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A Structure Theory for Jordan H^*-Pairs.

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Sunto. – *Il concetto di H^*-coppia di Jordan, appare, in modo naturale, nello studio degli H^*-sistemi tripli di Lie* ([3]). *Di fatto, nel* [4, Th. 3.1] *si prova che il problema della classificazione degli H^*-sistemi tripli di Lie si riduce a provare l'esistenza di certi inviluppi di L^*-algebre e in* [3] *si prova anche che è possibile associare H^*-coppie topologicamente semplici non quadratiche di Jordan ad un'ampia classe di H^*-sistemi tripli di Lie e che poi gli inviluppi precedenti possono essere ottenuti da un'opportuna classificazione, in termini di H^*-coppie associative, di queste coppie. In questo lavoro viene dato un teorema di classificazione delle H^*-coppie topologicamente semplici non quadratiche di Jordan in termini di H^*-coppie associative e di certuni loro anti-isomorfismi. Vengono anche enunciate alcune conseguenze di questa classificazione.*

Summary. – *Jordan H^*-pairs appear, in a natural way, in the study of Lie H^*-triple systems* ([3]). *Indeed, it is shown in* [4, Th. 3.1] *that the problem of the classification of Lie H^*-triple systems is reduced to prove the existence of certain L^*-algebra envelopes, and it is also shown in* [3] *that we can associate topologically simple non-quadratic Jordan H^*-pairs to a wide class of Lie H^*-triple systems and then the above envelopes can be obtained from a suitable classification, in terms of associative H^*-pairs, of these pairs. In this paper we give a classification theorem for topologically simple non-quadratic Jordan H^*-pairs in terms of associative H^*-pairs and certain of their anti-isomorphisms. Some consequences of this classification are also stated.*

1. – Introduction.

Recall that an H^*-pair $A = (A^+, A^-)$ is a pair of Hilbert spaces over the complex numbers with involution, in which the inner products $(\cdot|\cdot)$, the pair triple products $\langle\cdot,\cdot,\cdot\rangle$ and involution $*$ are «compatible». By applying the structure theory of Jordan H^*-triple systems developed by A. Castellón, J.A. Cuenca and C. Martín in [9, 11], one could describe topologically simple Jordan H^*-pairs, however, we use entirely different methods to classify topologi-

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cally simple Jordan H^*-pairs. In fact, we note that the introduction of techniques of tight evelopes, D'Amour's extension theorems, Zel'manov polynomials and dual spaces methods in the treatment of problems of Jordan H^*-theory, motivated in part by Rodriguez's paper [24], is perhaps the most interesting novelty in this paper.

The paper is organized as follows. In the second section we give the preliminary results on associative and Jordan pairs and we obtain in the third section a structure theory for topologically simple non-quadratic Jordan H^*-pairs in terms of topologically simple associative H^*-pairs, by forgetting their Hilbert space structures and starting with the remaining purely algebraic information available on them. Jordan H^*-pairs with zero annihilator are well related to hermitian Hilbert triples introduced and classified by W. Kaup in [17, 18], (see Remark 3). However, the new approach we give allows us to prove the existence of associative H^*-algebra envelopes for certain topologically simple Lie H^*-triple systems, (what implies the classification of the last ones). We also extend one of the results of [24] and [2] (see Remark 1).

2. – Definitions and preliminary results.

2.1. *On associative pairs.*

Let $A = (A^+, A^-)$ be a pair of modules over a commutative unitary ring K, and $\langle \cdot, \cdot, \cdot \rangle$: $A^\sigma \times A^{-\sigma} \times A^\sigma \to A^\sigma$, two trilinear maps written

$$(x, y, z) \mapsto \langle x, y, z \rangle$$

for $\sigma \in \{+, -\}$. Then A is called an *associative pair* if the following identities are satisfied:

$$\langle \langle x, y, z \rangle, u, v \rangle = \langle x, \langle y, z, u \rangle, v \rangle = \langle x, y, \langle z, u, v \rangle \rangle$$

for $x, z, v \in A^\sigma$ and $y, u \in A^{-\sigma}$.

The definitions of *homomorphism*, *epimorphism*, *monomorphism* and *isomorphism* are the usual ones. The opposite pair A^{op} of the pair $A = (A^+, A^-)$ is the pair (A^-, A^+) with the same triple products. An *anti-homomorphism* from A to B is a K-linear mapping $\nu = (\nu^+, \nu^-)$ from the pair A to the pair B^{op} satisfying $\nu^\sigma(\langle x, y, z \rangle) = \langle \nu^\sigma(z), \nu^{-\sigma}(y), \nu^\sigma(x) \rangle$ for all $x, z \in A^\sigma$ and $y \in A^{-\sigma}$. An anti-isomorphism $\nu = (\nu^+, \nu^-)$ will be called *involutive* if $\nu^{-\sigma}\nu^\sigma = Id$. An *ideal* $I = (I^+, I^-)$ of A is a couple of K-submodules such that

$$\langle I^\sigma, A^{-\sigma}, A^\sigma \rangle + \langle A^\sigma, I^{-\sigma}, A^\sigma \rangle + \langle A^\sigma, A^{-\sigma}, I^\sigma \rangle \subset I^\sigma.$$

A pair A will be called *simple* if and only if $\langle A^\sigma, A^{-\sigma}, A^\sigma \rangle \neq 0$ and its only ideals are 0 and A.

