possible to neglect the algebraic foundations of the discipline, considered an essential tool for a modern approach to algebraic geometry. When he went back to the University of Bologna in 1945, B. Segre wanted to spread his new ideas by including within his courses on projective geometry some topics of algebra, precisely: residue classes modulo p, groups, rings, division rings, fields, subsets, subgroups, subrings and ideals, subfields, zeros of polynomials, decomposition of polynomials, finite fields, Galois fields, linear spaces, graphs. Just mentioned topics are taken from the table of contents of the book by B. Segre *Lezioni di Geometria Moderna*, Vol I, *Fondamenti di Geometria sopra un corpo qualsiasi*, published in 1948 by Zanichelli, where Segre included the content of his lectures given in Bologna during the Academic year 1946/47. The subtitle of the book is eloquent: whereas in classical algebraic geometry the basic field (of coefficients of polynomials involved) was always the field C of complex numbers, in a modern vision it is often possible to remove this restriction. First of all, the opening sentence of the preface of the book is particularly significant: "*The high degree of generality and perfection recently reached by Algebra, today allows us to set questions on a new basis, much larger than the classic ones, at the same time providing elegant and powerful means fit to the treatment with the utmost generality of the geometric problems, particularly those related to the foundations of Mathematic*". So, Segre's point of view about the importance of algebra became public and shared. Few years later, B. Segre moved to Rome as a professor thanks to F. Severi who gave him a position, implicitly accepting that need of a renewal, which was not recognized in the recent past.

G. Zappa, who had been forced to work also in geometry, became in 1946 professor in Naples and taught there before moving to Florence. After 1946, Zappa had been working essentially in algebra, the field of his main scientific interest. In Naples Zappa could take advantage of the precious collaboration of his pupil Rodolfo Permutti and published (first edition 1950, second edition 1952)

the volume *Gruppi, corpi, equazioni* (the same title that appears in the textbook of the two authors published by Feltrinelli in 1963). In this textbook, in addition to the basic topics of algebra, the Galois theory of algebraic equations was also presented with the famous 'criterion of solvability of Galois', from which the impossibility of solving by radicals the general algebraic equation when the degree of the polynomial involved is greater than 4 follows. There is indeed no doubt about the contribution given by Zappa to the spread of algebra in the renewal of mathematical teaching in Italy.

A special attempt to convince the Italian mathematicians to pay attention to algebra is then due to Aldo Andreotti, who was always an aspiring scholar of algebraic geometry from his early years in the University of Pisa, when he was also a student at the Scuola Normale (years 1942/47). After graduating in 1947, Andreotti moved to Rome to be hired as an assistant professor by F. Severi. On the one hand Severi had a strong admiration for this young researcher who had shown an extraordinary talent combined with ambition and desire to study difficult issues in algebraic geometry; on the other hand, Severi was also trying to cool the attention and enthusiasm showed by his young assistant to study algebra and topology (general and algebraic) that appeared necessary to renew and upgrade the Italian school of algebraic geometry. Perhaps in this attachment to the old Italian school of classical algebraic geometry shown by Severi, one may see the will of the old teacher to defend a glorious tradition while it was showing its limits, as his excellent student had repeatedly shown. Severi did not share and tried to oppose Andreotti's proposal to spend a period of study and research in the United States. Finally Andreotti was allowed to go to Princeton, but he could never forgive Severi for the obstacles; the experience in USA was extremely fruitful and decisive for his future.

Andreotti, who was born in Florence, used to spend frequently in his family-house a few hours to talk with three Florentine friends: Francesco Gherardelli (an assistant professor of geometry in Firenze), Stefano Guazzone (a student at the Scuola Normale di Pisa until 1949) and myself (also a student at the Scuola Normale in 1948/53); Guazzone introduced me in those conversatioins with Andreotti and Gherardelli. These frequent and often lengthy conversations, almost always dominated by Andreotti, have left their mark. Gherardelli was included in the University of Florence, where the innovative ideas of Andreotti arrived immediately; Guazzone and I could spread the views of Andreotti not only within the University of Pisa but also within the Scuola Normale, an advanced cultural nationwide landmark.

