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Bollettino dell'Unione Matematica Italiana, Serie 3, Vol. **15** (1960), n.1, p. 38–39.
Zanichelli
<http://www.bdim.eu/item?id=BUMI_1960_3_15_1_38_0>

On the separation of exponentials.

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Summary. - *A method is given for finding the number of exponential components without first determining the amplitudes of frequencies of the components.*

1. Introduction.

In a number of different fields, it is known as a result of an a priori theoretical analysis that a sequence of numbers. $\{u_n\}$, obtained experimentally, has the representation

$$u_n = \sum_{k=1}^{N} c_k e^{\lambda_k n}. \tag{1}$$

If the dimension N is known, the parameters c_k and λ_k can be determined provided that enough of the u_n are given. If N is itself an unknown, as it is in many investigations where the determination of the number of components is one of the major objects, the problem is much more complex, and thus far more interesting.

The purpose of this note is to outline a method designed to yield the value of N, without a simultaneous determination of the c_k and λ_k. For other treatments of this problem, with some physical background, see [1], [2].

2. Recurrence Relations.

We start from the frequently used fact that a sequence such as that given above can be generated by means of the recurrence relation, or difference equation,

$$u_{t+N} = a_1 u_{t+N-1} + \dots + a_N u_t, \qquad t = 0, 1, \dots, \tag{1}$$

with $u_0, u_1, \dots, u_{N-1}$ prescribed. Consider the determinant

$$(2) \qquad C_k(t) = \begin{vmatrix} u_t & u_{t+1} & \cdots & u_{t+k} \\ u_{t+1} & u_{t+2} & \cdots & u_{t+k+1} \\ \vdots & & & \\ u_{t+k} & u_{t+k+1} & \cdots & u_{t+2k} \end{vmatrix}$$

related to the CASORATI determinant.

Assume as we may that in (1.1) $c_i \neq 0$, $i = 1, 2, \dots, N$, and that $\lambda_i \neq \lambda_j$ for $i \neq j$.

It is then easy to see that

$$(3) \qquad \begin{aligned} &\text{a)} \quad C_k(t) = 0, \ k \geq N, \ t = 0, 1, \dots . \\ &\text{b)} \quad C_{N-1}(t) = C_{N-1}(0) a_N^t \neq 0, \ t = 0, 1, \dots, \\ &\text{c)} \quad C_k(t) \neq 0, \ k < N-1, \text{ for large } t. \end{aligned}$$

It follows that the functions $C_k(t)$, $k = 1, 2, \dots$, can be used as a sequence of test functions to determine N, given the numerical values of u_n.

3. Discussion.

The most unsatisfactory of the criteria in (3) is (3c). In many cases, this can be considerably strengthened, making use of the fact that the λ_k are real and the c_k are positive. In this case, it follows from the known criteria for the positive definiteness of quadratic forms, or from the equivalent known properties of Gramian determinants, that $C_k(t) > 0$, $k < N-1$.

This problem was first proposed to the author PETER STEVENSON of LIVERMORE in connection with the study of fission processus.

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[2] C. LANCZOS, *Applied Analysis*, Prentice-Hall, 1956.