Bollettino
Unione Matematica Italiana

Filippo Cammaroto, Giovanni Lo Faro, Jack R. Porter

N-sets and near compact spaces

Bollettino dell'Unione Matematica Italiana, Serie 8, Vol. **2-B** (1999), n.2, p. 291–298.

Unione Matematica Italiana

<http://www.bdim.eu/item?id=BUMI_1999_8_2B_2_291_0>

Bollettino dell'Unione Matematica Italiana, Unione Matematica Italiana, 1999.

Bollettino U. M. I.
(8) 2-B (1999), 291-298

N-Sets and Near Compact Spaces.

FILIPPO CAMMAROTO (*) - GIOVANNI LO FARO - JACK R. PORTER

Sunto. – *Si provano nuovi risultati riguardanti gli «*N*-sets» e gli spazi «Near-compact». Si completano alcune ricerche pubblicate dai primi due autori nel 1978 e si risolvono due problemi recentemente posti da Cammaroto, Gutierrez, Nordo e Prada.*

1. – Introduction and preliminaries.

In this paper, some new results about N-sets and near compact spaces are presented. First, N-sets in Hausdorff spaces are characterized in terms of absolutes, thus, extending the work by Vermeer [V] in 1985. Near compact spaces are shown to be δ-closed (this solves Problem 2 in [CGNP]) and if A is a noncompact N-set in a Hausdorff space X, then A is θ-closed in X but there is a Hausdorff space Y in which X is embedded such that A is not θ-closed in Y. Finally, an example of a near compact space is developed which contains a nonconvergent, particularly closed ultrafilter; this result completes some research started in [CF] and solves Problem 1 in [CGNP].

All spaces under consideration in the first three sections of this paper are assumed to be Hausdorff. Let X be a space. The *semiregularization* of X, denoted as $X(s)$, is the underlying set of X with the topology generated by $\{\mathrm{int}_X(\mathrm{cl}_X U) : U \in \tau(X)\}$. It follows (see [PW]) that $\tau(X(s)) \subseteq \tau(X)$ and $\tau(X(s)(s)) = \tau(X(s))$. The space X is *semiregular* if $\tau(X) = \tau(X(s))$; in particular, $X(s)$ is semiregular. A subset A of X is *regular open* if $\mathrm{int}_X(\mathrm{cl}_X A) = A$; so, the topology $\tau(X(s))$ is generated by the regular open subsets of X. A set A of X is *regular closed* if $\mathrm{cl}_X(\mathrm{int}_X A) = A$, i.e., $X \setminus A$ is regular open.

Recall that a subset A of X is an N-*set* (resp. H-*set*) if for each cover $\mathcal{C}$ of A by sets open in X, there is a finite subfamily $\mathcal{F} \subseteq \mathcal{C}$ such that $A \subseteq \cup\{\mathrm{int}_X(\mathrm{cl}_X U) : U \in \mathcal{F}\}$ (resp. $A \subseteq \cup\{\mathrm{cl}_X U : U \in \mathcal{F}\}$). A space Y is *near compact* (resp. H-*closed*) if Y is an N-set (resp. H-set) of Y. An equivalent characterization of A being an N-set of X is that A with the topology inherited from $X(s)$ is compact. In particular, X is near compact if and only if $X(s)$ is compact;

(*) This research was supported by a grant from the C.N.R. (G.N.S.A.G.A.) and M.U.R.S.T. through «Fondi 40%» Italy.

now, using a result in [K] , X is near compact if and only if X is H-closed and Urysohn. The reader is referred to [PW] for properties of H-sets and H-closed spaces and for definitions and notations not specifically defined in this paper. A description of the well-known noncompact, minimal Hausdorff space (see 9.8(d) in [PW]) is included as it is frequently referenced in this paper.