Let us see a first example of an associative pair. A *dual pair* of vector spaces over a division K-algebra Δ is a triple (X, X', h) such that X is a left Δ-vector space, X' is a right Δ-vector space and h is a non-degenerate bilinear form $h: X \times X' \to \Delta$. One can consider the X'-topology of X (and the X-topology of X'), see [16, Chapter IV Section 6]. If we have two dual pairs (X, X', h) and (Y, Y', g), one can define $L(X, Y)$ as the set of all continuous linear maps from X to Y (and $F(X, Y)$ the subset of all finite rank elements of $L(X, Y)$). Any subpair of $(L(X, Y), L(Y, X))$ containing $(F(X, Y), F(Y, X))$ with the triple products $\langle x, y, z\rangle^\sigma := xyz$, is a prime associative pair with nonzero socle (see [12]). If $f \in L(X, Y)$, we define the *adjoint* of f, denoted by $f^\sharp$, as the only element $f^\sharp \in L(Y', X')$ such that $g(f(x), y') = h(x, f^\sharp(y'))$ for any $x \in X$ and $y' \in Y'$ (see [16, Chapter IV, Theorem 1, p. 72] for existence and uniqueness).

In [16, Proposition IV. 8.1], it is shown that $f \in F(X, Y)$ if and only if $f(t) = \sum_{i=1}^{n} h(t, x_i')\, y_i$ for all $t \in X$, with $\{x_i'\}_{i=1}^n$ in X' and $\{y_i\}_{i=1}^n$ in Y, (the map $t \mapsto h(t, x')\, y$ will be denoted by $x' \otimes y$). We note the following rules governing the «product» $\otimes$: For all $x \in X$, $x', x_1', x_2' \in X'$, $y, y_1, y_2 \in Y$, $y' \in Y'$ and $\mu \in \Delta$,

(i) $(x_1' + x_2') \otimes y = x_1' \otimes y + x_2' \otimes y$

(ii) $x' \otimes (y_1 + y_2) = x' \otimes y_1 + x' \otimes y_2$

(iii) $x' \mu \otimes y = x' \otimes \mu y$

(iv) $(y' \otimes x)(x' \otimes y) = x' g(y, y') \otimes x$

(v) $(x' \otimes y)^\sharp = y \otimes x'$.

The basic reference for definitions and notations about dual pairs theory will be [16, Chapter IV].

A couple $e = (e^+, e^-)$ of a pair $A = (A^+, A^-)$ is called an *idempotent* if

$$\langle e^\sigma, e^{-\sigma}, e^\sigma\rangle = e^\sigma.$$

We recall that the (11)-*Peirce space* of A associated to e, denoted by

$$A_{11}(e) = (A_{11}^+(e), A_{11}^-(e)),$$

is defined as

$$A_{11}^\sigma(e) = \{x^\sigma \in A^\sigma : \langle x^\sigma, e^{-\sigma}, e^\sigma\rangle = \langle e^\sigma, e^{-\sigma}, x^\sigma\rangle = x^\sigma\}.$$

We shall need the following result that can be found in [5]:

LEMMA 1 ([5, Lemma 1]). – *Let* (X, X', h), (Y, Y', g) *be dual pairs and denote*

$$R := (F(X, Y), F(Y, X)).$$

1. *For any* $e^+ \in F(X, Y)$, $e^+ \neq 0$, *there exists* $e^- \in F(Y, X)$ *such that* (e^+, e^-) *is an idempotent. Moreover, there is a unique* $n \in \mathbb{N}$ *such that* $e^+ = \sum_{i=1}^{n} x_i' \otimes y_i$ *and* $e^- = \sum_{i=1}^{n} y_i' \otimes x_i$, *with* $\{x_i\}$, $\{x_i'\}$, $\{y_i\}$ *and* $\{y_i'\}$ *systems of linearly independent vectors in* X, X', Y *and* Y' *respectively, satisfying* $h(x_i, x_j') = g(y_i, y_j') = \delta_{ij}$ *(Kronecker delta).*

2. *If* $e = (e^+, e^-)$ *is an idempotent of* R, *being* $e^+ = \sum_{i=1}^{n} x_i' \otimes y_i$ *and*

$$e^- = \sum_{i=1}^{n} y_i' \otimes x_i$$

as in (1), *then* $R_{11}^+(e)$ *is linearly generated by* $\{e_{i,j}^+ := x_i' \otimes y_j : i, j = 1, \ldots, n\}$ *and* $R_{11}^-(e)$ *is linearly generated by*

$$\{e_{i,j}^- := y_i' \otimes x_j : i, j = 1, \ldots, n\}.$$

3. *The sets* $\{e_{i,j}^+\}$ *and* $\{e_{i,j}^-\}$ *described in* (2) *are systems of linearly independent vectors of* $R_{11}^+(e)$ *and* $R_{11}^-(e)$ *respectively.*

4. *If* X *and* Y *are infinite dimensional vector spaces, then for* e_1, e_2 *idempotents of* R, *there exists* e_3, *another idempotent of* R, *satisfying*

$$R_{11}^+(e_1) \cup R_{11}^+(e_2) \subset R_{11}^+(e_3).$$

2.2. *On Jordan pairs.*

The basic reference for definitions and notations about Jordan pairs theory will be [20]. Let us see some examples of Jordan pairs. The simple Jordan algebra $V = V(f)$ of a nondegenerate symmetric bilinear form gives rise to a Jordan pair $J = (V, V)$ by defining $Q^\sigma(x) = U(x)$, these Jordan pairs are called of *quadratic type*.