In the selection of 1946 for chairs of geometry, as well as Zappa, also Giovanni Dantoni was a winner, and he was immediately called to teach geometry at the University of Pisa. Dantoni was a scholar of Algebraic geometry and he had also a special interest in algebra, particularly intensified after he moved to Catania (in 1956), as it is clearly shown by his last scientific articles, all strictly devoted to

algebra and not algebraic geometry. However, at that time (around 1950) Dantoni, even appreciating the young aspirants in research of algebraic subjects, sometimes mitigated his trend. For example, while Andreotti did not hide his sympathy for the textbook *Modern Algebra* by B.L. Van der Waerden (published by Springer in 1931 and later translated into English in 1949), recommending to read it, Dantoni complained particular difficulties in understanding that work, now universally regarded as a basic informative manual. Also for the book *Moderne Agebraische Geometrie-Die idealtheoretischen Grundlagen* by Wolfgang Groebner, published in 1949, Dantoni did not hide his antipathy, partially shared by Andreotti, who considered anyway important to deal with that textbook. We can however say that, despite some hesitations, also Dantoni contributed to the spread of algebra (with my own benefit). In an official *colloquio* (student-teacher) that I had with him at the Scuola Normale while attending my third year in Pisa, and also in my dissertation, discussed in 1952, I have been offered by Dantoni arguments in commutative algebra (according to a name already mentioned, later officially introduced by O. Zariski and P. Samuel at the end of the decade 1950) related to algebraic geometry, which satisfied me completely. In Pisa the innovative ideas of Dantoni and Andreotti about algebra (very often the second seemed to overcome the first one in enthusiasm, but the situation was the opposite in the most recent scientific works by the two mathematicians) were certainly not opposed by the two senior lecturers F. Cecioni and S. Cherubino (professors of analysis and geometry respectively), both interested in algebra. These ideas were also appreciated by G. Gemignani, assistant to Dantoni, and V. Checcucci, who taught courses in the third and fourth years with contents in algebra and topology. At that time also M. Sce (who later moved to Milan) and L. Cantoni were students in Pisa, both interested in algebraic topics. After graduating, Guazzone moved first to Padova and then to Trieste, where some scholars, as U. Morin, G. Zacher and M. Dolcher, were particularly interested in algebra. Thus our discussions on algebra could continue elsewhere and in many places the same problems were introduced independently; so the growing opinion of the need to introduce the teaching of algebra in Italy could develop on large scale in various universities.

The cities of Rome, Pisa, Florence, Milan, Padua and Trieste have been already mentioned. In Milan in particular, there was Oscar Chisini, a famous professor whose name is still associated with Enriques, as they wrote together a treaty in various volumes Teoria geometrica delle equazioni in which (as it is clearly stated in the title) the traditional point of view - that algebra is an aid to geometry - was completely reversed; now, a geometric interpretation may help the study of algebraic equations. Some students of Chisini, such as E. Marchionna and C. Tibiletti, were however aware of the importance of autonomous algebraic developments and one can also mention the presence in Milano of a young researcher, Edoardo Vesentini, who was oriented towards studies of

analytic and differential geometry, both subjects beeing different from the traditional themes of algebraic geometry. Since 1952 Vesentini, even keeping his permanent position as an assistant professor at the University of Milano, was called by Severi and Segre to teach at the University of Rome and also in the Istituto di Alta Matematica (chaired with a tenure by Severi). In 1959 Vesentini was then called in Pisa, as a professor of geometry, and there he had a long time scientific activity with Andreotti.

A particular case is that of Torino, where Andreotti has taught courses in geometry from 1951 to 1956.

3. – Teaching and research carried out by Andreotti in Torino in the period 1951/56. The work of Andreotti-Salmon " Anelli con unica decomponibilità in fattori primi ed un problema di intersezioni complete"

When Andreotti became professor of Geometry in the Italian University in 1951, he was only 27 (only E. Bombieri and few other scholars were younger professors). He had already achieved an international fame for his scientific results and was particularly respected by his (teacher-opponent) Severi; following some rumors about the job selection in 1951, Severi had influenced the committee to include the name of Andreotti in the winners' list.

Immediately after, Andreotti was called as a full professor of geometry at the University of Torino where he spent five years, until 1956. Andreotti was teaching an elementary course in one of the first two years and, simultaneously, a more advanced one in the second biennium. Andreotti's ideas for reform, particularly his open-minded teaching of algebra and topology (general and algebraic) were spread in Torino since 1951. His scientific interests, already mainly oriented towards algebraic geometry, then shifted to complex geometry and the shift became stronger in subsequent years. For his advanced course in geometry at third or fourth year of undergraduate degree in mathematics, first he focused on algebraic geometry (for example, a proof of Bzout's theorem in a field of characteristic p) but later focused on the geometric applications of complex analysis and in the last two years (1954/56) on Hodge's theory of harmonic integrals. The geometry course in the first two years was mainly related with elementary algebra and analytic geometry, while the part on topology could be completed in the second biennium.