EXAMPLE 1.1. – Let $Z = \{(1/n, 1/m): n, |m| \in \omega \setminus \{0\}\} \cup \{(1/n, 0): n \in \omega \setminus \{0\}\}$ with the topology inherited from the usual topology of the plane. Let $Y = Z \cup \{a, b\}$ and define a set $U \subseteq Y$ to be open if $U \cap Z$ is open in Z and $a \in U$ (resp. $b \in U$) implies there is some $k \in \omega \setminus \{0\}$ such that $\{(1/n, 1/m): n \geq k, m \in \omega \setminus \{0\}\} \subseteq U$ (resp. $\{(1/n, -1/m): n \geq k, m \in \omega \setminus \{0\}\} \subseteq U$). The space Y is minimal Hausdorff, i.e., H-closed and semiregular, but not compact.

There are some differences between the theory of H-sets and H-closed spaces and the theory of N-sets and near compact spaces. For example, if $A \subseteq X \subseteq Y$ where Y is a space and A is an H-set of X, it is easy to show that A is an H-set of Y. However, if A is an N-set of X, it is not necessarily true that A is an N-set of Y. In 1.1, let $A = \{a\} \cup \{(1/n, 0): n \in \omega \setminus \{0\}\} \cup \{(1/n, 1/m): n, m \in \omega \setminus \{0\}\}$. Then A is an N-set of A (in particular, A is near-compact) but A is not an N-set of Y. This also shows that a near compact subspace of a space is not necessarily an N-set. On the other hand, if A is an H-closed subspace of X, then A is an H-set.

For a space X, let (EX, k_X) denoted the Iliadis absolute of X and (PX, π_X) the Banaschewski absolute of X. The space EX is extremally disconnected and Tychonoff whereas PX is only extremally disconnected; however $\pi_X: PX \to X$ is continuous and a perfect irreducible surjection whereas $k_X: EX \to X$ is only θ-continuous and a perfect irreducible surjection. The underlying set of EX and PX is the set $\{\mathcal{U}: \mathcal{U}$ is a fixed open ultrafilter on $X\}$; the topology on EX is generated by $\{OU: U \in \tau(X)\}$ where $OU = \{\mathcal{U} \in EX: U \in \mathcal{U}\}$ and the topology on PX is the finer topology generated by $\tau(EX) \cup \{k_X^{\leftarrow}[U]: U \in \tau(X)\}$. Some of the properties of the Iliadis and Banaschewski absolutes which are needed in this paper are listed below.

PROPOSITION 1.2 [PW]. – *Let X be a space and $B \subseteq PX$.*

(a) *For $U \in \tau(X)$, $OU = O(\mathrm{int}_X(\mathrm{cl}_X U))$ and $EX \setminus OU = O(X \setminus \mathrm{cl}_X U)$.*

(b) *For $U \in \tau(X)$, $k_X[OU] = \mathrm{cl}_X U$ and for $x \in X$, $k_X^{\leftarrow}(x) \subseteq OU$ if and only if $x \in \mathrm{int}_X(\mathrm{cl}_X U)$.*

(c) $(PX)(s) = EX$.

(d) *A subspace B of EX is compact if and only if B is* N-*set of PX if and only if B is* H-*set of PX.*

When X is H-closed and Urysohn, we have this characterization of N-sets.

PROPOSITION 1.3. – *Let X be* H-*closed and Urysohn, i.e., X is near compact, and $A \subseteq X$. The following are equivalent*:

(a) *A is an* N-*set of X.*

(b) *A is an* H-*set of X.*

(c) *$k_X^{\leftarrow}[A]$ is a compact subspace of EX.*

(d) *$\pi_X^{\leftarrow}[A]$ is an* H-*set of PX.*

(e) *A is a compact subspace of $X(s)$.*

2. – Absolutes and N-sets.

Vermeer [V] characterized N-sets of H-closed, Urysohn spaces in terms of the absolute (see 1.3 (c, d)). In this section, we extend his characterization to N-sets of Hausdorff spaces. A useful lemma is presented first.