If A is an associative pair, then A^J will denote the *symmetrized* Jordan pair of A, that is, the Jordan pair whose underlying K-module agrees with that of A, and whose quadratic operators are given by $Q^\sigma(x)(y) = \langle x, y, x\rangle^\sigma$. Let (X, Y, h) be a dual pair over a K-division algebra with involution $(\Delta, -)$, we can define the opposite dual pair (Y, X, h^{op}), considering Y and X as left and right Δ-vector spaces respectively, for the actions $\lambda y := y\overline{\lambda}$, $x\lambda := \overline{\lambda}x$ for all $x \in X$, $y \in Y$ and $\lambda \in \Delta$, and defining $h^{op}: Y \times X \rightarrow \Delta$ as $h^{op}(y, x) := \overline{h(x, y)}$ for $(y, x) \in Y \times X$. Then, other examples of Jordan pairs are any subpair of

$$(Sym(L(X, Y), \sharp), Sym(L(Y, X), \sharp)),$$

(where $Sym(L(X, Y), \sharp) = \{f \in L(X, Y) : f^\sharp = f\}$ and $Sym(L(Y, X), \sharp)$ is defined similarly), containing

$$(Sym(F(X, Y), \sharp), Sym(F(Y, X), \sharp)),$$

where the quadratic operators are $Q^\sigma(x)(y) = xyx$ and $\sharp$ is the adjoint operator

$$L(X, Y) \to L(X, Y) \quad \text{or} \quad L(Y, X) \to L(Y, X);$$

and any subpair of

$$(Skw(L(X, Y), \sharp), Skw(L(Y, X), \sharp)),$$

(where $(Skw(L(X, Y), \sharp) = \{f \in L(X, Y) : f^\sharp = -f\}$ and $Skw(L(Y, X), \sharp)$ is defined similarly), containing

$$(Skw(F(X, Y), \sharp), Skw(F(Y, X), \sharp)),$$

with the quadratic operators and $\sharp$ as above.

2.3. *On H^*-pairs.*

Let $V = (V^+, V^-)$ be a, non necessarily associative, complex pair with triple products denoted by $\langle \cdot, \cdot, \cdot \rangle^\sigma$, and let $* = (*^+, *^-)$ be a couple of conjugate-linear mappings $*^\sigma : V^\sigma \to V^{-\sigma}$ for which $*^\sigma \circ *^{-\sigma} = Id$ and

$$\langle x^\sigma, y^{-\sigma}, z^\sigma \rangle^{*^\sigma} = \langle (z^\sigma)^{*^\sigma}, (y^{-\sigma})^{*^{-\sigma}}, (x^\sigma)^{*^\sigma} \rangle$$

for x^σ, $z^\sigma \in V^\sigma$ and $y^{-\sigma} \in V^{-\sigma}$. Then $* = (*^+, *^-)$ is called an *involution* of V. We say that V is an *H^*-pair* if V^+ and V^- are also Hilbert spaces over the complex numbers with inner products $(\cdot|\cdot)_\sigma : V^\sigma \times V^\sigma \to \mathbb{C}$, endowed with an involution $* = (*^+, *^-)$ such that

$$(2.1) \qquad (\langle x^\sigma, y^{-\sigma}, z^\sigma \rangle | t^\sigma)_\sigma = (x^\sigma | \langle t^\sigma, (z^\sigma)^{*^\sigma}, (y^{-\sigma})^{*^{-\sigma}} \rangle)_\sigma =$$

$$(y^{-\sigma} | \langle (x^\sigma)^{*^\sigma}, t^\sigma, (z^\sigma)^{*^\sigma} \rangle)_{-\sigma} = (z^\sigma | \langle (y^{-\sigma})^{*^{-\sigma}}, (x^\sigma)^{*^\sigma}, t^\sigma \rangle)_\sigma$$

for x^σ, z^σ, $t^\sigma \in V^\sigma$ and $y^{-\sigma} \in V^{-\sigma}$. The complete notation for an H^*-pair would be $(V, *, (\cdot|\cdot))$ but we will frequently speak of the H^*-pair V (omitting the involution and inner products).

We also recall that an H^*-pair V is said to be *topologically simple* when

$$\langle V^\sigma, V^{-\sigma}, V^\sigma \rangle \neq 0$$

and its only closed ideals, with respect to the norm topology, are $\{0\}$ and V.

A $*$-*homomorphism* $f\colon V \to W$ of H^*-pairs is a homomorphism such that $f^{-\sigma}((x^\sigma)^*) = (f^\sigma(x^\sigma))^*$ for any $x^\sigma \in V^\sigma$ and $\sigma \in \{+, -\}$. An $*$-isomorphism f is said to be *k-isogenic* whenever $(f^\sigma(x^\sigma) | f^\sigma(y^\sigma)) = k(x^\sigma | y^\sigma)$ for any $x^\sigma, y^\sigma \in V^\sigma$, and $\sigma \in \{+, -\}$, with k a positive real number. The pair

$$Ann(V) = (Ann(V^+), Ann(V^-))$$

is a self-adjoint closed ideal of V that we call the *Annihilator* of V. Following [7] it is easy to prove that any H^*-pair V with continuous involution splits into the orthogonal direct sum $V = Ann(V) \perp U$, where $U = (U^+, U^-)$ is an H^*-subpair of V with zero annihilator. Moreover, each H^*-pair V with zero annihilator satisfies $V = \overline{\perp I_\alpha}$ where $\{I_\alpha\}_\alpha$, $(I_\alpha = (I_\alpha^+, I_\alpha^-))$, denotes the family of minimal closed ideals of V, each of them being a topologically simple H^*-pair. This reduces the study of this pairs to the study of the topologically simple ones.

3. – Jordan H^*-pairs.

3.1. *Previous results.*

We are primarily interested in infinite-dimensional pairs since any finite-dimensional topologically simple H^*-pair is simple, and it can be proved that any simple finite-dimensional complex Jordan pair can be endowed with an (essentially unique) structure of H^*-pair. The existence of an H^*-structure can be seen as a consequence of [22, 3.3 Satz] and [23], while the essential uniqueness of this H^*-structure follows from [6] and [23]. As the classification of simple finite-dimensional Jordan pairs over $\mathbb{C}$ has been previously considered (see [20]) we shall confine ourselves to the infinite dimensional case.

