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About mathematical models of phase transitions.

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About Mathematical Models of Phase Transitions.

Phase transitions occur in many relevant physical and engineering processes. The mathematical research on these phenomena dates back to the nineteenth century and was started by G. Lamé and B. P. Clapeyron in a paper (1831) unfortunately later forgotten, in which the basic problem was clearly formulated and an explicit solution was written: the classical model is named after the physicist Josef Stefan, who in 1889 proposed a model for the melting of the polar ices. The Stefan problem is a remarkable example of a *free boundary problem,* since the evolution of the surface which separates the phases is a priori unknown.

The basic Stefan model represents phase transitions in a rather simplified way, and a number of physically justified generalizations have been considered. These include undercooling and superheating effects, kinetic laws, phase relaxation, the Cahn-Hilliard equation for phase separation, phase-field models, surface tension effects, mean curvature flow, phase nucleation, and so on.

Many evolution phenomena with one or more free boundaries can be represented by similar mathematical models. A large community of researchers works on these and related topics; several meetings are organized every year, with the partecipation of mathematicians, physicists, material scientists, engineers, and so on. In Italy the research on the Stefan problem began with G. Sestini in the 1950s, and was then continued by a number of mathematicians in several directions. The scientific production on this topic has grown exponentially (a huge bibliography has been prepared by D. A. Tarzia and will be soon available), and nowadays it seems extremely hard and far beyond our task to provide a complete picture of the past and ongoing research.

Instead we realized that we were given a splendid opportunity to review some rather specific subjects in this field, all concerning different generalizations of the basic Stefan problem, and their numerical aspects.

The article of Visintin reviews the standard strong and weak formulations of the classical Stefan problem in several space dimensions and then deals with a fine length-scale model of *undercooling, nucleation* and *surface tension*, providing also the natural framework for the other three papers. The contribution of Fasano concerns undercooling and the onset of singularities in the evolution of the phase interface, especially in one-dimensional systems. Magenes considers phase transitions in systems in which the thermal capacity is large in small (possibly lower-dimensional) regions, the so-called *concentrated capacities.* Finally Verdi surveys the methods for the numerical solution of the classical two-phase Stefan problem in several space dimensions, including local refinement techniques and *a posteriori* error analysis, and of various generalizations such as phase relaxation, surface tension effects and mean curvature flow.

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