LEMMA 2.1. – *If X is a space, $U \in \tau(X)$, and $A \subseteq X$ such that $k_X^{\leftarrow}[A]$ is compact, then $A \cap \mathrm{cl}_X U$ is an* H-*set of X.*

PROOF. – Since OU is clopen by 1.2 (a), $OU \cap k_X^{\leftarrow}[A]$ is compact. So, $k_X[OU \cap k_X^{\leftarrow}[A]] = k_X[OU] \cap A = \mathrm{cl}_X U \cap A$ is an H-set of X. ■

THEOREM 2.2. – *Let X be a space and $A \subseteq X$. The following are equivalent*:

(a) *A is an* N-*set of X.*

(b) *A is an* H-*set of X and for each H-set B of X with $B \subseteq A$, $k_X^{\leftarrow}[B]$ is compact.*

(c) *For each $U \in \tau(X)$, $k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is compact.*

PROOF. – Suppose (a) is true and A is an N-set of X. Clearly, A is also an H-set of X. Let B be an H-set of X with $B \subseteq A$. Since B is an H-set of X, B is also an H-set of $X(s)$; so, B is closed in $X(s)$. As A is a compact subspace of $X(s)$, B is a compact subspace of $X(s)$. But $k_X\colon EX \to X(s)$ is also perfect, so, $k_X^{\leftarrow}[B]$ is compact. Thus, (a) implies (b). By 2.1, (b) implies (c). To show (c) implies (a), suppose for each $U \in \tau(X)$, $k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is compact. Using $U = X$, we have that $k_X^{\leftarrow}[A]$ is compact. There is a compact subset $C \subseteq k_X^{\leftarrow}[A]$ such that $k_X | C\colon C \to A$ is irreducible (and onto). The function $f = k_X | C$ is compact. If $D \subseteq C$ is closed in C, then D is compact and $f[D] = k_X[D]$ is an H-set of X. In particular, $f[D]$ is closed in X. So, $f[D]$ is closed in A. Let τ be the topology on A generated by the base $\{A \setminus f[D]: D$ is closed in $C\}$. By 2.3 in [V], $f: C \to (A, \tau)$ is θ-continuous and (A, τ) is minimal Hausdorff, i.e., H-closed and semiregular.

Now, let ϱ be the topology on A induced by $\tau(X(s))$. Next, we show that $\varrho \subseteq \tau$. A closed base for $\tau(X(s))$ are the sets of the form $\mathrm{cl}_X U$ where $U \in \tau(X)$. By (c), $k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is compact. Now, $C \cap k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is closed in C. Thus, $f[C \cap k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]] = k_X[C] \cap A \cap \mathrm{cl}_X U = A \cap \mathrm{cl}_X U$ is closed in (A, τ). This shows that $\varrho \subseteq \tau$. Note that since X is Hausdorff, so is $X(s)$; hence, (A, ϱ) is a Hausdorff space. As (A, τ) is minimal Hausdorff, it follows that $\varrho = \tau$. Finally, we show that (A, ϱ) is Urysohn. Let $x, y \in A$ such that $x \neq y$. Now, $k_X^{\leftarrow}(x)$ and $k_X^{\leftarrow}(y)$ are disjoint compact subsets of EX. There are disjoint regular open sets $U, V \in \tau(X)$ such that $k_X^{\leftarrow}(x) \subseteq OU$, $k_X^{\leftarrow}(x) \subseteq OV$, and $OU \cap OV = \emptyset$. By (c), $k_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is compact. So, there are disjoint regular open sets $W, T \in \tau(X)$ such that $k_X^{\leftarrow}[A \cap \mathrm{cl}_X U] \subseteq OW$, $k_X^{\leftarrow}(y) \subseteq OT$ and $OW \cap OT = \emptyset$. By 1.2 (b), $A \cap \mathrm{cl}_X U \subseteq \mathrm{int}_X(\mathrm{cl}_X W) = W$. Also, $y \in T$ and $W \cap T = \emptyset$. So, $W \cap \mathrm{cl}_X T = \emptyset$ implies $A \cap \mathrm{cl}_X(U) \cap \mathrm{cl}_X(T) = \emptyset$. But $x \in W \cap A \in \varrho$ and $y \in T \cap A \in \varrho$. Also, $\mathrm{cl}_{(A, \varrho)}(W \cap A) \subseteq \mathrm{cl}_{X(s)}(W \cap A) \subseteq \mathrm{cl}_{X(s)}(W) \cap \mathrm{cl}_{X(s)}(A) = \mathrm{cl}_X(W) \cap A$; likewise, $\mathrm{cl}_{(A, \varrho)}(T \cap A) \subseteq \mathrm{cl}_X(T) \cap A$. Thus, $\mathrm{cl}_{(A, \varrho)}(W \cap A) \cap \mathrm{cl}_{(A, \varrho)}(T \cap A) = \emptyset$. This shows that (A, ϱ) is Urysohn. By 7.5(b)(1) in [PW], an Urysohn, minimal Hausdorff space is compact. This completes the proof that A is an N-set of X. ■