If $J = (J^+, J^-)$ is a Jordan H^*-pair, we define the *polarized Jordan triple system of* J as the Jordan H^*-triple system T with Hilbert space $J^+ \perp J^-$ whose quadratic operator P and involution are given by

$$P(x)(y) = (Q^+(x^+)(y^-), Q^-(x^-)(y^+))$$

and $(x^+, x^-)^* := ((x^-)^*, (x^+)^*)$ for all $(x^+, x^-), (y^+, y^-) \in T$ (see [6] or [8] for definition of H^*-triple system). As we proved in [2, Proposition 1] that any topologically simple Jordan H^*-pair is prime, non-degenerate and with non-zero socle, and its polarized triple system inherits these «characteristics», we can derive from the classification of prime, non-degenerate Jordan triple systems with non-zero socle in [15, Theorem 7], that the underlying Jordan pair J of an infinite dimensional topologically simple non quadratic Jordan H^*-pair is one of the following:

Type (i): J is a subpair of $(L(X, Y), L(Y, X))^J$ containing $(F(X, Y),$

$F(Y, X))^J$ with the quadratic operators $Q^\sigma(x)(y) = xyx$, being (X, X') and (Y, Y') dual pairs over $\mathbb{C}$.

Type (ii): J is a subpair of

$$(Sym(L(X, Y), \sharp), Sym(L(Y, X), \sharp))$$

containing $(Sym(F(X, Y), \sharp), Sym(F(Y, X), \sharp))$, where (X, Y, h), (Y, X, h^{op}) are a dual pair and its opposite over $\mathbb{C}$, the quadratics operators are

$$Q^\sigma(x)(y) = xyx$$

and $\sharp$ is the adjoint operator.

Type (iii): J is a subpair of

$$(Skw(L(X, Y), \sharp), Skw(L(Y, X), \sharp))$$

containing $(Skw(F(X, Y), \sharp), Skw(F(Y, X), \sharp))$, with (X, Y, h), (Y, X, h^{op}), the quadratic operators and $\sharp$ as in the previous type.

Topologically simple Jordan H^*-pairs of types (i) and (ii) have been studied in [2] and [5] respectively. It is proved in these references that the ones of type (i) are k-isogenic, ($*$-isometrically isomorphic up to a positive factor of the inner product), to A^J with A a topologically simple associative H^*-pair, and that the ones of type (ii) are k-isogenic to

$$J = (Sym(A^+, \xi^+), Sym(A^-, \xi^-))$$

with $A = (A^+, A^-)$ as above and $\xi = (\xi^+, \xi^-)$ an involutive $*$-anti-isomorphism from A to A^{op}.

We proceed to study the remaining type.

3.2. *Study of Jordan H^*-pairs such that their underlying Jordan pairs are of Type* (iii).

We are going to prove in this section that if J is a subpair of

$$(Skw(L(X, Y), \sharp), Skw(L(Y, X), \sharp))$$

containing $(Skw(F(X, Y), \sharp), Skw(F(Y, X), \sharp))$, where (X, Y, h), (Y, X, h^{op}) are a dual pair and its opposite over $\mathbb{C}$, the quadratic operators are $Q^\sigma(x)(y) = xyx$ and $\sharp$ is the adjoint operator, then J is k-isogenic to

$$(Skw(A^+, \xi^+), Skw(A^-, \xi^-))$$

with $A=(A^+, A^-)$ a topologically simple associative H^*-pair and $\xi=(\xi^+, \xi^-)$ an involutive $*$-anti-isomorphism from A to A^{op}. The techniques we shall use are the dual vector spaces methods in Jordan pairs theory introduced in [2] and [5]. Many technical differences with the situations that we find in these references lead us to develop § 3.2 in detail.

We recall that any complex pair $V=(V^+, V^-)$ is a real pair restricting the field of scalars to $\mathbb{R}$. This real pair is denoted by $V^{\mathbb{R}}=((V^+)^{\mathbb{R}}, (V^-)^{\mathbb{R}})$.

We shall also need the following result due to A. D'Amour:

We say that a $\mathbb{Z}_2$-graded associative algebra $A=A_0\oplus A_1$ with graded involution δ is a *$\mathbb{Z}_2$-graded δ-envelope* for a Jordan triple system T if $T\subset Sym(A_1, \delta)$ and T generates A.

A $\mathbb{Z}_2$-graded δ-envelope A is *δ-tight* if every nonzero graded δ-ideal $I=I^\delta$ of A satisfies $I\cap T\neq 0$. The hypothesis $Z(T_i)\neq 0$ imposed in the next theorem means that the Zel'manov polynomials, (defined in [14]), do not vanish on T_i. As a consequence T_i is of hermitian type (following McCrimmons's terminology). The Zelmanov polynomial were applied in [21] to the study of strongly prime quadratic Jordan algebras, and then in [1] to Jordan triple systems and pairs. The nature of such polynomials permits to discern whether a given Jordan system has a hermitian part or not [21, p. 143].

THEOREM 2 ([13, Theorem B]). – *For $i=1, 2$ let T_i be a prime Jordan triple system with $Z(T_i)\neq 0$ and $\mathbb{Z}_2$-graded δ-tight algebra envelope A_i. Then any isomorphism $f: T_1\to T_2$ extends uniquely to a graded δ-isomorphism*

$$F: A_1\to A_2.$$

LEMMA 3. – *Let (M_i, M_i, h_i), $i=1, 2$, be complex selfdual pairs relative to hermitian forms h_i. Assume that for any $0\neq m_i\in M_i$ $(i=1, 2)$ such that $h_i(m_i, m_i)=0$, we have that $h_i(m_i, m_i')\neq 0$ for all m_i' in M_i such that $m_i'\notin \mathbb{C}m_i$.*

1. *If e is an idempotent of $R:=(F(M_1, M_2), F(M_2, M_1))$ with $rank(e)\geqslant 3$ (as linear map), then there exist two idempotents of R, f_1 and f_2, with the property of being selfadjoint, $((f_i^\sigma)^\sharp=f_i^{-\sigma})$, and such that $R_{11}^+(e)=R_{11}^+(f_1)$ and $R_{11}^-(e)=R_{11}^-(f_2)$.*