The teaching of algebra by Andreotti, in the first two years in Torino, particularly of the basic elements of group theory, was really remarkable, as I could personally verify, during the fall of 1953, comparing my training on the subject with that of Luigi Petrone, who graduated in Torino a few months before under the direction of Andreotti. I graduated in autumn 1952 with Dantoni and then I spent a year (1952/53) of postgraduate specialization at the

Scuola Normale. Both, Petrone and myself, were recruited as fellows at the Istituto di Alta Matematica in Roma, during the year 1953/54; we became friends right away deciding, for economic reasons, to share a twin room. During the daily mathematical conversations, it was immediately evident how Petrone overcame me in in the field of group theory. My dissertation too, as Petrone's, was indeed devoted to the topics of algebraic geometry, and furthermore, preparing the colloquio at the third year of the Scuola Normale, I had studied ideal theory on a textbook by O. Perron. So there was no doubt that the superiority of Petrone on group theory was a consequence of the approach given by Andreotti in Torino in his first course, particulary focused on algebraic foundations. I met later Petrone in Torino in the fall of 1954, when Andreotti could manage to offer us there a temporary teaching grant; in the meantime, however, the scientific interests of Petrone were moving towards applied mathematics.

During the academic year spent in Rome, I had unsuccessfully tried, as during the previous year, to extend to surfaces and algebraic varieties of higher dimension the elementary technique of proof of the postulation formula of a simple algebraic curve, presented in my dissertation. As suggested by Dantoni, I showed my work to B. Segre, who right away decided to publish it (even if it appeared a little light) in a short article on the Journal Bollettino dell'Unione Matematica Italiana. Also Segre agreed on the need to extend the procedure to higher dimensions and, when I failed, after months of fruitless attempts, I lost my hopes to get any tenure in the Italian University and decided to study for the job selections in the high schools I had already applied for. I was very surprised, in September 1954, when Andreotti called me by phone in Florence with the proposal to joint him at Torino university in order to work under his direction. I should get a little salary around 25.000 liras per month and my parents declared themselves immediately available to support me at their expenses, encouraging me to accept. But yet I showed my resistance in talks with Andreotti. I first said that I was unable to understand certain works of algebraic geometry and his immediate reply was: A clever mathematician cannot understand them. I added that I had not succeeded in extending my proof on the postulation of a curve to higher dimensions and he replied: I too did appreciate your proof and tried to extend it but without success. Finally, I thought that I had a decisive argument, saying that my time for research was limited, because I wanted to spend part of it in political activities as a member of the Communist Party. Andreotti immediately replied that all this was excellent, because he considered himself a *democratico cristiano* and so we would have neutralized each other. Andreotti emphasized his need to get my help in taking written notes on his advanced course about Hodge theory, adding that I was certainly able to move again, under his leadership, to a new scientific research. Finally he convinced me and I did accepte; a few days later I was notified by him that I had been given a course

(as Petrone) and the salary was about 40.000 liras per month, more than enough to make a living; I had to teach *Istituzioni di Matematica* to the students in Chemistry.

So I was in Torino in 1954, settling alone, without Petrone, who lived with his family. During my first year in Torino, I just respected the agreement made with Aldo in Firenze of taking notes during his second course on harmonic integrals, revising and discussing them with him, but I gave up completely any scientific research. Later (summer 1955), meeting Aldo in his country-house residence in the Tuscany-Emilian Mountains, I told him all my concerns for my future in the University, as a single scientific publication was not sufficient for going on in my career. I also added to Aldo that next summer (1956) I had to serve in the army for eighteen months (no further delay after the age of 26 years was allowed and I had no hope to skip it for health reasons). If before this unfortunate but inevitable event, I would fail to publish at least a second paper, Aldo would not succeede in helping me to get a position with the hope to carry out my scientific work not yet materialized. Aldo listened with the utmost attention and told me that in the coming fall, when we would meet again in Torino, he would suggest me some problem in algebraic geometry suitable to my ability. He was well aware that without his initial support, I could not personally find interesting and not difficult open problems to be solved and then published by me. Keeping his word, Aldo received me in his house in Torino in the autumn of 1955 to offer me some problem. He began his conversation talking about a partial personal failure in one of his works.