Corollary 2.3. – *Let X be a space and $A \subseteq X$. The following are equivalent*:

(a) *A is an* N-*set of X.*

(b) *A is an* H-*set of X and for each* H-*set B of X with $B \subseteq A$, $\pi_X^{\leftarrow}[B]$ is an* H-*set of $P(X)$.*

(c) *For each $U \in \tau(X)$, $\pi_X^{\leftarrow}[A \cap \mathrm{cl}_X U]$ is an* H-*set of $P(X)$.*

Proof. – Follows from 1.2 (d) and 2.2. ■

3. – Near compact spaces.

Recall that a set A of a space X is *θ-closed* (resp. *δ-closed*) in X if for each $p \in X \setminus A$, there is $U \in \tau(X)$ such that $p \in U$ and $A \cap \mathrm{cl}_X U = \emptyset$ (resp. $A \cap \mathrm{int}_X(\mathrm{cl}_X U) = \emptyset$). Dikranjan and Giuli [DG] investigated those spaces X which are θ-closed in every space Y in which X can be embedded (Y is called a *superspace* of X) and proved that such spaces are compact. Here is a characterization of these spaces when θ-closed is replaced by δ-closed.

Proposition 3.1. – *A space X is* H-*closed if and only if X is δ-closed in every superspace Y of X.*

PROOF. – If X is δ-closed in every superspace Y of X, then X is H-closed as a δ-closed set is always closed. Conversely, suppose X is H-closed and Y is a superspace of X. Fix $p \in Y \backslash X$. For each $x \in X$, there is an open set $U_x \in \tau(X)$ such that $x \in U_x$ and $p \notin \mathrm{cl}_X U_x$. As X is an H-set in Y, there is a finite set $F \subseteq X$ such that $X \subseteq \cup \{\mathrm{cl}_X U_x : x \in F\}$. Now, $p \in V = \cap \{X \backslash \mathrm{cl}_X U_x : x \in F\}$ and $V \cap X = \emptyset$. Also, note that V is a regular open set. So, X is δ-closed in Y. ■

Proposition 3.1 provides another characterization of near compactness and solves Problem 2 in [CGNP].

COROLLARY 3.2. – *A space X is near compact if and only if X is Urysohn and δ-closed in every superspace of X.*

It would seem that if every regular open cover of a space X has a finite subcover (i.e., X is near compact), it would follow that X is θ-closed in every superspace Y of X. However, the result by Dikranjan and Giuli implies that each noncompact, near compact spaces is not θ-closed in some superspace. In 1.1, the near compact space A is not θ-closed in Y. On the other hand, N-sets behave very nicely in the space housing them as the next result indicates.