2. *If M_i, $i=1, 2$, are infinite dimensional vector spaces, then for f_1, f_2 selfadjoint idempotents of R, there exists another selfadjoint idempotent of R, f_3, satisfying $R_{11}(f_1)\cup R_{11}(f_2)\subset R_{11}(f_3)$.*

PROOF. – (1) For simplicity of notation we write $(\cdot, \cdot)$ instead of h_1 throughout the Proof. Fix $e=(e^+, e^-)$ an idempotent of $(F(M_1, M_2), F(M_2, M_1))$, such that $e^+=\sum_{i=1}^n x_i\otimes y_i$, $n\geqslant 3$, with $\{x_1, \ldots, x_n\}$ and $\{y_1, \ldots, y_n\}$ free sets of vectors of M_1 and M_2 respectively (see Lemma 1). Since $(\cdot, \cdot)$ is a hermitian form, we have linearly independent vectors $\{\tilde{x}_1, \dots, \tilde{x}_n\}$ in M_1 such that $\mathcal{L}(\{x_1, \dots, x_n\}) = \mathcal{L}(\{\tilde{x}_1, \dots, \tilde{x}_n\})$, $(\tilde{x}_i, \tilde{x}_j) = 0$ if $i \neq j$ and $(\tilde{x}_i, \tilde{x}_i) \in \{1, -1\}$. We claim that there are no i, k such that $(\tilde{x}_i, \tilde{x}_i) = 1$ and $(\tilde{x}_k, \tilde{x}_k) = -1$. Indeed, in the opposite case, the elements $\tilde{x}_i + \tilde{x}_k$ and $\tilde{x}_j, j \notin \{i, k\}$, satisfy $(\tilde{x}_i + \tilde{x}_k, \tilde{x}_i + \tilde{x}_k) = 0$ and $(\tilde{x}_i + \tilde{x}_k, \tilde{x}_j) = 0$, being

$$\{\tilde{x}_i + \tilde{x}_k, \tilde{x}_j\}$$

linearly independent vectors, a contradiction with the hypothesis. Therefore,

$$(\tilde{x}_i, \tilde{x}_j) = k\delta ij$$

with k a fixed element of $\{1, -1\}$. In a similar way, we can take $\{\tilde{y}_1, \dots, \tilde{y}_n\}$ linearly independent vectors in M_2 such that $\mathcal{L}(\{y_1, \dots, y_n\}) = \mathcal{L}(\{\tilde{y}_1, \dots, \tilde{y}_n\})$ and $(\tilde{y}_i, \tilde{y}_j) = p\delta ij$ with p a fixed element of $\{1, -1\}$. Taking into account Lemma 1-(2), it is easy to prove that $f_1 = (f_1^+, f_1^-)$ with $f_1^+ = \sum_{i=1}^{n} k\tilde{x}_i \otimes p\tilde{y}_i$ and $f_1^- = \sum_{i=1}^{n} p\tilde{y}_i \otimes k\tilde{x}_i$ is the first selfadjoint idempotent which we are looking for. In the same manner, we can find another selfadjoint idempotent f_2 such that $R_{11}^-(e) = R_{11}^-(f_2)$.

(2) According to Lemma 1-(4), we have e an idempotent of R such that $R_{11}^+(f_1) \cup R_{11}^+(f_2) \subset R_{11}^+(e)$, moreover, by Lemma 1-(2) we always can take e such that $rank(e) \geqslant 3$. We conclude from applying (1) to e that there exists a selfadjoint idempotent f_3 of R satisfying $R_{11}^+(f_1) \cup R_{11}^+(f_2) \subset R_{11}^+(f_3)$, hence that

$$(R_{11}^+(f_1))^\sharp \cup (R_{11}^+(f_2))^\sharp \subset (R_{11}^+(f_3))^\sharp,$$

and finally that $R_{11}^-(f_1) \cup R_{11}^-(f_2) \subset R_{11}^-(f_3)$ which is our claim.

LEMMA 4. – *Let J be a complex topologically simple Jordan H^*-pair such that J is a subpair of $(Skw(L(X, Y), \sharp), Skw(L(Y, X), \sharp))$ containing*

$$(Skw(F(X, Y), \sharp_1), Skw(F(Y, X), \sharp_1)),$$

where (X, Y, h), (Y, X, h^{op}) are an infinite-dimensional dual pair and its opposite, and $\sharp_1$ is the adjoint operator, $(\sharp_1 : L(X, Y) \to L(X, Y)$, $\sharp_1 : L(Y, X) \to L(Y, X))$, then:

1. *There exist two selfdual pairs (M_i, M_i, h_i) $i = 1, 2$, an isomorphism Φ of associative pairs and a conjugate-linear involutive anti-automorphism $*'$ onto the associative pair $(F(X, Y), F(Y, X))$ extending $*$, such that the*

following diagram commutes:

$$
\begin{array}{ccccc}
 & (F(X, Y), F(Y, X)) & \overset{*'}{\longrightarrow} & (F(X, Y), F(Y, X)) & \\
\Phi & \downarrow & & \downarrow & \Phi \\
 & (F(M_1, M_2), F(M_2, M_1)) & \underset{\natural_2}{\longrightarrow} & (F(M_1, M_2), F(M_2, M_1)) &
\end{array}
$$

$\natural_2$ *being the adjoint operator*

$(\natural_2 \colon F(M_1, M_2) \to F(M_2, M_1),\ \natural_2 \colon F(M_2, M_1) \to F(M_1, M_2))$.

2. *If* $0 \neq m_i \in M_i$ $(i = 1, 2)$ *is such that* $h_i(m_i, m_i) = 0$, *then* $h_i(m_i, m_i') \neq 0$ *for all* m_i' *in* M_i *such that* $m_i' \notin \mathbb{C} m_i$.