A few months earlier Aldo had sent Groebner, a professor in Insbruck, an article to be published in the Austrian magazine *Monatshefte fur Mathematik*. This work was entitled *Anelli con unica decomponibilità in fattori primi e un problema di intersezioni complete* which initially showed the equivalence, under certain hypothesis, of two properties, the first one algebraic and the second one geometric. The algebraic property concerned the so-called factorial ring, i.e. the commutative integral domain in which every non-zero and not invertible element admits an essentially unique decomposition in the product of irreducible factors (a particularly simple example of a factorial ring is the ring Z of integers for which every number is decomposed into the product of prime factors in an essentially unique way: two factors may differ by -1, as in this case: $6 = 2 \cdot 3 = (-2) \cdot (-3)$. The geometric property concerned those particular varieties (such as the planes of ordinary space) that contain in codimension 1 complete intersections only (a counterexample in the ordinary projective space is the quadric given by the equation $xy - zt = 0$ containing the cubic curve of parametric equations: $x = u^3$, $y = v^3$, $z = u^2v$, $t = uv^2$; this cubic cannot be a complete intersection of the quadric with a surface of degree m, as the equality $3 = 2m$ is impossible). Groebner said that it was not possible at that time to accept the article because the referee had found a gap in the new algebraic proof

(based on the above mentioned equivalence) of a result proved by Max Noether using geometric techniques in 1882: a generic non-singular surface of order not less than 4 contains only complete intersections (in the previous counterexample the quadric surface is not singular, but has order 2 and it is not generic since it is associated with the polynomial xy-zt that can be written as a homogeneous determinant of order 2). Aldo proposed me to overcome this gap and also improve the statement (and proof) of another theorem (concerning ideals generated by some minors extracted from a matrix) included in his article. Andreotti added that, after the conclusion of this work, his tastes had changed, from the geometric applications of algebra to the geometric applications of complex analysis. For him it was not easy to start again with old problems, so, he would be happy if I would succeed.

Of course, I could not rejoice that his paper had met obstacles and I was also a little worried on Aldo's changement in his tastes; my research interest was still in algebra or algebraic geometry and I thought (rightly) that it would remain unchanged in the future. But at the same time, I realized the propitious chance that I had from Aldo. He suggested to me precise and circumscribed problems in a field I particularly liked. A systematic reading of the book of Groebner (as well as the textbook of Northcott Ideal Theory appeared in 1953), gave me a complete satisfaction and I could even more appreciate the research suggested by Aldo. First of all, I decided to work in the year 1955/56 only in scientific researches; in his advanced course Aldo could receive a good help from Davide Demaria, who was working essentially in topology and also in differential and algebraic geometry. I was able to give, within a few weeks, an answer to the second question on the ideal of minors of a matrix; the algebraic proof of the theorem of Noether, very elaborate, has had my conclusion only late in the spring of 1956. Aldo, who was like me very satisfied with the outcome, made the proposal, unexpected and surprising for me, to write a joint paper with him. My contribution appeared to be so little compared to his one that I felt obliged in a first moment not to accept the proposal. But, at the same time, I did realize that a separate publication of my results would present many difficulties and so I accepted the attractive solution, easily finding an agreement with Aldo on an introductory note to specify the limit of my contribution.

Of course I could benefit of this paper on several occasions during my career and my research, where my name was near the name of a great mathematician, who was also a true innovator. A few years later, in 1961, Pierre Samuel was invited to give a lecture at a meeting in Florence, and he chose to speak about our work (Andreotti and I were also invited). I was there and told Samuel that I would go for two years in Paris (1961/63) and I would be happy to do research under his direction, just at a time when he was working intensely on factorial rings. Samuel accepted and the works I wrote in Paris under his suggestion were decisive for my further career.

The research interests of Andreotti for algebra and algebraic geometry were almost exhausted after our work in Torino; sometimes he was saying that it was too much difficult to work at a high level in algebraic geometry, because of the big competition all over the world (expecially in USA with so big names as O. Zariski, D. Mumford, A .Weil, J. Tate, M. Artin , etc. and in France, where J. P. Serre and A, Grothendieck were not less famous). But the importance he always gave to the teaching and the development of algebra had a confirmation after he moved to Pisa in 1956. Andreotti immediately invited Iacopo Barsotti an Italian mathematician who was living in the States to come back to Italy as a professor at the University of Pisa; Barsotti accepted, and for many years gave his high contribution to the study of algebra and algebraic geometry in Italy. Andreotti has then also worked hard to call to Pisa as teachers of algebra two scholars of great prestige such as Corrado De Concini and Claudio Procesi (and he would have liked that also Enrico Arbarello could go to Pisa).

In conclusion, the actual positive renewal of Italian algebra and algebraic geometry is a consequence of many positive facts, among which there is, explicitly or implicitly, the high contribution tenaciously given by Aldo Andreotti in the course of a life too short, but very intense and fruitful.

Università di Bologna, Dip.to di Matematica
Via Zamboni, 33 - 40126 Bologna

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