PROPOSITION 3.3. – *Let A be an* N-*set in a space X. Then A is θ-closed in X.*

PROOF. – Since A is a compact subspace of $X(s)$ and $X(s)$ is Hausdorff, it follows that if $p \in X \backslash A$, there is a regular open set U in X such that $p \in U$ and $A \cap \mathrm{cl}_{X(s)} U = \emptyset$. But $\mathrm{cl}_{X(s)} U = \mathrm{cl}_X U$. The proof is completed. ■

The result in 3.3 motivates this related question: if A is an N-set in a space X and X is a subspace of a space Z, then is A also θ-closed in Z? Using the example 1.1, we know it is not possible to show that A is an N-set in Z (A is an N-set in A but A is not θ-closed in Y). In fact we can extend the result in [DG] to this result.

PROPOSITION 3.4. – *Let A be an* N-*set in a space X. Then A is θ-closed in every superspace Y of X if and only if A is compact.*

PROOF. – The result is clear if A is compact. Conversely, suppose A is θ-closed in every superspace Y of X. Assume A is not compact. Then there is a closed filter $\mathcal{F}$ on A such that $\cap \mathcal{F} = \emptyset$. Let $Y = X \times [0, 1)$. Points $\{(x, r)\}$ are isolated whenever $r > 0$. For $(x, 0) \in Y$, a basic open neighborhood of $(x, 0)$ is of the form $U \times [0, a)$ where $x \in U \in \tau(X)$ and $0 < a \leq 1$. Now, Y is Hausdorff, X is homeomorphic to $X \times \{0\}$ and $X \times \{0\}$ is a closed, nowhere dense closed subset of Y. Let $Z = Y \cup \{\infty\}$. A set $V \subseteq Z$ is defined to be open if $V \cap Y \in \tau(Y)$ and $\infty \in V$ implies there is some $F \in \mathcal{F}$ and $0 < a \leq 1$ such that $F \times (0, a) \subseteq U$. Since $\mathrm{cl}_Y(F \times (0, a)) = F \times [0, a)$, it follows that Z is also Hausdorff. However, in Z, A is not θ-closed in Z. This is a contradiction. So, A is compact. ■

The first two authors used particularly closed filters to characterized near compact spaces in [CF]. Recall that a nonempty family $\mathcal{F}$ of regular closed sets of a space X is called *particularly closed* if $\mathcal{F}$ has finite intersection property (it may happen that the intersection of two elements of $\mathcal{F}$ is not a regular closed subset of X even though the intersection would be nonempty). A *particularly closed filter* (resp *particularly closed ultrafilter*) is the closed filter generated by a particularly closed family (resp. a maximal particularly closed family).

PROPOSITION 3.5 [CF]. – *Let X be a space.*

(a) *The space X is near compact if and only if every particularly closed filter has nonempty intersection.*

(b) *If every particularly closed ultrafilter on X converges, then X is near compact.*

We now show that the converse of 3.5 (b) is false.

EXAMPLE 3.6. – A particularly closed ultrafilter $\mathcal{U}$ on a near compact space X which does not converge.

The space X is the underlying set of $\beta\omega$ with a finer topology where ω is the discrete set of nonnegative integers. First partition ω into infinite sets $\{A_i\colon i\in\omega\}$ and let $p_i \in \mathrm{cl}_{\beta\omega}(A_i)\setminus\omega$ for each $i\in\omega$. Now, $S=\{p_i\colon i\in\omega\}\subseteq\beta\omega\setminus\omega$ and $C=\mathrm{cl}_{\beta\omega}(S)\subseteq\beta\omega\setminus\omega$. For $q\in\mathrm{cl}_{\beta\omega}(S)\setminus S$, if $q\in U\in\tau(\beta\omega)$, then $U\cap S$ is an infinite set. Now, $D=\beta\omega\setminus S$ is dense in $\beta\omega$ and X has the topology generated by $\tau(\beta\omega)\cup D$. Since D is dense in X, $X(s)=\beta\omega$ and, hence, X is near compact. Now, $\mathcal{F}=\{V\in\tau(\beta\omega)\colon q\in\mathrm{cl}_{\beta\omega}([V])\}$ is a particularly closed family. Let $\mathcal{U}=\{A\subseteq X\colon A$ is closed and $A\supseteq F$ for some $F\in\mathcal{F}\}$. Clearly, $q\in\cap\,\mathcal{U}$ and since $\beta\omega$ is Hausdorff, $\cap\,\mathcal{U}=\{q\}$.