3. h_i $(i = 1, 2)$ *is hermitian.*

PROOF. – (1) Consider the associative algebra

$$A = \begin{pmatrix} F(X)^{\mathbb{R}} & F(X, Y)^{\mathbb{R}} \\ F(Y, X)^{\mathbb{R}} & F(Y)^{\mathbb{R}} \end{pmatrix}$$

with the grading $A_0 = \begin{pmatrix} F(X)^{\mathbb{R}} & 0 \\ 0 & F(Y)^{\mathbb{R}} \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & F(X, Y)^{\mathbb{R}} \\ F(Y, X)^{\mathbb{R}} & 0 \end{pmatrix}$ and with the product

$$\begin{pmatrix} \alpha_1 & f_1 \\ g_1 & \beta_1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_2 & f_2 \\ g_2 & \beta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \cdot \alpha_2 + f_1 \cdot g_2 & \alpha_1 \cdot f_2 + f_1 \cdot \beta_2 \\ g_1 \cdot \alpha_2 + \beta_1 \cdot g_2 & g_1 \cdot f_2 + \beta_1 \cdot \beta_2 \end{pmatrix}$$

where $x \cdot y := y \circ x$.

We claim that (A, δ) and (A^{op}, δ) are $\mathbb{Z}_2$-graded δ-tight algebra envelopes of the real polarized Jordan triple system $T = Skw(F(X, Y), \natural)^{\mathbb{R}} \oplus Skw(F(Y, X), \natural)^{\mathbb{R}}$, with

$$\begin{pmatrix} \alpha & f \\ g & \beta \end{pmatrix}^{\delta} := \begin{pmatrix} \beta^{\natural} & -f^{\natural} \\ -g^{\natural} & \alpha^{\natural} \end{pmatrix}.$$

Indeed, it is clear that $T \subset Sym(A_1, \delta)$. In order to prove that T generates A (and A^{op}), let us consider $x_1 \otimes x_2 \in F(Y, X)^{\mathbb{R}}$, $x_1, x_2 \neq 0$. If $x_2 \neq \lambda x_1$ with $\lambda \in \mathbb{C}$, we can always find $x_1', x_2' \in X$ and $y_1', y_2' \in Y$, such that $\{x_1, x_2, x_1', x_2'\}$ is a linearly independent set of vectors in X and $h(x_i, y_j') = 0$, $h(x_i', y_j') = \delta_{ij}$ for $i, j \in \{1, 2\}$, (by the density of Y in the conjugate space X^* of X). Then,

$$x_1 \otimes x_2 = (x_1' \otimes x_2 - x_2 \otimes x_1')(y_2' \otimes y_1' - y_1' \otimes y_2')(x_1 \otimes x_2' - x_2' \otimes x_1), \tag{3.1}$$

with

$$y_2' \otimes y_1' - y_1' \otimes y_2' \in Skw(F(X, Y), \sharp)^{\mathbb{R}}$$

and

$$x_1' \otimes x_2 - x_2 \otimes x_1', \; x_1 \otimes x_2' - x_2' \otimes x_1 \in Skw(F(Y, X), \sharp)^{\mathbb{R}}.$$

If $x_2 = \lambda x_1$ with $\lambda \in \mathbb{C}$, (3.1) can be followed from the previous case, taking into account that we can find, as before, $x_2, x_3 \in X$ and $y_2, y_3 \in Y$ such that $\{x_2, x_3\}$ and $\{y_2, y_3\}$ are linearly independent set of vectors satisfying $h(x_i, y_i) = 1$, $i \in \{2, 3\}$, and then $x_1 \otimes \lambda x_1 = (x_2 \otimes \lambda x_1)(y_3 \otimes y_2)(x_1 \otimes x_3)$. Therefore T generates $F(Y, X)^{\mathbb{R}}$. In a similar way we prove that T generates $F(X, Y)^{\mathbb{R}}$ and so T generates A_1. As $A \subset A_1^2 \oplus A_1$, then T generates A. Finally, as A is a simple algebra because A is isomorphic to the simple algebra $F(X \oplus Y, X \oplus Y)$ being this one simple by [16, p. 75], the δ-tight condition holds clearly.

The infinite dimensional nature of T forces its hermitian character, and this implies that the Zel'manov polynomials do not vanish on T. Thus $Z(T) \neq 0$ and by D'Amour's theorem (Theorem 2), the involution $*$ of J extends to a δ-isomorphism of two-graded algebras $\nu : A \to A^{op}$, hence by an easy argument $*' = ((*')^+, (*')^-)$ with

$$(*')^+ := \nu|_{F(X, Y)^{\mathbb{R}}} : F(X, Y)^{\mathbb{R}} \to F(Y, X)^{\mathbb{R}}$$

and

$$(*')^- := \nu|_{F(Y, X)^{\mathbb{R}}} : F(Y, X)^{\mathbb{R}} \to F(X, Y)^{\mathbb{R}}$$

is an involutive anti-automorphism of the associative pair $(F(X, Y)^{\mathbb{R}}, F(Y, X)^{\mathbb{R}})$. With [Theorem 4] we complete the proof of (1).

(2) Our proof starts with the observation that the isomorphism of associative pairs Φ given by (1), implies the existence of a conjugate-linear involutive anti-isomorphism of associative pairs $\varphi = (\varphi^+, \varphi^-)$ such that the following diagram commutes:

$$\begin{array}{ccccc} & (F(X, Y), F(Y, X)) & \xrightarrow{\sharp_1} & (F(Y, X), F(X, Y)) & \\ \Phi & \downarrow & & \downarrow & \Phi \\ & (F(M_1, M_2), F(M_2, M_1)) & \xrightarrow[\varphi]{} & (F(M_2, M_1), F(M_1, M_2)) & \end{array}$$

The identities $\sharp_1 \circ *' = *' \circ \sharp_1$ (consequence of the fact that ν is a δ-isomorphism of two-graded algebras), $\Phi \circ \nu = \natural_2 \circ \Phi$ and $\Phi \circ \natural_1 = \varphi \circ \Phi$ imply $\varphi \circ \natural_2 = \natural_2 \circ \varphi$.

