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On the Riesz representation theorem.

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Zanichelli

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On the Riesz representation theorem

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Summary. - *A variational technique is used to obtain a short proof of a famous result of F. Riesz.*

On the Riesz representation theorem.

In [1], HOFFMAN and MCANDREW indicate how variational techniques can be used to obtain constructive proofs of some of the fundamental results in the theory of linear inequalities. In this note we wish to show how a similar technique may be used to obtain a short proof of a famous result of F. RIESZ. This states that any continuous linear functional, $L(f)$, defined for functions $f \in L^2(0, 1)$ has the form

$$L(f) = \int_0^1 gf\,dx,$$

where $g \in L^2(0, 1)$.

To establish this, consider the convex functional

$$F(f) = 2L(f) - \int_0^1 f^2 dx.$$

Let g be the function in $L^2(0,1)$ which maximizes $F(f)$. Then, if u is an arbitrary function in $L^2(0,1)$, the scalar function of t,

$$F(g + tu) = 2L(g + tu) - \int_0^1 (g + tu)^2 dx$$

has its maximum at $t = 0$. Setting the derivative with respect to t equal to zero, we have

$$L(u) - \int_0^1 gu dx = 0$$

for all u, the required representation.

RÈFERENCE

[1] HOFFMAN. A. J., and M. H. MCANDREW, *Linear Inequalities and Analysis*, « American Mathematical Monthly », Vol. 71, 1964, pp. 416-418.

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