First, we show that $\mathcal{U}$ is a particularly closed ultrafilter on X, i.e., that $\mathcal{F}$ is a maximal particularly closed family on X. Let $\emptyset\neq W\in\tau(X)$. Note that $\mathrm{cl}_X W=\mathrm{cl}_X T$ where $T=\mathrm{int}_X(\mathrm{cl}_X W)\in\tau(\beta\omega)$ and that $\mathrm{cl}_X W=\mathrm{cl}_{\beta\omega} T$ (see 2.2(f)) in [PW]) is clopen in $\beta\omega$. If $q\notin\mathrm{cl}_X W$, then $q\in\beta\omega\setminus\mathrm{cl}_{\beta\omega}T$ which is clopen in $\beta\omega$; so, $\beta\omega\setminus\mathrm{cl}_{\beta\omega}T\in\mathcal{F}$ and $\mathrm{cl}_{\beta\omega}T\notin\mathcal{F}$. If $q\in\mathrm{cl}_X W=\mathrm{cl}_{\beta\omega}T$, then $\mathrm{cl}_X W\in\mathcal{F}$. This shows that $\mathcal{F}$ is a maximal particularly closed family.

Since the members of $\mathcal{F}$ are clopen in $\beta\omega$, it follows that $\mathcal{F}$ is a filter base. So, to show $\mathcal{U}$ does not converge to the point q, it suffices to show that $\mathcal{F}$ does not converge to q. Now, $q\in D\in\tau(X)$. Suppose $V\in\tau(\beta\omega)$ and $q\in\mathrm{cl}_{\beta\omega}V$. Then $S\cap\mathrm{cl}_{\beta\omega}V$ is an infinite set. But $D=\beta\omega\setminus S$. So, $\mathrm{cl}_{\beta\omega}V\nsubseteq D$ for each $\mathrm{cl}_{\beta\omega}V\in\mathcal{F}$.

This shows that no member of $\mathcal{F}$ is contained in D. Hence $\mathcal{U}$ does not converges to q.

4. – R-compact and R-near compact spaces.

In this section, no separation axiom on spaces are assumed. In [CN], a concept related to near compactness is introduced. An open cover $\mathcal{C} = \{U_\alpha : \alpha \in J\}$ of a space X is called a *R-cover* [CN] if there is an open cover $\{V_\alpha : \alpha \in J\}$ of X such that $\mathrm{cl}_X(V_\alpha) \subseteq U_\alpha$ for $\alpha \in J$. A space X is R-*compact* (resp. R-*near compact*) if each R-cover $\mathcal{C}$ of X has a finite subcover (resp. has a finite subfamily $\mathcal{F} \subseteq \mathcal{C}$ such that $X = \cup\{\mathrm{int}_X(\mathrm{cl}_X U): U \in \mathcal{F}\}$). In [DG], Urysohn, R-compact spaces are characterized as Urysohn spaces which are θ-closed in every Urysohn superspace (called Urysohn-θ-closed in [DG]). Clearly, a near compact space is R-near compact and a R-near compact space is weakly compact (every R-cover has a finite subfamily whose closures cover). A space is *almost compact* (sometimes called *quasi-H-closed*) if every open cover has a finite subfamily whose closures cover. The relationships of these concepts are best viewed in this diagram:

$$\begin{array}{ccccc}
\text{Compact} & \longrightarrow & \text{Near compact} & \longrightarrow & \text{Almost compact} \\
\downarrow & & \downarrow & & \downarrow \\
\text{R-compact} & \longrightarrow & \text{R-near compact} & \longrightarrow & \text{Weakly compact}
\end{array}$$

The properties of the top row are usually studied in the setting of Hausdorff spaces and those in the bottom row in the Urysohn setting. Little is know about R-near compact spaces. In addition to the two problems about R-near compact space asked in [CGNP], here is another problem.