Applying [12, Theorem 3] to Φ, we obtain linear or conjugate-linear homeomorphisms $\alpha : X \to M_1$, and $\beta : Y \to M_2$ such that $\Phi^-(f) := \alpha f \beta^{-1}$.

For simplicity of notation we write $(\cdot, \cdot)$ instead of h_1. If $(m_1, m_1) = 0$ and $(m_1, m_1') = 0$ with $m_1' \notin \mathbb{C} m_i$, there exist $0 \neq x_1$, $0 \neq x_2 \in X$ such that $\alpha(x_1) = m_1'$ and $\alpha(x_2) = m_1$, the density of the dual pairs, (see [16, Chapter IV, Section 6]), gives us the existence of $0 \neq y_1$, $0 \neq y_2 \in Y$ such that $(x_i, y_j) = \delta_{ij}$. As

$$x_1 \otimes x_2 - x_2 \otimes x_1 \in Skw(F(Y, X), \natural_1)$$

and

$$\langle x_1 \otimes x_2 - x_2 \otimes x_1,\ y_2 \otimes y_1 - y_1 \otimes y_2,\ x_1 \otimes x_2 - x_2 \otimes x_1 \rangle = x_1 \otimes x_2 - x_2 \otimes x_1$$

then

$$\Phi(x_1 \otimes x_2 - x_2 \otimes x_1) \in Skw(F(M_2, M_1), \varphi^-),$$

$$\Phi(y_2 \otimes y_1 - y_1 \otimes y_2) \in Skw(F(M_1, M_2), \varphi^+)$$

and

$$\langle \Phi(x_1 \otimes x_2 - x_2 \otimes x_1),\ \Phi(y_2 \otimes y_1 - y_1 \otimes y_2),\ \Phi(x_1 \otimes x_2 - x_2 \otimes x_1) \rangle = \Phi(x_1 \otimes x_2 - x_2 \otimes x_1).$$

Since $\Phi^-(x_1 \otimes x_2)(z) = (\beta^{-1}(z), x_1)\, \alpha(x_2)$ and

$$\Phi^-(x_2 \otimes x_1)(z) = (\beta^{-1}(z), x_2)\, \alpha(x_1)$$

for all $z \in M_2$, then [16, Lemma on page 72] implies the existence of $0 \neq m_2''$, $0 \neq m_2''' \in M_2$ such that $\Phi^-(x_1 \otimes x_2) = m_2'' \otimes m_1$ and $\Phi^-(x_2 \otimes x_1) = m_2''' \otimes m_1'$. The commutativity $\Phi \circ \natural_1 = \varphi \circ \Phi$ and the identity $\varphi \circ \natural_2 = \natural_2 \circ \varphi$ yield

$$\varphi(m_2'' \otimes m_1) = \Phi^-(x_2 \otimes x_1) = m_2''' \otimes m_1'$$

and $\varphi(m_1 \otimes m_2'') = m_1' \otimes m_2'''$.

We can consider the completion

$$(\overline{Skw(F(M_1, M_2), \varphi^+)},\ \overline{Skw(F(M_2, M_1), \varphi^-)})$$

of the pre-Hilbert space structure induced on

$$(Skw(F(M_1, M_2), \varphi^+),\ Skw(F(M_2, M_1), \varphi^-))$$

by the isomorphism Φ. This completion has an H^*-structure induced by the unique extension

$$\widetilde{\Phi}: J \to (\overline{Skw(F(M_1, M_2), \varphi^+)}, \overline{Skw(F(M_2, M_1), \varphi^-)}),$$

by continuity of Φ. The identities (2.1) of the Jordan H^*-pair

$$((\overline{Skw(F(M_1, M_2), \varphi^+)}, \overline{Skw(F(M_2, M_1), \varphi^-)}); \natural_2; (\cdot \,|\, \cdot))$$

give

$$\|m_2'' \otimes m_1 - \varphi(m_2'' \otimes m_1)\|^2 = (b - \varphi(b) \,|\, b - \varphi(b)) =$$

$$(\langle b - \varphi(b), \Phi(y_2 \otimes y_1 - y_1 \otimes y_2), b - \varphi(b)\rangle \,|\, b - \varphi(b)) =$$

$$(\Phi(y_2 \otimes y_1 - y_1 \otimes y_2) \,|\, \langle a - \varphi(a), b - \varphi(b), a - \varphi(a)\rangle)$$

being $a = m_1 \otimes m_2''$ and $b = m_2'' \otimes m_1$. The facts that

$$\langle a, b, a\rangle = \langle a, b, \varphi(a)\rangle = \langle a, \varphi(b), a\rangle = \langle \varphi(a), b, a\rangle = 0$$

and the involutive anti-isomorphism character of φ, enable us to write

$$\langle a - \varphi(a), b - \varphi(b), a - \varphi(a)\rangle = 0$$

and then $\|m_2'' \otimes m_1 - \varphi(m_2'' \otimes m_1)\| = 0$, hence $m_2'' = 0$ and $m_2''' = 0$, a contradiction.

(3) The bilinear forms h_1 and h_2 are alternate or hermitian, (see [12, Theorem 4]). Suppose h_1 is alternate then $\lambda(m_1 \otimes m_2) = \lambda m_1 \otimes m_2 = m_1 \otimes \lambda m_2$, $\lambda \in \mathbb{C}$, $m_1 \in M_1$ and $m_2 \in M_2$. By (1), $\natural$ is the involution of a complex H^*-pair then $(\lambda(m_1 \otimes m_2 - m_1' \otimes m_2'))^\natural = \bar{\lambda}(m_1 \otimes m_2 - m_1' \otimes m_2')^\natural$ for

$$m_1 \otimes m_2 - m_1' \otimes m_2' \in Skw(F(M_1, M_2), \varphi^+),$$

now taking $m_1 \otimes m_2 - m_1' \otimes m_2' \neq 0$ and $\lambda = i$ we obtain a contradiction, and consequently h_i is hermitian.

THEOREM 5. – *Let J be a topologically simple Jordan H^*-pair such that J is a subpair of $(Skw(L(X, Y), \natural), Skw(L(Y, X), \natural))$ containing*

$$(Skw(F(X, Y), \natural), Skw(F(Y, X), \natural))$$

where (X, Y, h), (Y, X, h^{op}) are an infinite dimensional dual pair and its opposite pair, and $\natural$ is the adjoint operator. Then J is k-isogenic to

$$(Skw(A^+, \xi^+), Skw(A^-, \xi^-))$$

with $A = (A^+, A^-)$ *a topologically simple associative* H^**-pair and* $\xi = (\xi^+, \xi^-)$ *an involutive* $*$*-anti-isomorphism from* A *to* A^{op}.

PROOF. – As $(Skw(F(X, Y), \natural), Skw(F(Y, X), \natural))$ is a non-zero ideal of J, the topological simplicity of J and Lemma 4 allow us to suppose that $(J, ^*, (\cdot|\cdot))$ is a subpair of

$$(Skw(L(M_1, M_2), \varphi), Skw(L(M_2, M_1), \varphi))$$

containing $(Skw(F(M_1, M_2), \varphi), Skw(F(M_2, M_1), \varphi))$, where (M_i, M_i, h_i), $i = 1, 2$, are under the hypothesis of Lemma 3 and φ is a conjugate-linear involutive anti-isomorphism of associative pairs from

$$(L(M_1, M_2), L(M_2, M_1))$$

to $(L(M_2, M_1), L(M_1, M_2))$, and that $(a^\sigma)^* = (a^\sigma)^\natural$, $\natural$ being the adjoint of a^σ with respect to h_i for

$$a^+ \in Skw(F(M_1, M_2), \varphi) \quad \text{and} \quad a^- \in Skw(F(M_2, M_1), \varphi).$$

For simplicity of notation we write R instead of

$$(F(M_1, M_2), F(M_2, M_1)).$$

Taking into account Lemma 3-(2), we can refine Loos' result in [19, Theorem 3] as in [2, Theorem 2] or [5, Theorem 2], so as to prove that the families $\{Skw(R_{11}(f), \varphi)\}$ and $\{R_{11}(f)\}$ are direct systems of finite dimensional Jordan and associative H^*-pairs (relative to inclusion), respectively, when f ranges over the directed set of the selfadjoint idempotents of R. Finally, by applying direct limits arguments as in [2, Theorem 2] or [5, Theorem 2] we complete the proof.

It follows easily as in [5, Corollary 1] the following

COROLLARY 1. – *Let* $A = (A^+, A^-)$ *be a prime associative pair and* $\xi = (\xi^+, \xi^-)$ *an involutive anti-isomorphism from* A *to* A^{op}. *Suppose that*

$$J = (Skw(A^+, \xi^+), Skw(A^-, \xi^-)),$$

(*where* $Skw(A^\sigma, \xi^\sigma)$ *denotes* $\{a \in A^\sigma : \xi^\sigma(a) = -a\}$), *is an infinite dimensional and topologically simple Jordan* H^**-pair. Then* A *is a topologically simple associative* H^**-pair.*

Summarizing we can claim:

THEOREM 6 (Main Theorem). – *Let* $J = (J^+, J^-)$ *be a topologically simple infinite-dimensional non-quadratic Jordan* H^**-pair. Then,* J *is k-isogenic* *(isometrically $*$-isomorphic up to a positive factor of the inner products) to one of the followings:*

1. *A^J, where A is a topologically simple associative H^*-pair.*

2. *$Sym(A, \xi)$, where A is a topologically simple associative H^*-pair and ξ is an involutive $*$-anti-isomorphism from A to A^{op}.*

3. *$Skw(A, \xi)$, with A and ξ as in the previous case.*

We finally note that the classification of topologically simple associative H^*-pairs can be obtained easily from ([10, Main theorem]).

REMARK 1. – It is proved in [2] that if J is a topologically simple Jordan H^*-pair such that J is the symmetrized Jordan pair of A, for an associative pair A, then A is necessarily an associative topologically simple H^*-pair and the inner products and involution of J agree with the ones in A. This result supposes a version for Jordan pairs of a previous result for Jordan algebras given in [24]. Corollary 1 extends the above results.

REMARK 2. – In [3, Theorem 3.1] we describe the equivalence between the categories of topologically simple polarized L^*-triples, (see [3] for definitions), and topologically simple Jordan H^*-pairs. From here, Theorem 6 allows us to obtain easily a complete classification of topologically simple polarized L^*-triples.

REMARK 3. – Theorem 6 gives us a new approach to the structure theory of infinite dimensional hermitian Hilbert triples given by W. Kaup in [17, 3.9] and [18] (see the same references for definitions and details), taking into account that

(i) Any hermitian Hilbert triple V gives us a Jordan H^*-pair $J(V) = (V^+, V^-)$ defining $V^+ := V$, V^- as V up to the scalar and inner products defined by $\lambda v := \bar{\lambda} \cdot v$ and $(u|v)' := \overline{(u|v)}$, where $\cdot$ and $(\cdot|\cdot)$ are the scalar and inner products of V. The triple products as an V and the involutions are the identity.

(ii) If (V^+, V^-) is a Jordan H^*-pair, then $T((V^+, V^-)) := V^+$ with the triple product defined by $\{x, y, z\} := \{x, y^*, z\}^+$ is a hermitian Hilbert triple, and

(iii) It is easy to prove that if V is a hermitian Hilbert triple then

$$TJ(V) = V,$$

and that if (V^+, V^-) is a Jordan H^*-pair with zero annihilator then

$$JT((V^+, V^-))$$

is a Jordan H^*-pair $*$-isometrically isomorphic to (V^+, V^-).

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