PROBLEM. – Is there a Urysohn, R-near compact space which is not R-compact?

The example of a Urysohn-closed space (i.e., Urysohn and weakly-compact) presented in [DG] which is not Urysohn-θ-closed is also not R-near compact.

The diagram suggests that Urysohn, R-near compact spaces might be precisely those Urysohn spaces which are δ-closed in every Urysohn superspace. This is not the case as noted in the following result which is the Urysohn analog of 3.1.

PROPOSITION 4.1. – *An Urysohn space X is Urysohn-closed if and only if X is δ-closed in every Urysohn superspaces Y containing X.*

PROOF. – Clearly, if X is δ-closed in every Urysohn superspace, then X is Urysohn-closed. Conversely, suppose X is Urysohn-closed. Let Y be a Urysohn superspace containing X. Fix $p \in Y \setminus X$. For each $x \in X$, there are open sets U_x, $V_x \in \tau(Y)$ such that $x \in U_x \subseteq \mathrm{cl}_Y U_x \subseteq Y \setminus \mathrm{cl}_Y V_x \subseteq Y \setminus V_x \subseteq Y \setminus \{y\}$. Since X is Urysohn-closed, there is a finite set $F \subseteq X$ such that

$$X = \cup \{\mathrm{cl}_X(X \cap Y \setminus \mathrm{cl}_Y V_x) \colon x \in F\} \subseteq$$
$$\cup \{\mathrm{cl}_Y(Y \setminus \mathrm{cl}_Y V_x) \colon x \in F\} \subseteq \cup \{Y \setminus \mathrm{int}_Y(\mathrm{cl}_Y V_x) \colon x \in F\}.$$

Now, $p \in V = \cap \{\mathrm{int}_Y(\mathrm{cl}_Y V_x) \colon x \in F\}$ is a regular open set in Y and $V \cap X = \emptyset$. So, X is δ-closed in Y. ■

REFERENCES

[CF] F. CAMMAROTO - G. LO FARO, *Proprietà dei filtri particolarmente chiusi e nuove caratterizzazioni degli spazi nearly-compact*, Boll. U.M.I. (5), **15**-B (1978), 638-648.

[CGNP] F. CAMMAROTO - J. GUTIERREZ, G. NORDO - M. A. DE PRADA, *Introduccion a los epaciós H-cerrados – Principales contribuciones a las formas debiles de compacidad – Problemas abiertos* (Submitted).

[CN] F. CAMMAROTO - T. NOIRI, *On* R-*compact spaces*, Math. Vesnik, **41** (1989), 141-147.

[DG] D. DIKRANJAN - E. GIULI, *S(n)-θ-closed spaces*, Top. and its Appl., **28** (1988), 59-74.

[K] M. KATĔTOV, *Über* H-*abgeschlossene und bikompacte Raüme*, Casopis Pest. Mat., **69** (1940), 36-49.

[PW] J. PORTER - R. G. WOODS, *Extensions and Absolutes of Hausdorff Spaces*, Springer-Verlag, Berlin (1988).

[V] J. VERMEER, *Closed subspaces of* H-*closed spaces*, Pac. J. Math., **118** (1985), 229-247.

F. Cammaroto e G. Lo Faro: Dipartimento di Matematica, Università di Messina, Contrada Papardo, Salita Sperone 31, 98166 Sant'Agata, Messina (Italy)

J. R. Porter: Department of Mathematics, University of Kansas, Lawrence, KS 66045 (U.S.A.).

Pervenuta in Redazione il 15 gennaio 1998.